ONE-DIMENSIONAL KINEMATICS

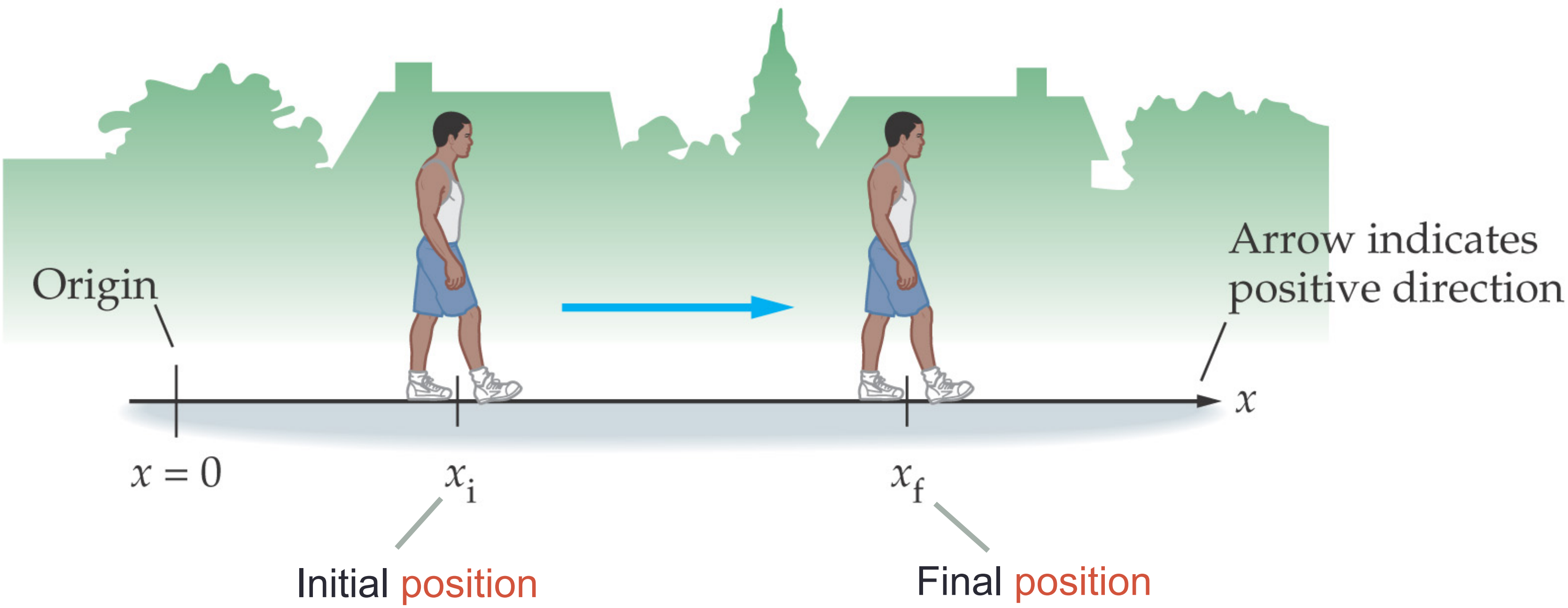
Chapter 2

Units of Chapter 2

- Position, Distance, and Displacement
- Average Speed and Velocity
- Instantaneous Velocity
- Acceleration
- Motion with Constant Acceleration
- Applications of the Equations of Motion
- Freely Falling Objects

2-1 Position, distance and displacement

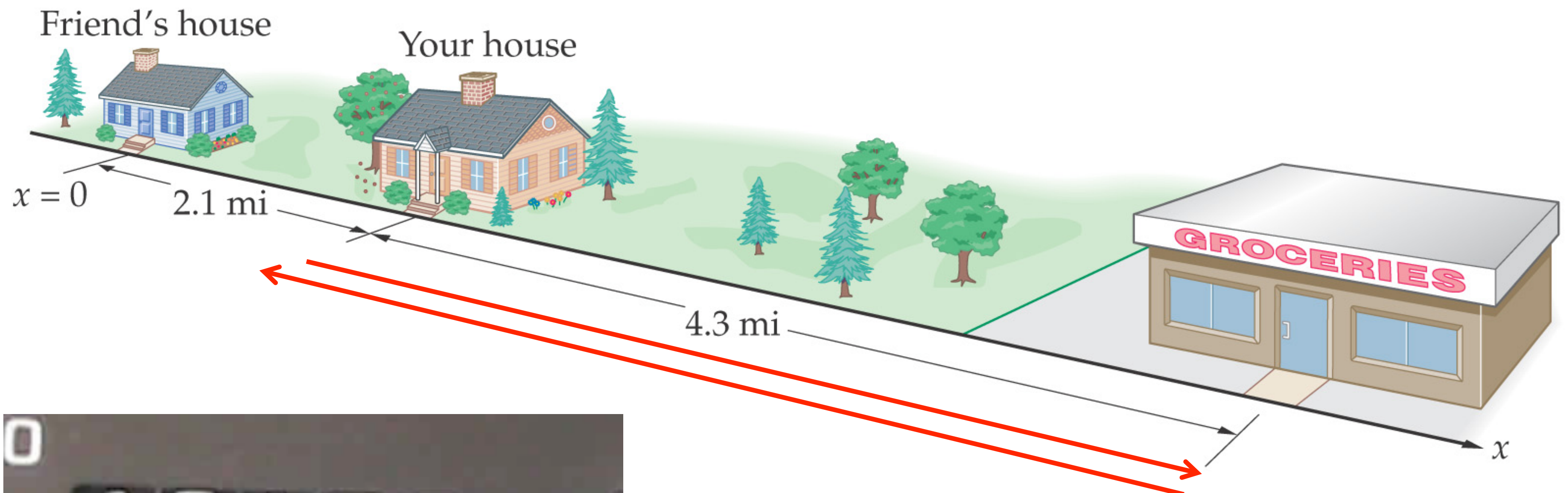
- Before describing motion, you must set up a **coordinate system** – define an origin and a positive direction.

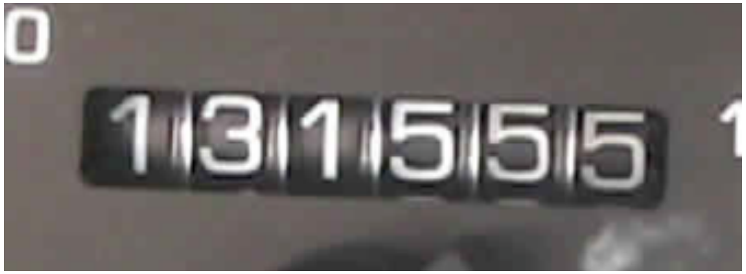


You can choose it as you like, but then you have to be consistent with it.

2-1 Position, distance and displacement

- The distance is the total length of travel; if you drive from your house to the grocery store and back, you have covered a distance of 8.6 mi.



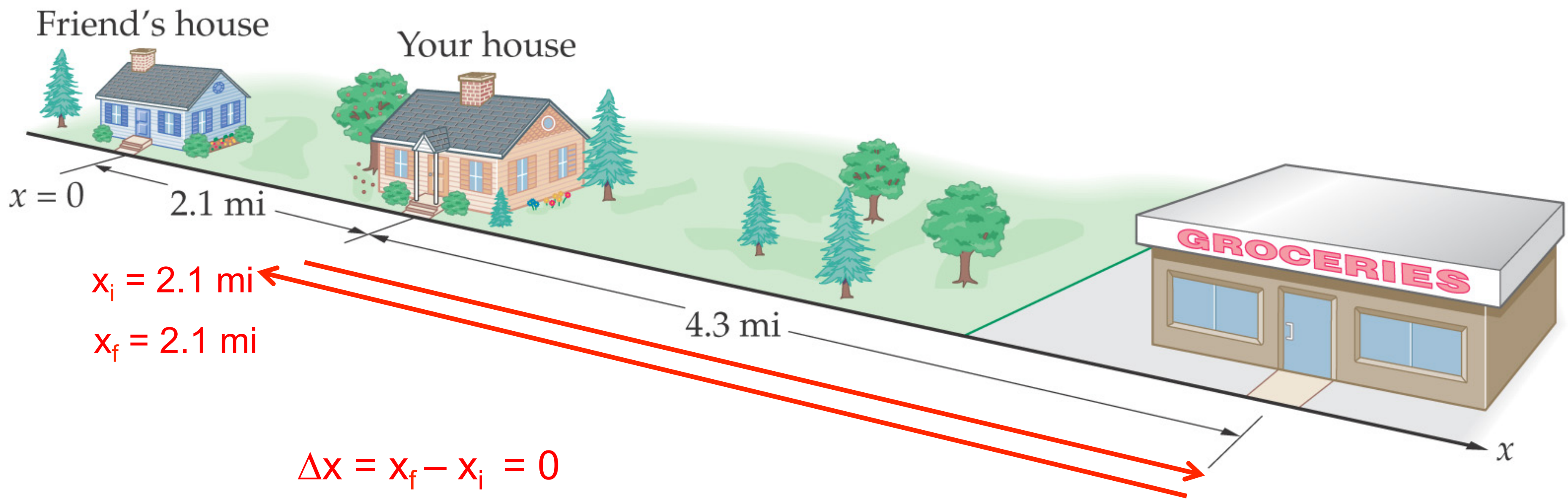


The odometer measures the total distance covered by the car during its lifetime

2-1 Position, distance and displacement

- The displacement is the change in position: final position – initial position

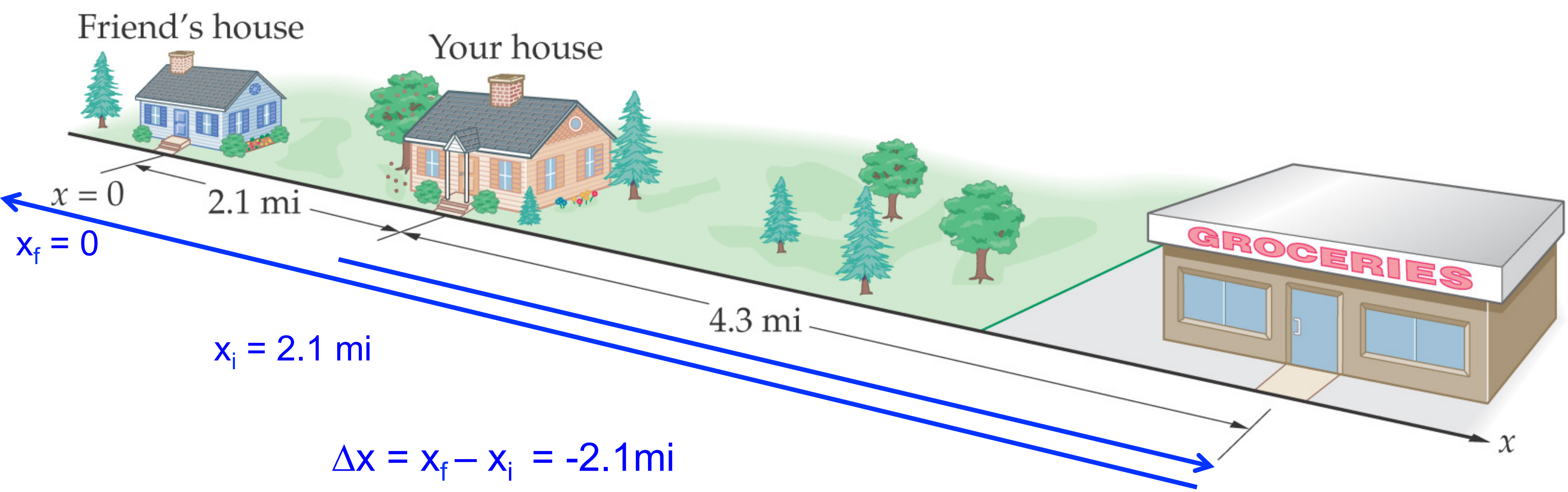
$$\Delta x = x_f - x_i$$



2-1 Position, distance and displacement

- The displacement is the change in position: final position – initial position

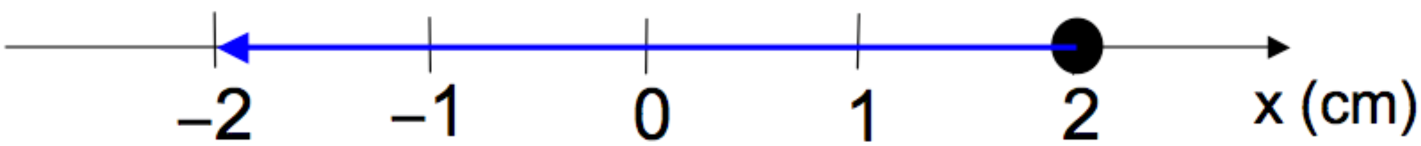
$$\Delta x = x_f - x_i$$



The displacement can be positive, negative or zero

Example

- A ball is initially at x = +2 cm and is moved to x =-2 cm. What is the displacement of the ball?



