GRAVITY

Chapter 12
Units of Chapter 12

- Newton’s Law of Universal Gravitation
- Gravitational Attraction of Spherical Bodies
- Kepler’s Laws of Orbital Motion
- Gravitational Potential Energy
- Energy Conservation
- Tides
Newton’s insight:
The force accelerating an apple downward is the same force that keeps the Moon in its orbit.
Hence, Universal Gravitation.

**Newton's Law of Universal Gravitation**
The force of gravity between any two point objects of mass $m_1$ and $m_2$ is attractive and of magnitude

$$F = G \frac{m_1 m_2}{r^2}$$  \hspace{1cm} 12–1

In this expression, $r$ is the distance between the masses and $G$ is a constant referred to as the **universal gravitation constant**. Its value is

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$$  \hspace{1cm} 12–2
12-1 Newton’s Law of Universal Gravitation

The gravitational force is always attractive, and points along the line connecting the two masses:

The two forces shown are an action-reaction pair.
Newton’s Law of Universal Gravitation

• $G$ is a very small number; this means that the force of gravity is negligible unless there is a very large mass involved (such as the Earth).

• If an object is being acted upon by several different gravitational forces, the net force on it is the vector sum of the individual forces.

• This is called the principle of superposition.
Gravitational force between a point mass and a sphere: the force is the same as if all the mass of the sphere were concentrated at its center.
What about the gravitational force on objects at the surface of the Earth? The center of the Earth is one Earth radius away, so this is the distance we use:

\[ F = G \frac{mM_E}{R_E^2} = m \left( \frac{GM_E}{R_E^2} \right) \]

Therefore,

\[ m \left( \frac{GM_E}{R_E^2} \right) = mg \]
12-2 Gravitational Attraction of Spherical Bodies

- The acceleration of gravity decreases slowly with altitude:

![Graph showing the decrease in acceleration of gravity with height]

- Acceleration of gravity $\delta g_h (\text{m/s}^2)$
- Height, $h$ (mi)
Example

What is the weight of a 100 kg astronaut on the surface of the Earth (force of the Earth on the astronaut)? How about in low Earth orbit? This is an orbit about 300 km above the surface of the Earth.

On Earth: \[ w = mg = 980 \text{ N} \]

In low Earth orbit: \[ w = mg(r_o) = m \left( \frac{GM_E}{(R_E + h)^2} \right) = 890 \text{ N} \]

Their weight is reduced by about 10%. The astronaut is NOT weightless!
12-2 Gravitational Attraction of Spherical Bodies

- Once the altitude becomes comparable to the radius of the Earth, the decrease in the acceleration of gravity is much larger:
Johannes Kepler made detailed studies of the apparent motions of the planets over many years, and was able to formulate three empirical laws:

1. Planets follow **elliptical orbits**, with the Sun at one focus of the ellipse.
2. As a planet moves in its orbit, it sweeps out an equal amount of area in an equal amount of time.

(b) Equal areas in equal times for highly elliptical orbit
3. The period, $T$, of a planet increases as its mean distance from the Sun, $r$, raised to the 3/2 power.

$$T = (\text{constant}) r^{3/2}$$

This can be shown to be a consequence of the inverse square form of the gravitational force.
A geosynchronous satellite is one whose orbital period is equal to one day. If such a satellite is orbiting above the equator, it will be in a fixed position with respect to the ground.

These satellites are used for communications and weather forecasting.
GPS satellites are not in geosynchronous orbits; their orbit period is 12 hours. Triangulation of signals from several satellites allows precise location of objects on Earth.
Gravitational potential energy of an object of mass $m$ a distance $r$ from the Earth’s center:

$$U = -G \frac{mM_E}{r}$$
Very close to the Earth’s surface, the gravitational potential increases linearly with altitude:

\[ \Delta U = mgh \]

Gravitational potential energy, just like all other forms of energy, is a scalar. It therefore has no components; just a sign.
12-5 **Energy Conservation**

- Total mechanical energy of an object of mass $m$ a distance $r$ from the center of the Earth:

\[ E = K + U = \frac{1}{2}mv^2 - G\frac{mM_E}{r} \]

- This confirms what we already know – as an object approaches the Earth, it moves faster and faster.
Energy Conservation
12-5 Energy Conservation

• Escape speed: the initial upward speed a projectile must have in order to escape from the Earth’s gravity

\[ v_e = \sqrt{\frac{2GM_E}{R_E}} = 11,200 \text{ m/s} \approx 25,000 \text{ mi/h} \]
Additional interesting effects due to gravity

- **Black holes:**
  If an object is sufficiently massive and sufficiently small, the escape speed will equal or exceed the speed of light – light itself will not be able to escape the surface.

- **Gravitational lensing:**
  Light will be bent by any gravitational field; this can be seen when we view a distant galaxy beyond a closer galaxy cluster. This is called gravitational lensing, and many examples have been found.

- **Tides:**
  When two large objects exert gravitational forces on each other, the force on the near side is greater than the force on the far side, because the near side is closer. This difference in gravitational force across an object due to its size is called a tidal force.
Summary of Chapter 12

• Force of gravity between two point masses:

\[ F = G \frac{m_1 m_2}{r^2} \]

• \( G \) is the universal gravitational constant:

\[ G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2 \]

• In calculating gravitational forces, spherically symmetric bodies can be replaced by point masses.
Summary of Chapter 12

- Acceleration of gravity:
  \[ g = \frac{GM_E}{R_E^2} \]

- Mass of the Earth:
  \[ M_E = \frac{gR_E^2}{G} \]

- Kepler’s laws:
  1. Planetary orbits are ellipses, Sun at one focus
  2. Planets sweep out equal area in equal time
  3. Square of orbital period is proportional to cube of distance from Sun
Summary of Chapter 12

- Orbital period:

\[ T = \left( \frac{2\pi}{\sqrt{GM_s}} \right) r^{3/2} = (\text{constant}) r^{3/2} \]

- Gravitational potential energy:

\[ U = -G \frac{m_1 m_2}{r} \]

- \( U \) is a scalar, and goes to zero as the masses become infinitely far apart
Summary of Chapter 12

- Total mechanical energy:

\[ E = K + U = \frac{1}{2}mv^2 - \frac{GmM_E}{r} \]

- Escape speed:

\[ v_e = \sqrt{\frac{2GM_E}{R_E}} \]

- Tidal forces are due to the variations in gravitational force across an extended body