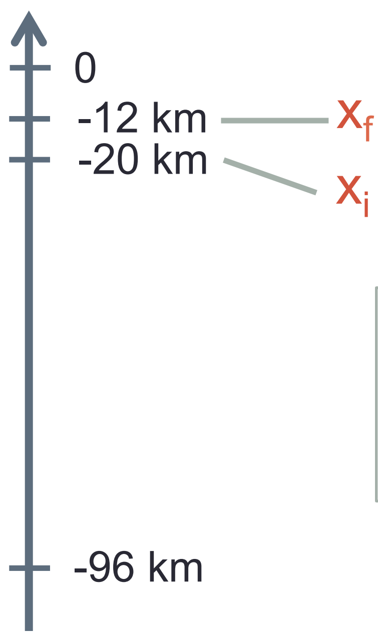
$$\Delta x = x_f - x_i = -2\text{cm} - 2\text{cm} = -4\text{cm}$$

Example

- At 3 PM a car is located 20 km south of its starting point. One hour later its is 96 km farther south. After two more hours it is 12 km south of the original starting point.

(a) What is the displacement of the car between 3 PM and 6 PM?

Use a coordinate system where north is positive:

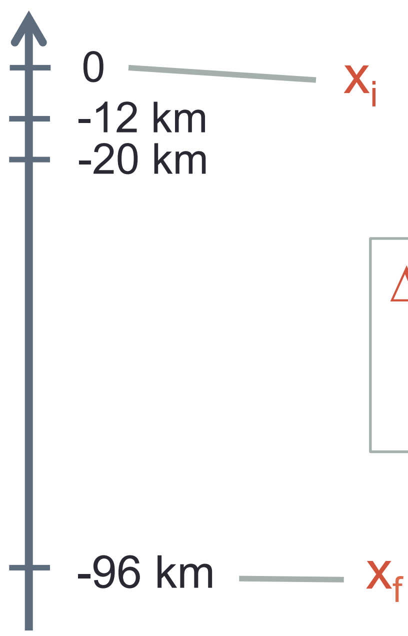


$$\Delta x = x_f - x_i$$
$$= -12\text{ km} - (-20\text{ km}) = +8\text{ km}$$

Example continued

(b) What is the displacement of the car from the starting point to the location at 4 pm?

Use a coordinate system where north is positive:



$$\Delta x = x_f - x_i$$
$$= -96\text{ km} - (0\text{ km}) = -96\text{ km}$$

Example continued

(c) What is the displacement of the car from 4 pm to 6 pm?

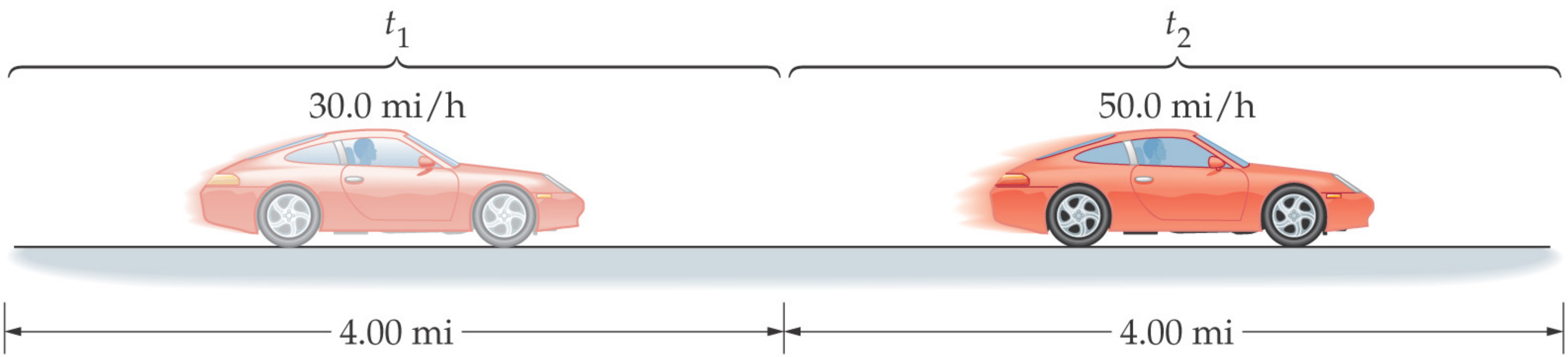
Use a coordinate system where north is positive:

$$\Delta x = x_f - x_i$$
$$= -12 \text{ km} - (-96 \text{ km}) = +84 \text{ km}$$

2-2 Average speed and velocity

- The average speed is defined as the distance traveled divided by the time the trip took:

Average speed = distance / elapsed time



Is the average speed of the car 40.0 mi/h, more than 40.0 mi/h, or less than 40.0 mi/h?

Answer: It takes more time to travel 4.00 mi at 30.0 mi/h than at 50.0 mi/h. You will be traveling at the lower speed for longer. The average speed will be less than 40.0 mi/h.

2-2 Average speed and velocity

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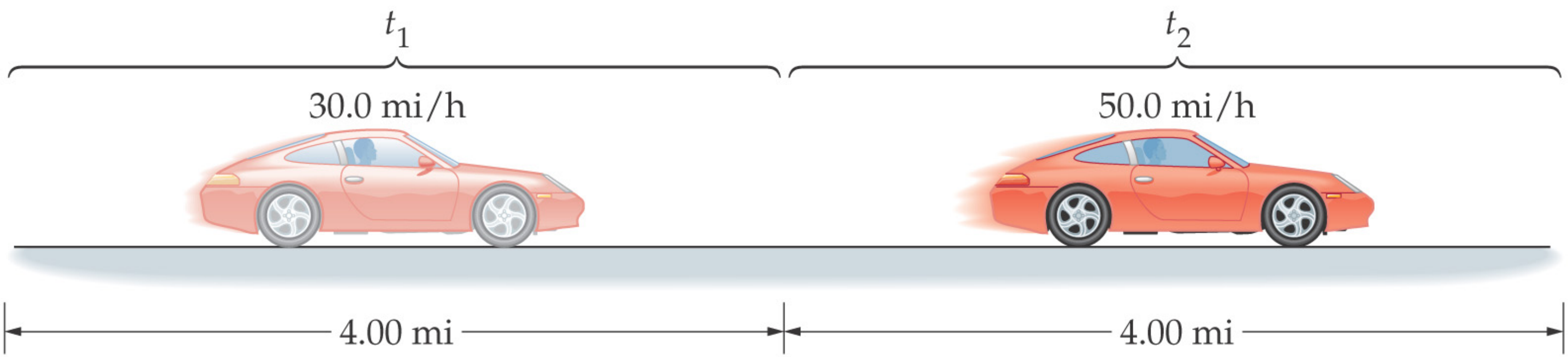

Is the average speed of the car 40.0 mi/h, more than 40.0 mi/h, or less than 40.0 mi/h?

Proof: let us calculate the average speed explicitly. The total distance is 8.00 mi.

t_1= (4.00mi)/(30.0 mi/h)=(4.00/30.0) h t_2= (4.00mi)/(50.0 mi/h)=(4.00/50.0) h

$t = t_1 + t_2$ =0.213 h average speed = (8.00 mi)/(0.213 h)=37.6 mi/h

2-2 Average speed and velocity

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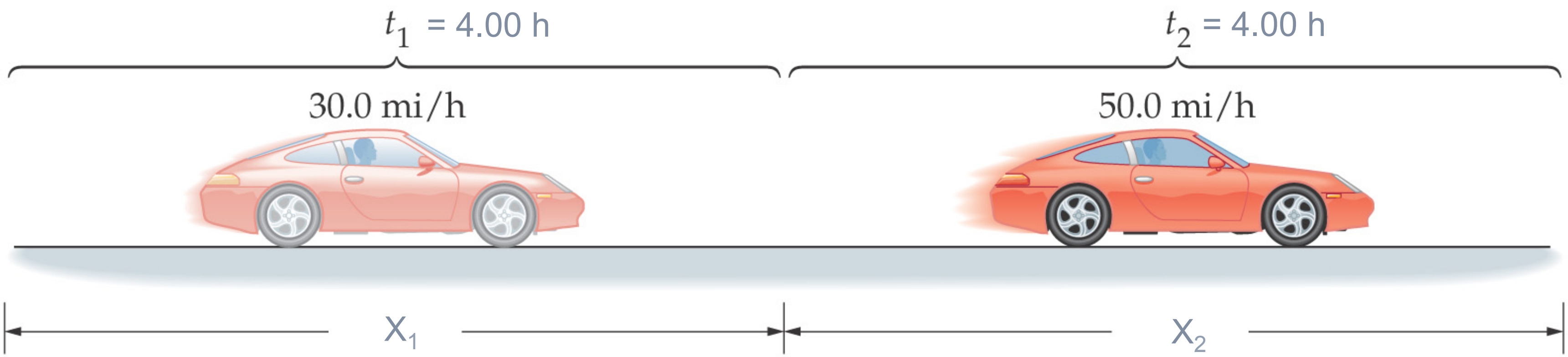


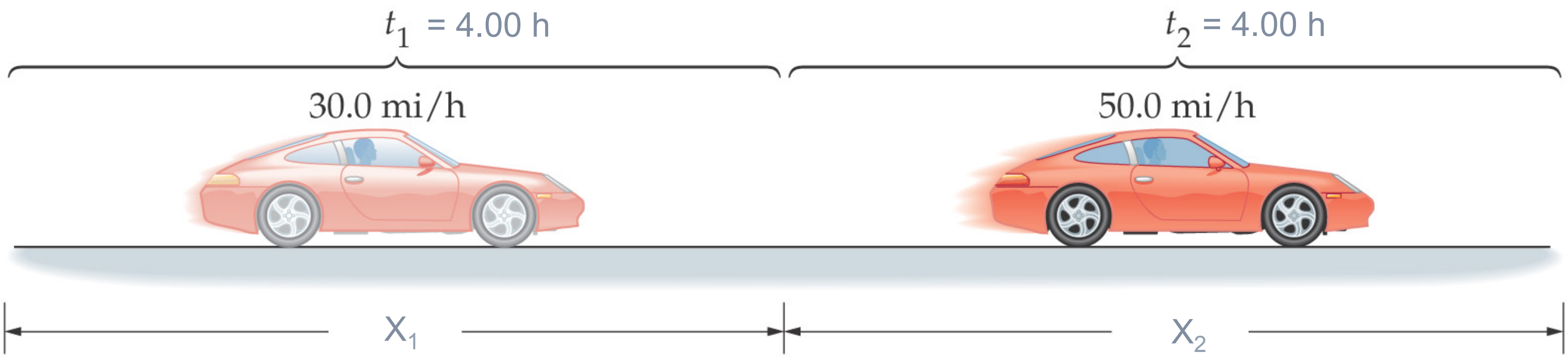
Is the average speed of the car 40.0 mi/h, more than 40.0 mi/h, or less than 40.0 mi/h?

Answer: The average speed is exactly 40.0 mi/h in this case: the car travels for equal amounts of time at 30.0 mi/h and at 50.0 mi/h

2-2 Average speed and velocity

- The average speed is defined as the distance traveled divided by the time the trip took:

Average speed = distance / elapsed time



Is the average speed of the car 40.0 mi/h, more than 40.0 mi/h, or less than 40.0 mi/h?

Proof: let us calculate the average speed explicitly. The elapsed time is 8.00 h.

x_1= (30.0 mi/h)x(4.00 h)=120 mi x_2= (50.0 mi/h)x(4.00 h)=200 mi

$x = x_1 + x_2$ =320 mi   average speed = (320 mi)/(8.00 h)=40.0 mi/h

Example

- A car is making a 12-mile trip. It travels the first 6.0 miles at 30 miles per hour and the last 6.0 miles at 60 miles per hour. What is the car's average speed for the entire trip?

t_1=6.0mi/(30 mi/h)=0.2 hours

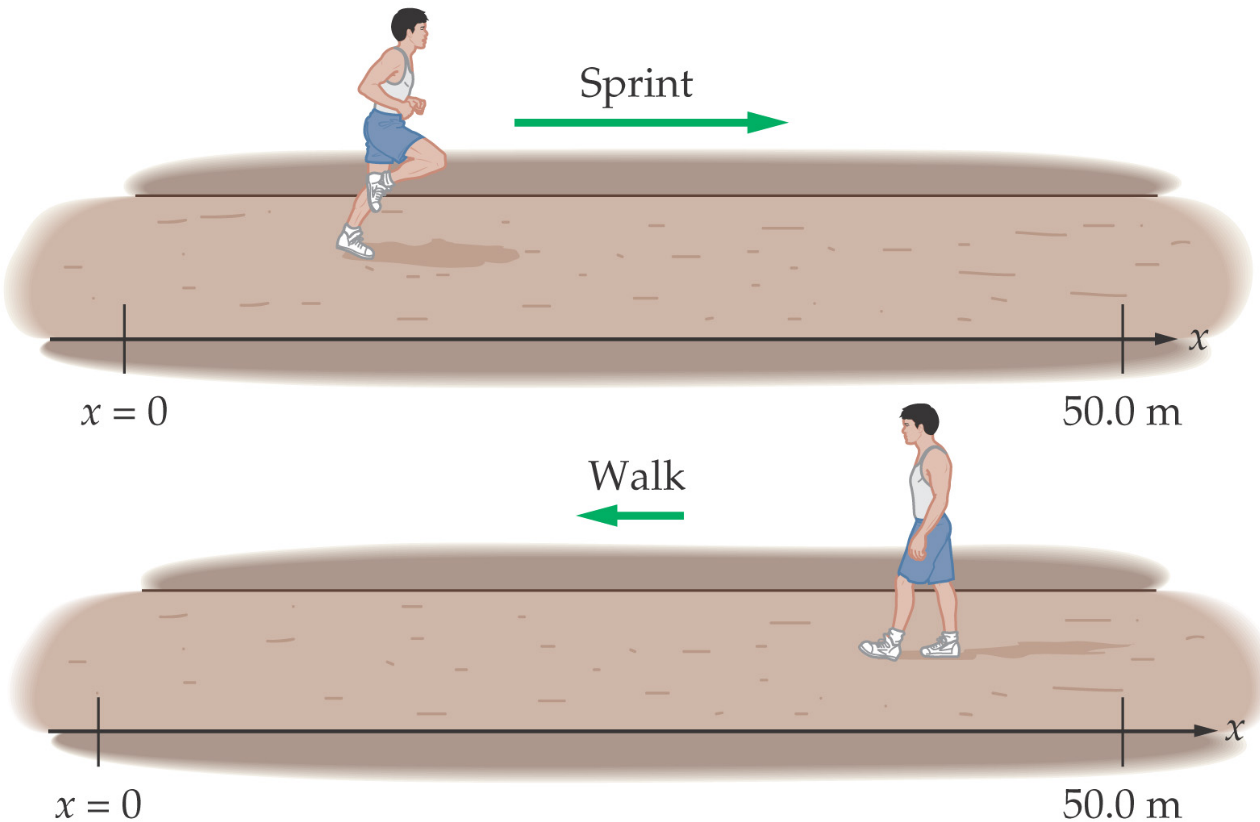
t_2=6.0mi/(60 mi/h)=0.1 hours.

t_{TOT} =t_1+t_2=0.3 hours.

v_{AV}=12mi/(0.3hours)=40 mi/h.

2-2 Average speed and velocity

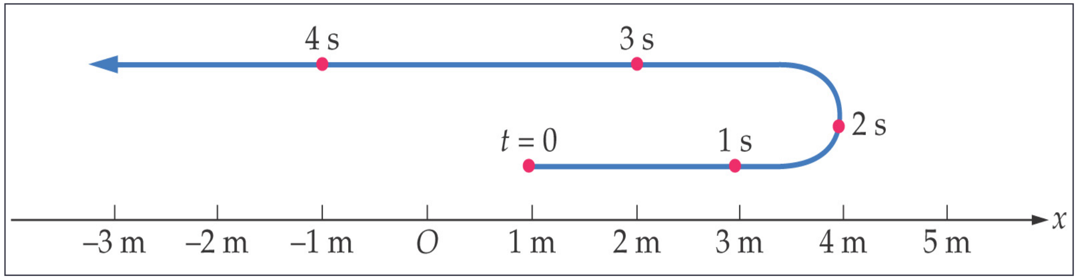
Average velocity = displacement / elapsed time

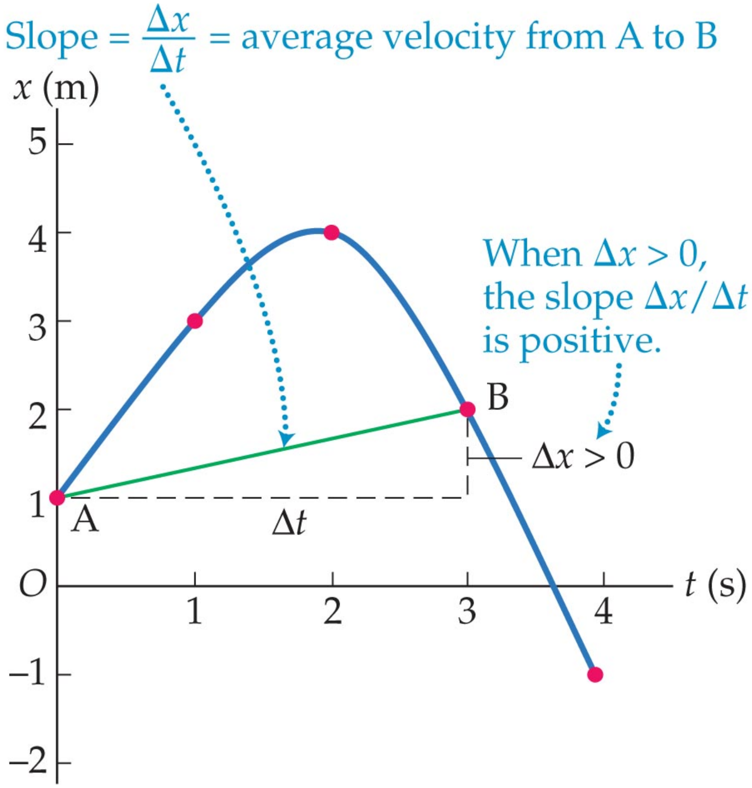


If you return to your starting point, your average velocity is zero.

2-2 Average speed and velocity

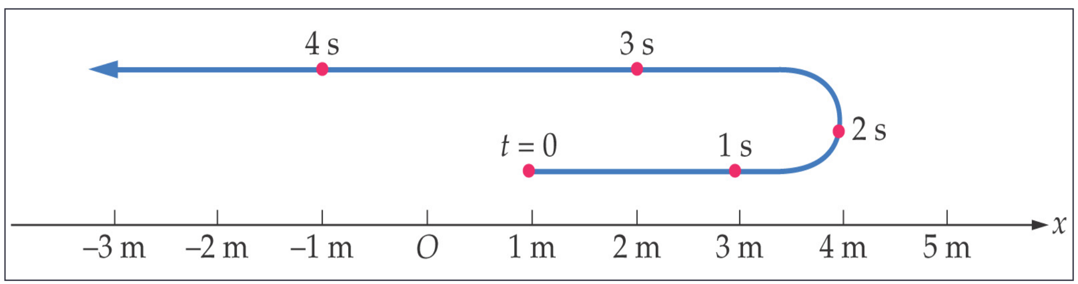
Graphical interpretation of **Average velocity**

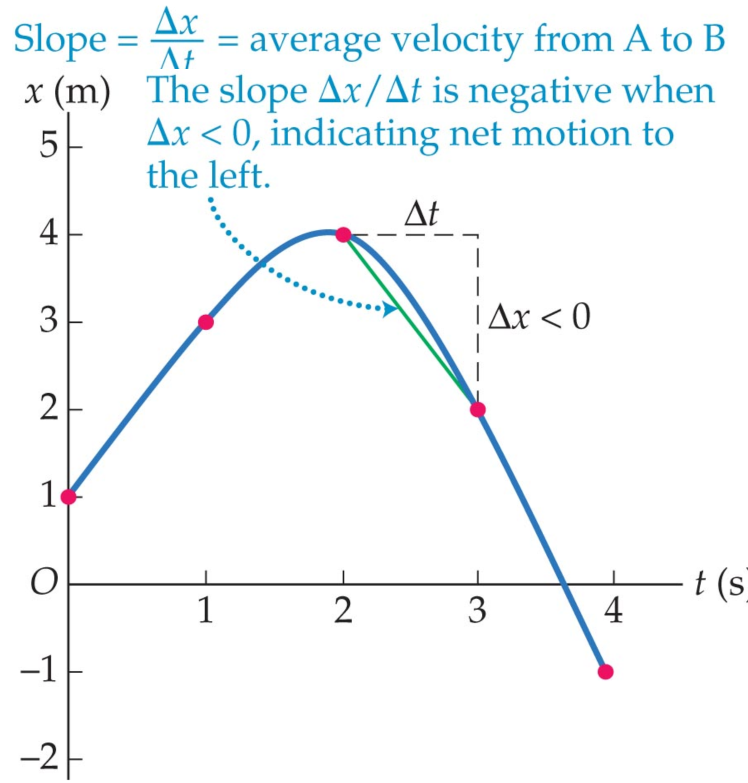




2-2 Average speed and velocity

Graphical interpretation of **Average velocity**





(b) Average velocity between $t = 2$ s and $t = 3$ s

2-3 Instantaneous velocity

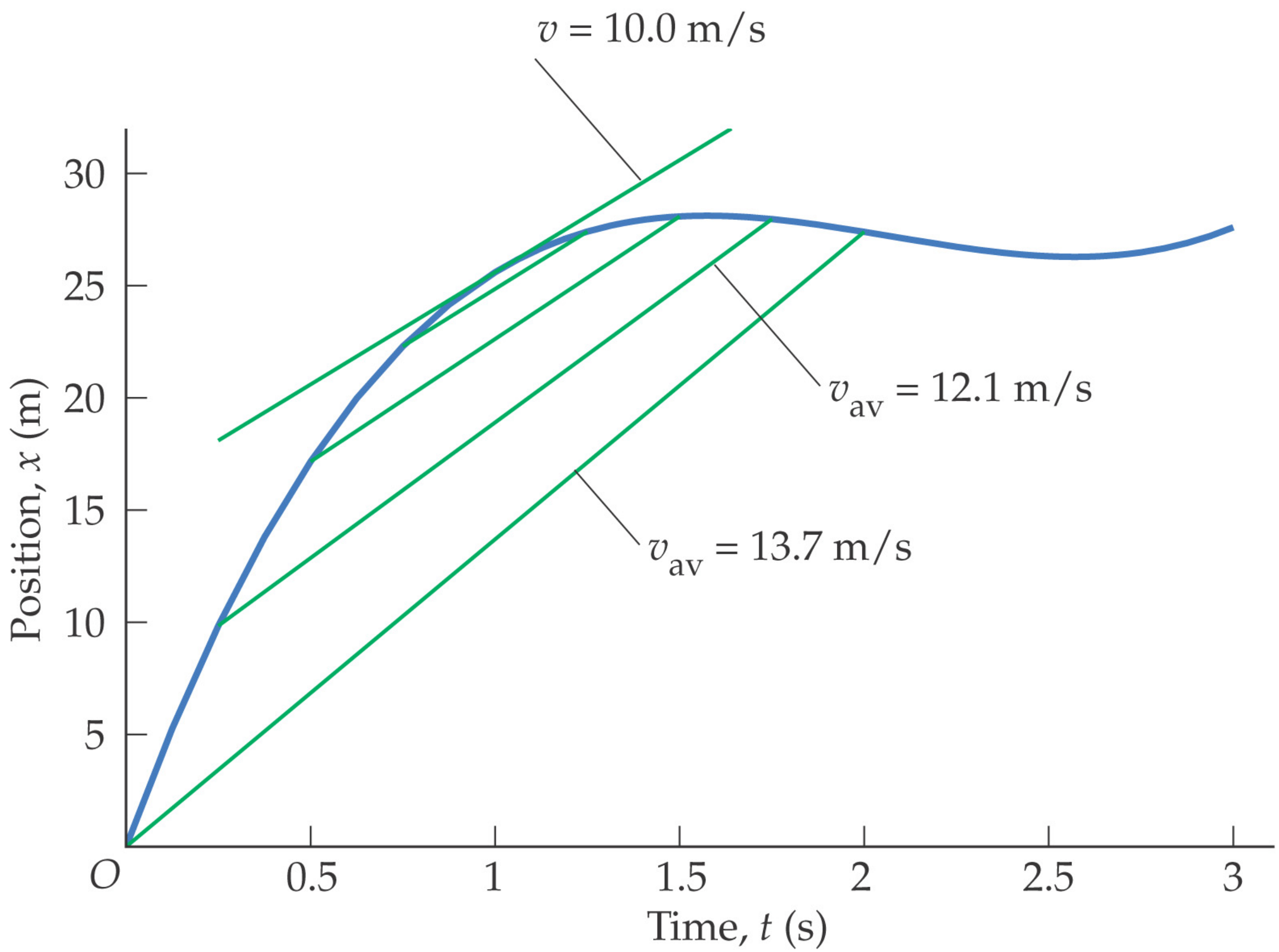
Definition:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

We evaluate the average velocity over a shorter and shorter period of time, approaching zero in the limit.

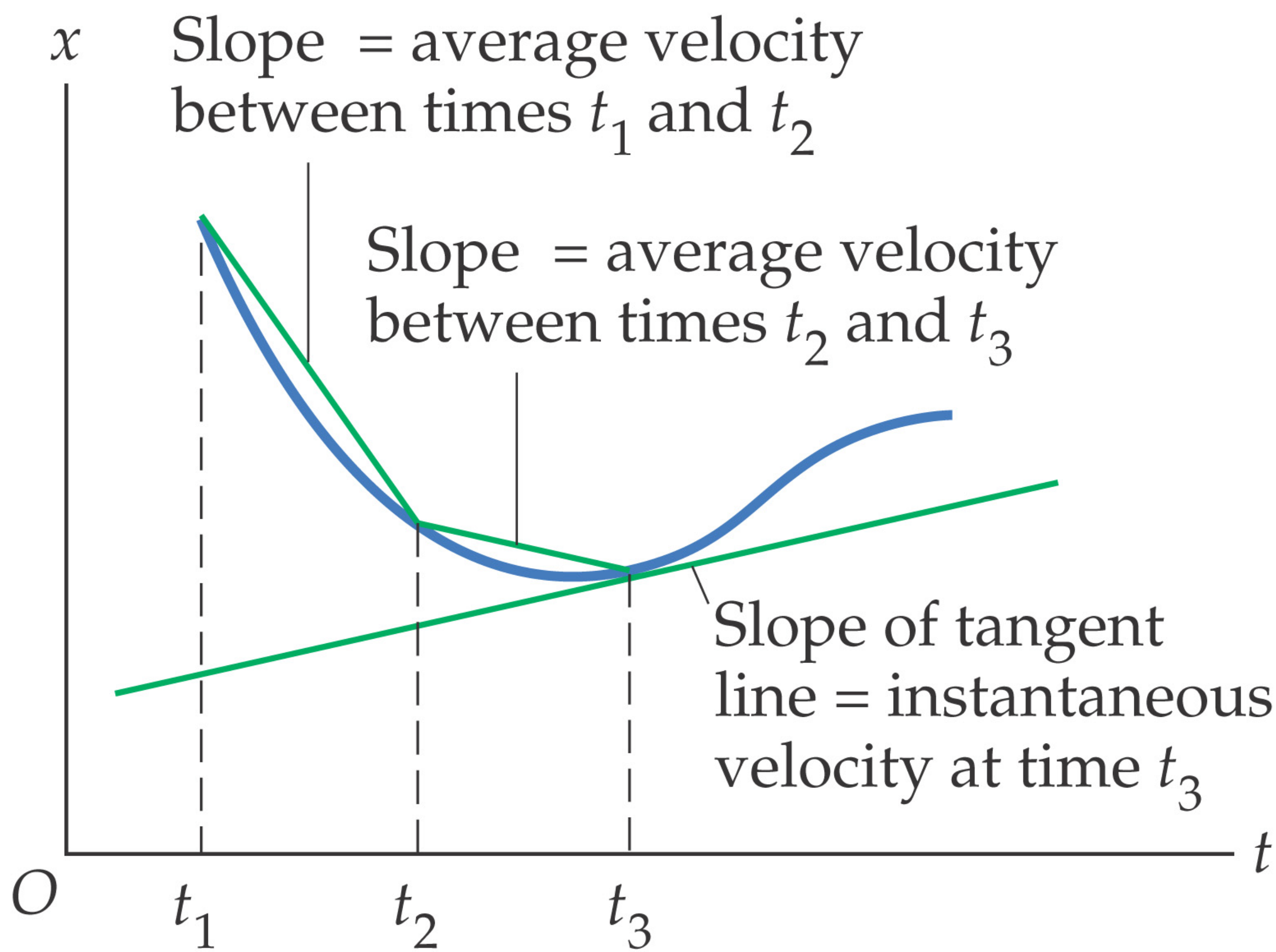
2-3 Instantaneous velocity

- This plot shows the average velocity being measured over shorter and shorter intervals. The instantaneous velocity is tangent to the curve.



2-3 Instantaneous velocity

Graphical interpretation of **Average** and **instantaneous velocity**



2-4 Acceleration

Average acceleration: rate of change of velocity with time

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

TABLE 2–3 Typical Accelerations (m/s^2)

Ultracentrifuge	3×10^6
Bullet fired from a rifle	4.4×10^5
Batted baseball	3×10^4
Click beetle righting itself	400
Acceleration required to deploy airbags	60
Bungee jump	30
High jump	15
Acceleration of gravity on Earth	9.81
Emergency stop in a car	8
Airplane during takeoff	5
An elevator	3
Acceleration of gravity on the Moon	1.62

2-4 Acceleration

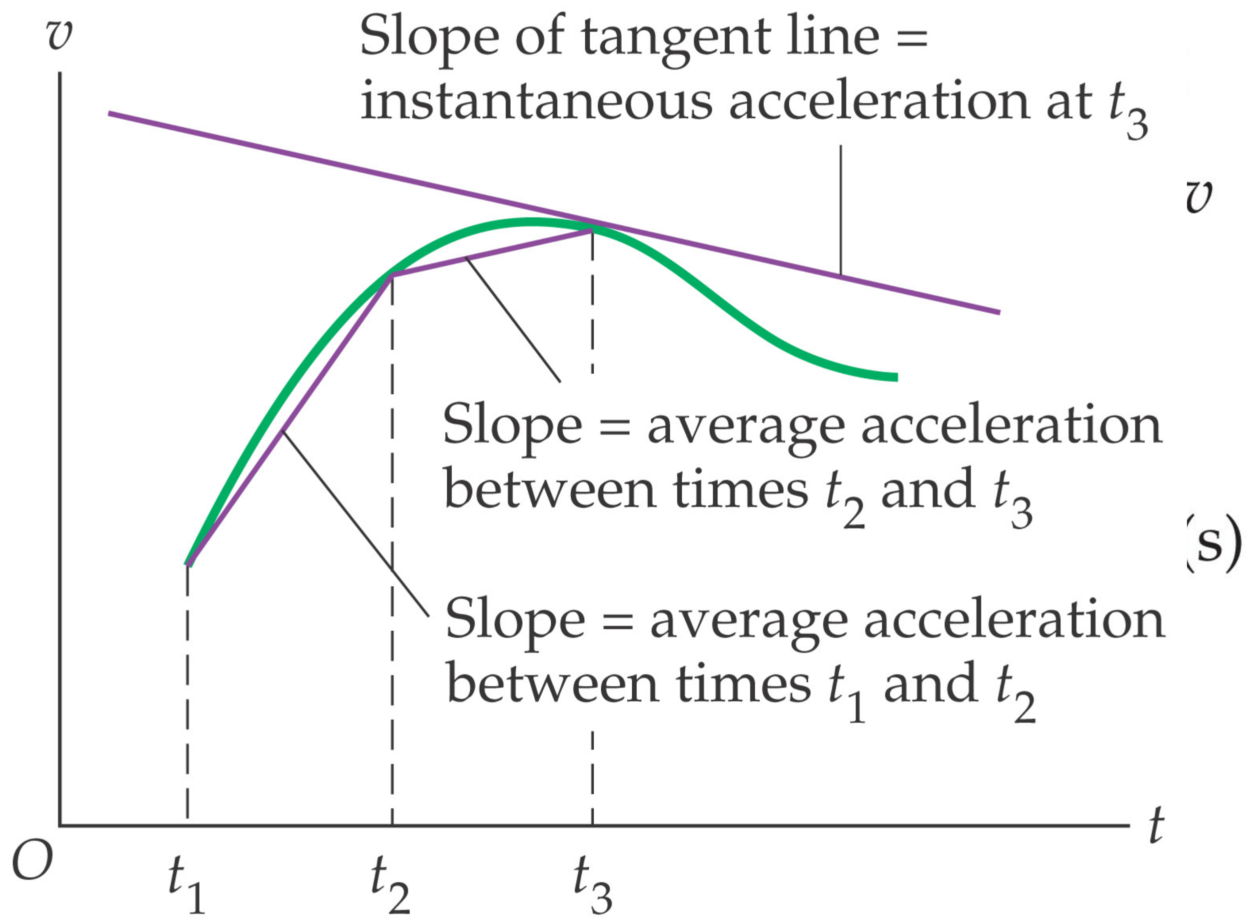
Instantaneous acceleration:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$$

- When acceleration is constant, the instantaneous and average accelerations are the same.

2-4 Acceleration

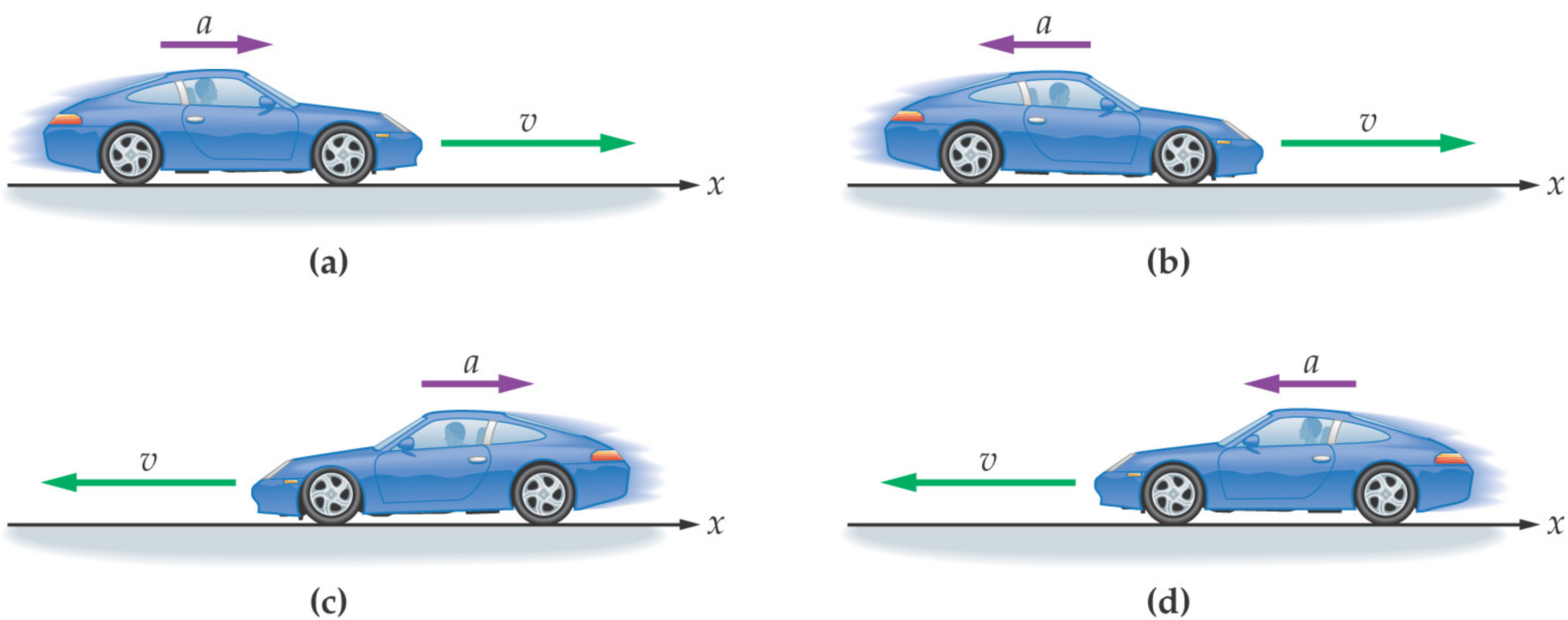
Graphical interpretation of **Acceleration**



Constant acceleration: v-versus-t plot is a straight line

2-4 Acceleration

Acceleration (increasing speed) and deceleration (decreasing speed) should not be confused with the directions of velocity and acceleration:



(a) and (d): the car's speed increases.
(b) and (c): the car's speed decreases.

2-4 Acceleration

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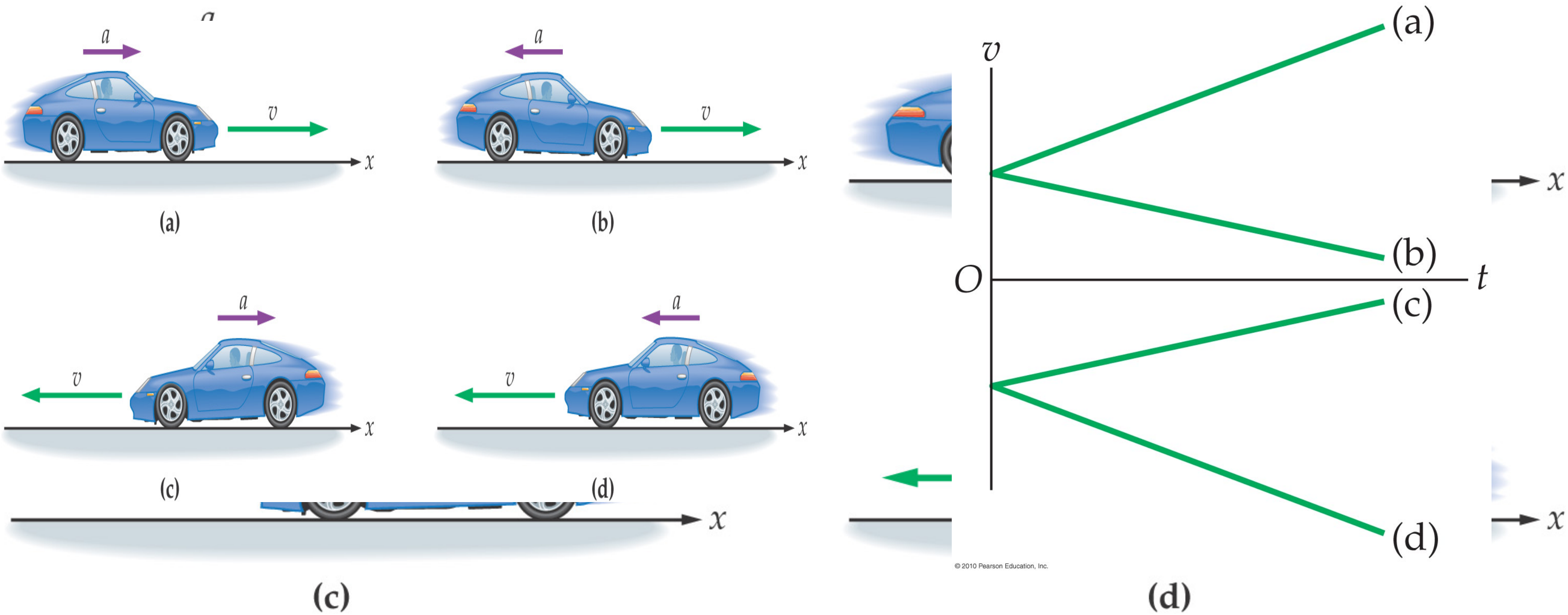


(a) and (d): the car’s speed increases.
(b) and (c): the car’s speed decreases.

Example

- The graph shows speedometer readings as a car comes to a stop. What is the magnitude of the acceleration at t = 7.0 s?

The slope of the graph at t = 7.0 s is:

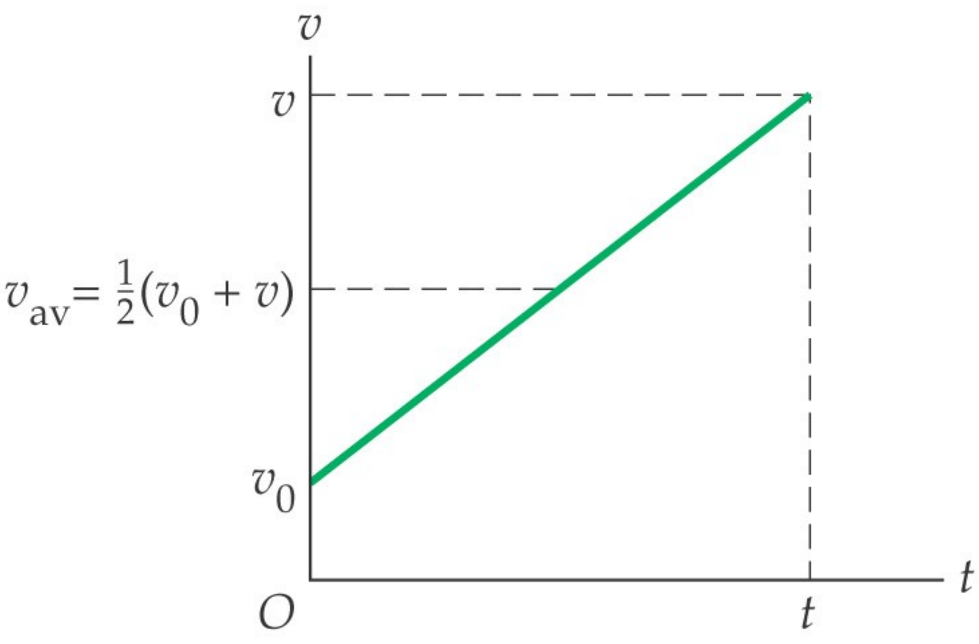
$$|a_{av}| = \left|\frac{\Delta v_x}{\Delta t}\right| = \left|\frac{v_2 - v_1}{t_2 - t_1}\right| = \left|\frac{(0-20)\text{m/s}}{(12-4)\text{s}}\right| = 2.5 \text{ m/s}^2$$

2-5 Motion with Constant Acceleration

- If the acceleration is constant, the velocity changes linearly:

$$v = v_0 + at$$

Average velocity



(a)

2-5 Motion with Constant Acceleration

- Average velocity:

$$v_{av} = \tfrac{1}{2}(v_0 + v)$$

- Position as a function of time:

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t$$

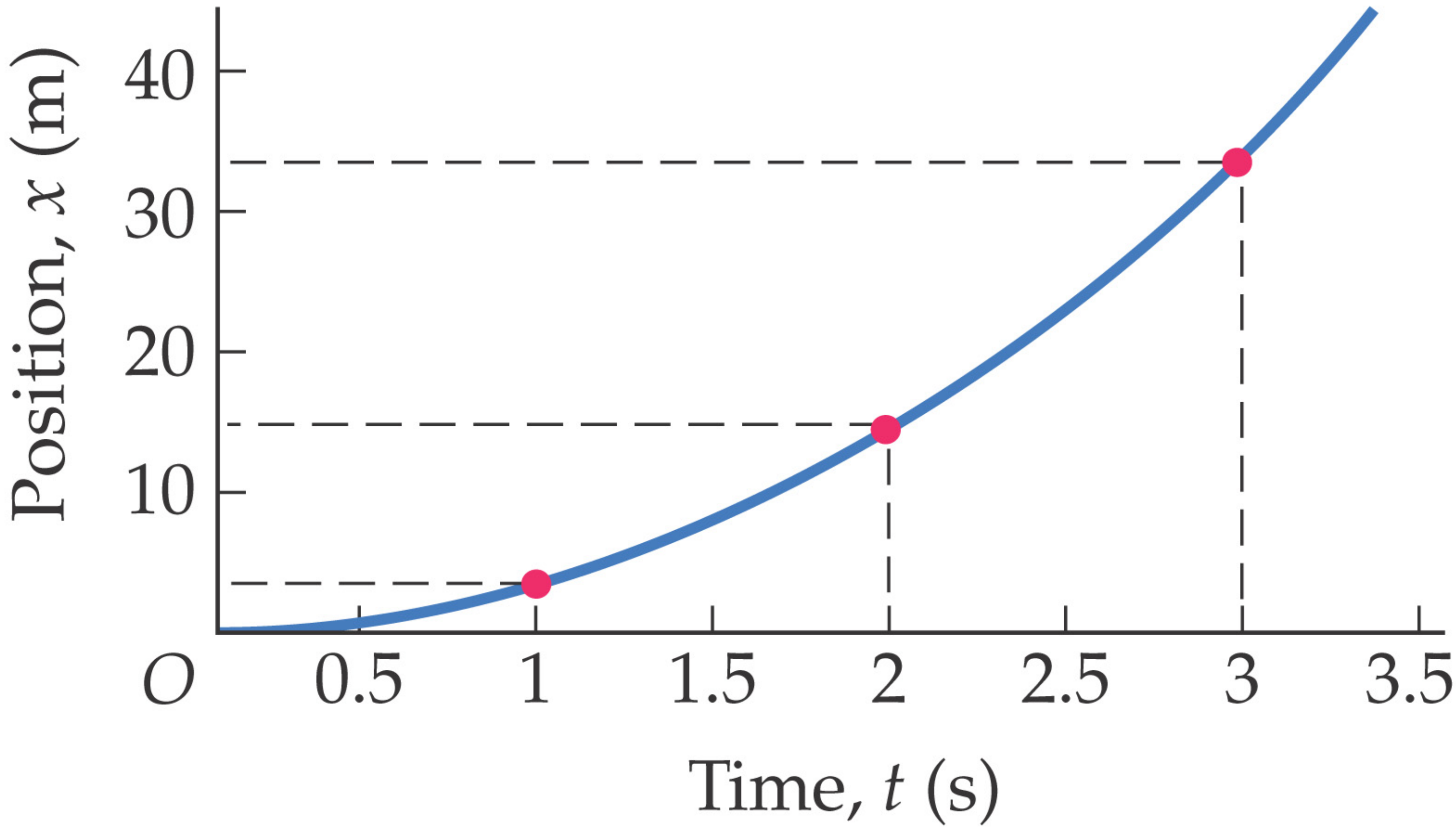
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

- Velocity as a function of position:

$$v^2 = v_0{}^2 + 2a(x - x_0) = v_0{}^2 + 2a\Delta x$$

2-5 Motion with Constant Acceleration

- The relationship between position and time follows a characteristic curve:



2-5 Motion with Constant Acceleration

TABLE 2–4 Constant-Acceleration Equations of Motion

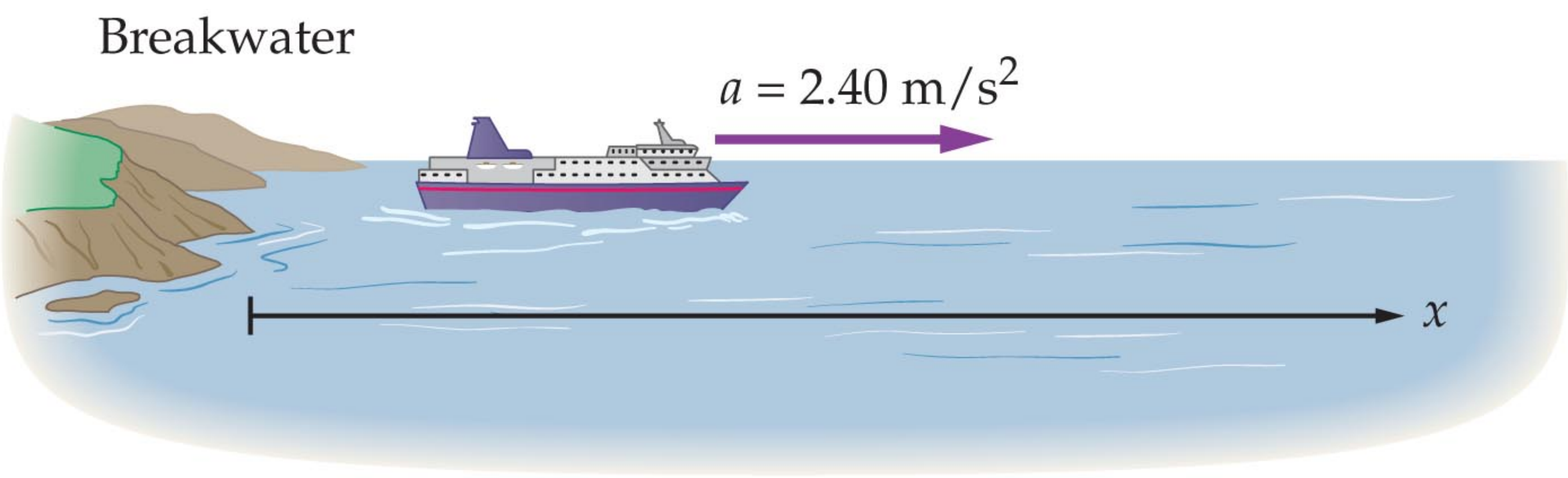
Variables related	Equation	Number
velocity, time, acceleration	$v = v_0 + at$	2–7
initial, final, and average velocity	$v_{av} = \frac{1}{2}(v_0 + v)$	2–9
position, time, velocity	$x = x_0 + \frac{1}{2}(v_0 + v)t$	2–10
position, time, acceleration	$x = x_0 + v_0 t + \frac{1}{2}at^2$	2–11
velocity, position, acceleration	$v^2 = v_0^2 + 2a(x - x_0) = v_0^2 + 2a\Delta x$	2–12

Example

- A boat moves slowly inside a marina with a constant speed of 1.50 m/s. As soon as it leaves the marina, it accelerates at 2.40 m/s^2.

(a) How fast is the boat moving after accelerating for 5.00 s?

(b) How far has the boat traveled in this time?



Example continued

(a)

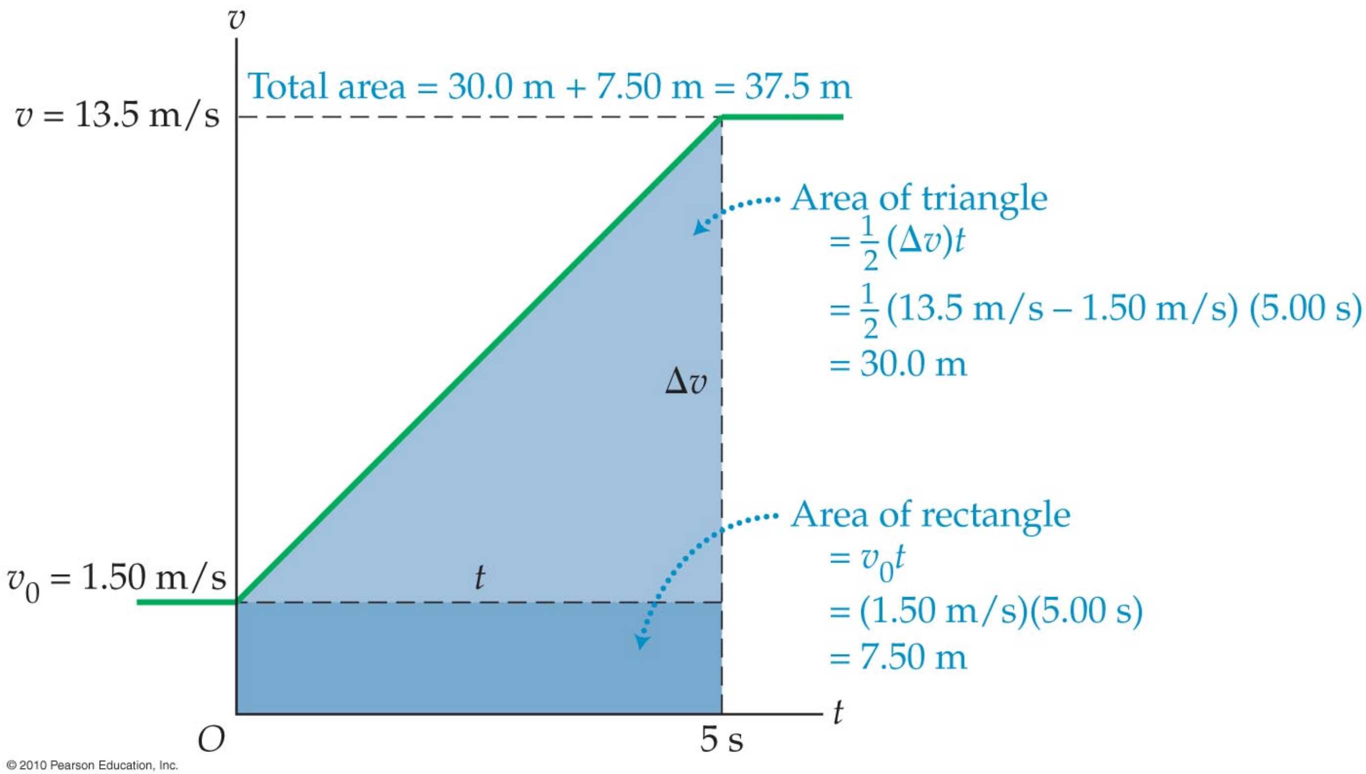
$$v = v_0 + at = 1.50\text{ m/s} + (2.40\text{ m/s}^2)(5.00\text{ s}) = 13.5\text{ m/s}$$

(b)

$$x = x_0 + \tfrac{1}{2}(v_0 + v)t = 0 + \frac{1}{2}(1.50\text{ m/s} + 13.5\text{ m/s})(5.00\text{ s}) = 37.5\text{m}$$

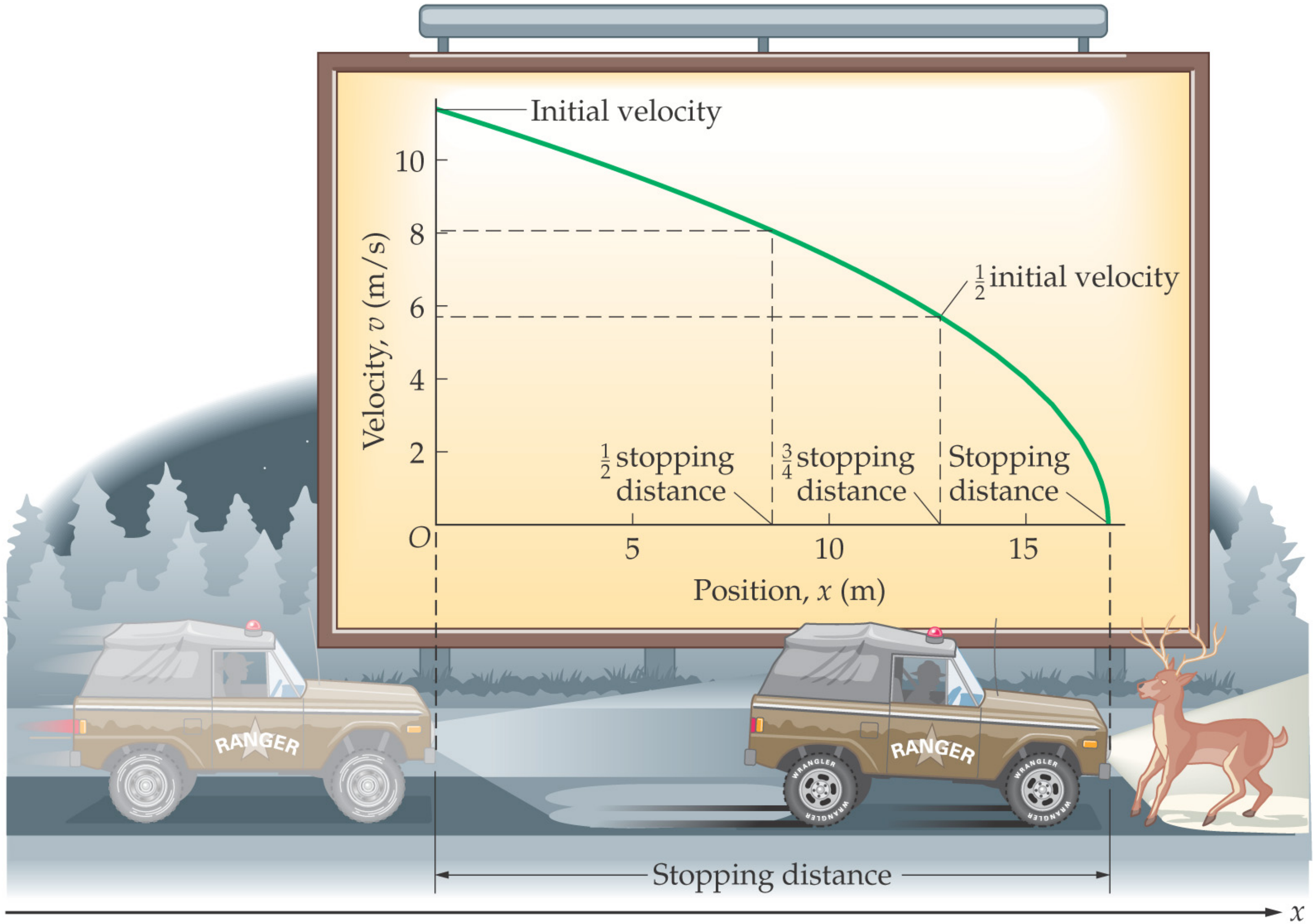
Example continued

- Velocity versus time for the boat:



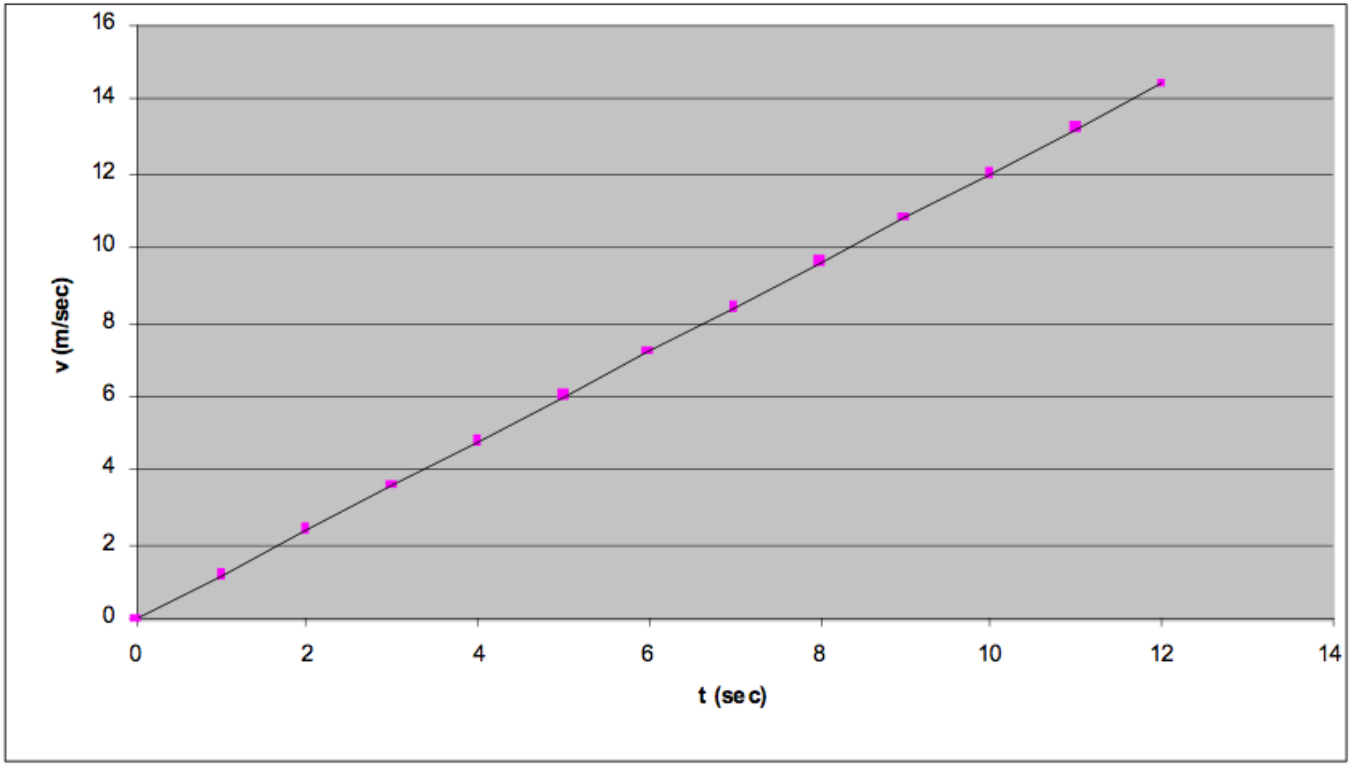
- The distance traveled by an object from time t_1 to time t_2 is equal to the **area under the velocity curve** between those two times.

2-6 Applications of the Equations of Motion



Example

- A trolley car in New Orleans starts from rest at the St. Charles Street stop and has a constant acceleration of 1.20 m/s^2 for 12.0 seconds.
- (a) Draw a graph of v versus t.



Example continued

(b) How far has the train traveled at the end of the 12.0 seconds?

The area between the curve and the time axis represents the distance traveled.

$$\Delta x = \frac{1}{2} v(t = 12\ \text{sec}) \cdot \Delta t$$
$$= \frac{1}{2}(14.4\ \text{m/s})(12\ \text{s}) = 86.4\ \text{m}$$

(c) What is the speed of the train at the end of the 12.0 s?

This can be read directly from the graph: v = 14.4 m/s.

Example

- A train of mass 55,200 kg is traveling along a straight, level track at 26.8 m/s. Suddenly the engineer sees a truck stalled on the tracks 184 m ahead. If the maximum possible braking acceleration has magnitude of 1.52 m/s^2, can the train be stopped in time?

We know: $a = -1.52\ m/s^2$, $v_i = 26.8\ m/s$, $v_f = 0$.

Using the given acceleration, compute the distance traveled by the train before it stops

$$v_f^2 = v_i^2 + 2\,a\,\Delta x$$

$$\Delta x = -\frac{v_i^2}{2a} = 236\ m$$

The train cannot be stopped in time.

Example

- A car with good tires on a dry road can decelerate at about 5.0 m/s^2 when breaking. Suppose a car is initially traveling at 55 mi/h. What is the stopping distance? (1609 meters/mile)

$$55 \text{ mi/h} = 55 \text{miles/h} \frac{(1609 \text{m/miles})}{(3600 \text{s/h})} = 24.5 \text{ m/s}$$

$$v^2 = v_0{}^2 + 2a\Delta x$$

We want the car to stop -> $v^2=0$

$$\Delta x = -v_0{}^2/2a = -(24.5^2 m^2/s^2)/(-10 \text{ m/s}^2) = 60 \text{ m}$$

2-7 Free Falling Objects

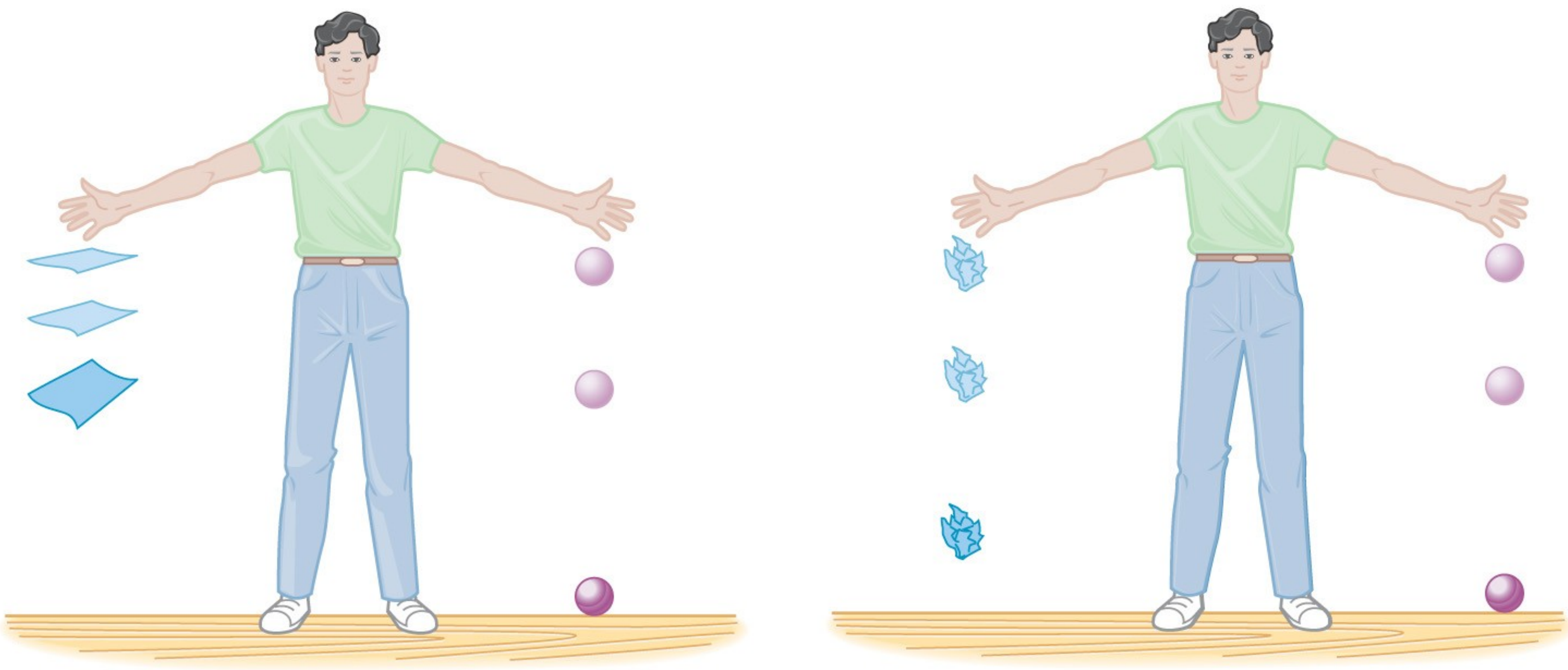
Free fall is the motion of an object subject only to the influence of gravity. The acceleration due to gravity is a constant, *g*.

TABLE 2–5 Values of *g* at Different Locations on Earth (m/s^2)

Location	Latitude	*g*
North Pole	90° N	9.832
Oslo, Norway	60° N	9.819
Hong Kong	30° N	9.793
Quito, Ecuador	0°	9.780

2-7 Free Falling Objects

An object falling in air is subject to air resistance (and therefore is not freely falling).



(a)

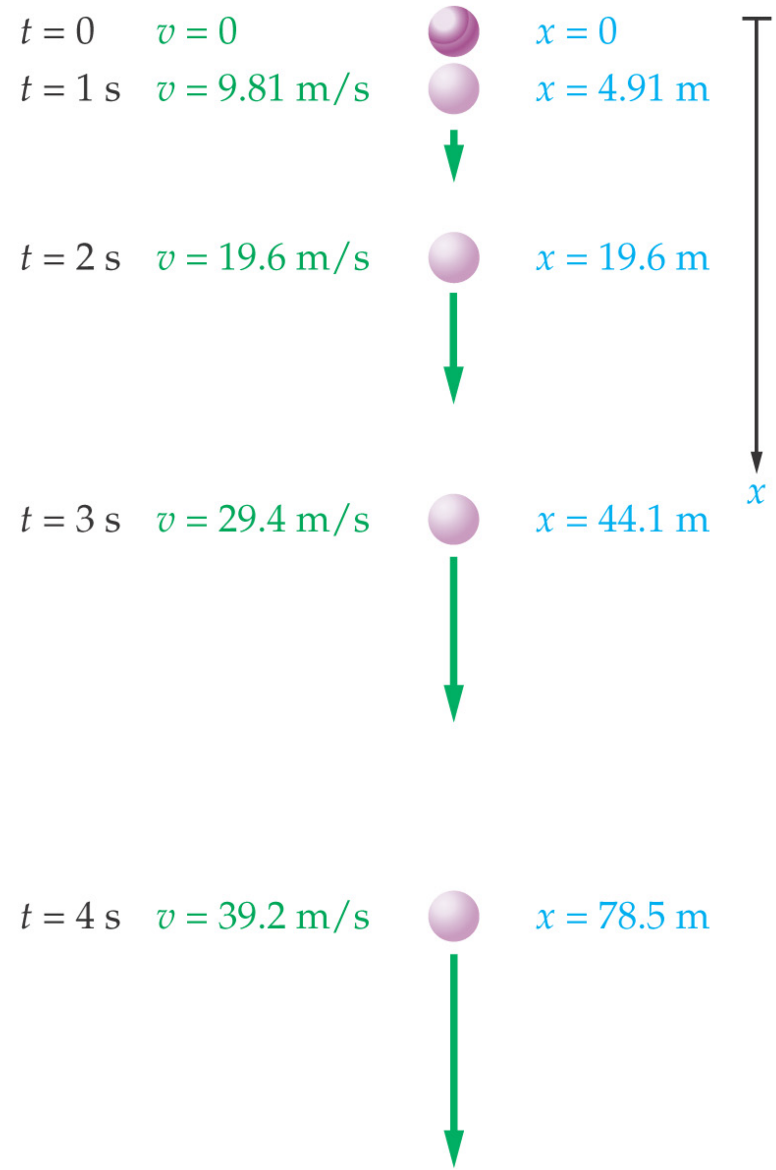
(b)

Experiment

In the near-perfect vacuum on the Moon's surface, there is no air resistance

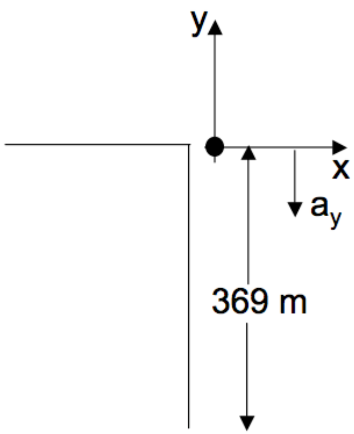
2-7 Free Falling Objects

Free fall from rest:



Example

- A penny is dropped from the observation deck of the Empire State Building 369 m above the ground. With what velocity does it strike the ground? Ignore air resistance.



Given: v_i=0 m/s; a = -9.81m/s^2; Δy = - 369 m

Unknown: v_f

Use: $v_f^2 = v_i^2 + 2a\,\Delta y = 2a\,\Delta y$

$$v_f = \sqrt{2a_y\Delta y} = \sqrt{2\left(-9.8\ \text{m/s}^2\right)\left(-369\right)\text{m}} = 85.0\ \text{m/s}$$ (downward)

How long does it take for the penny to strike the ground?

Given: v_i=0 m/s; a = -9.81m/s^2; Δy = - 369 m

Unknown: Δt

Use: $\Delta y = v_i\,\Delta t + 1/2a\,\Delta t^2 = 1/2a\,\Delta t^2$

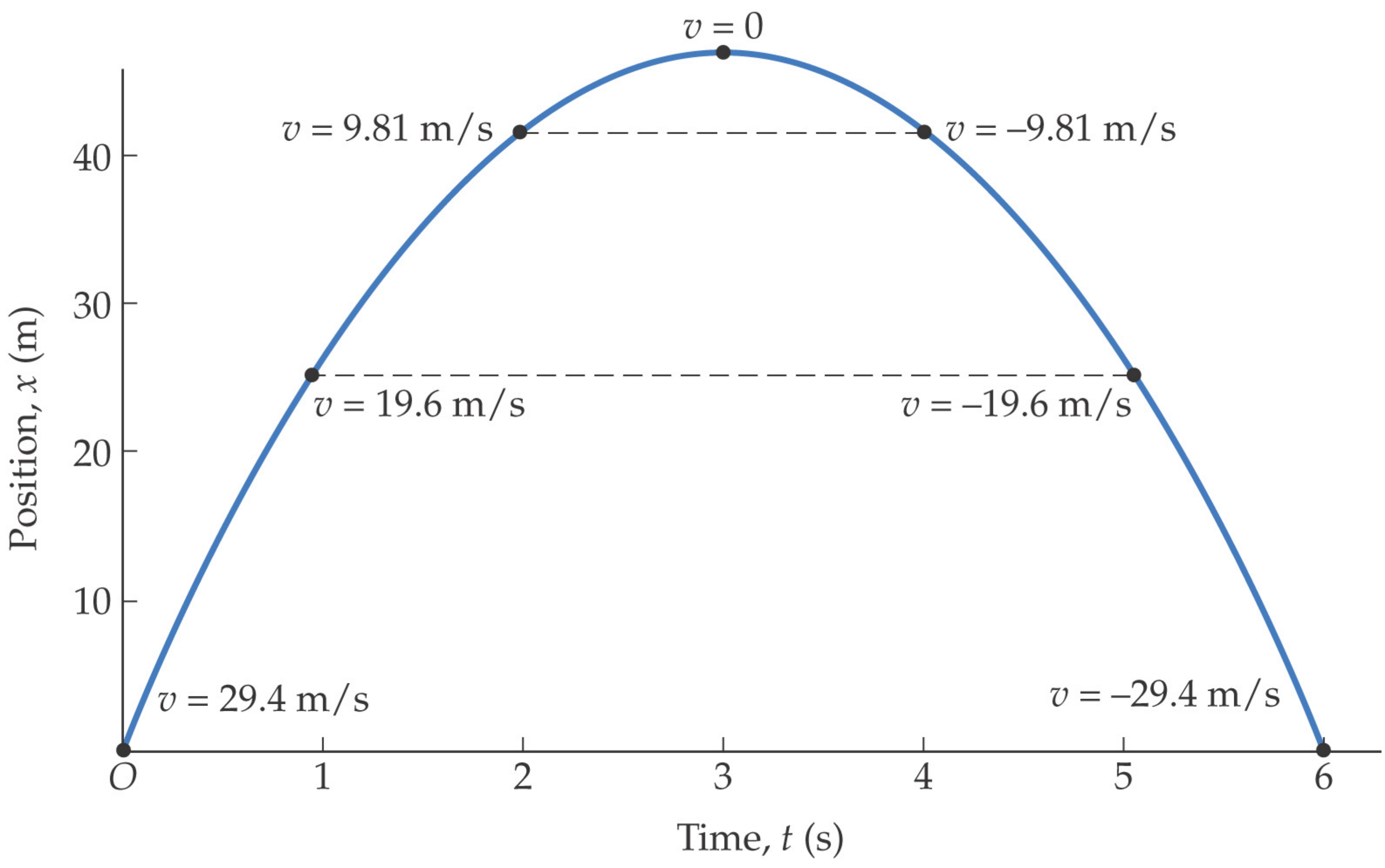
$$\Delta t = \sqrt{\frac{2\Delta y}{a}} = 8.7\ \text{sec}$$

Example

- An astronaut stands by the rim of a crater on the moon, where the acceleration of gravity is 1.62 m/s^2. To determine the depth of the crater, she drops a rock and measures the time it takes for it to hit the bottom. If the depth of the crater is 120 m, how long does it take for the rock to fall?

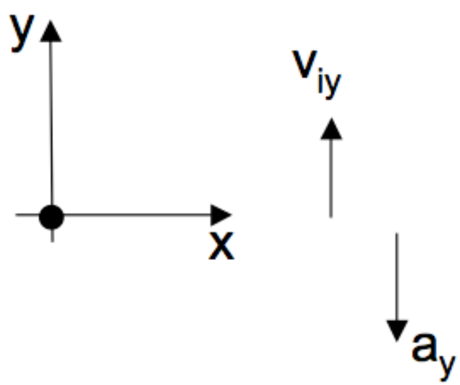
2-7 Free Falling Objects

Trajectory of a projectile:



Example

- You throw a ball into the air with speed 15.0 m/s; how high does the ball rise?



Given: $v_i = +15.0$ m/s; $a = -9.81 \text{m/s}^2$; $v_f = 0$

Unknown: Δy

Use: $v_f^2 = v_i^2 + 2a\,\Delta y = 2a\,\Delta y$

$\Delta y = -\ v_i^2/2a = 11.5$ m

Example

- A ball is thrown upwards with a speed of 16 m/s. How long does it take it to reach a height of 7.0 m on the way up?

Summary of Chapter 2

- Distance: total length of travel
- Displacement: change in position
- Average speed: distance / time
- Average velocity: displacement / time
- Instantaneous velocity: average velocity measured over an infinitesimally small time

Summary of Chapter 2

- Instantaneous acceleration: average acceleration measured over an infinitesimally small time
- Average acceleration: change in velocity divided by change in time
- Deceleration: velocity and acceleration have opposite signs
- Constant acceleration: equations of motion relate position, velocity, acceleration, and time
- Freely falling objects: constant acceleration $g = 9.81\ m/s^2$