WORK AND KINETIC ENERGY

Chapter 7

Units of Chapter 7

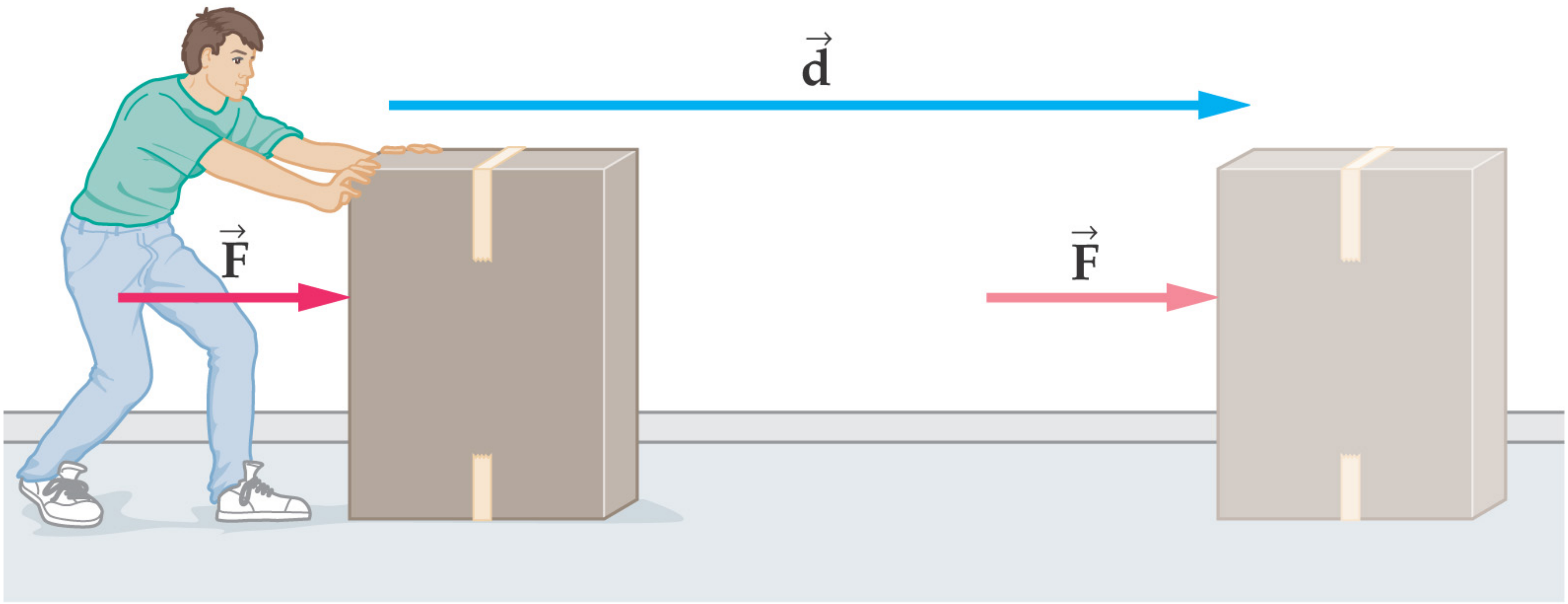
- Work Done by a Constant Force
- Kinetic Energy and the Work-Energy Theorem
- Work Done by a Variable Force
- Power

7-1 Work done by a Constant Force

- The definition of work, when the force is parallel to the displacement:

$$W = Fd$$

SI unit: newton-meter (N·m) = joule, J



7-1 Work Done by a Constant Force

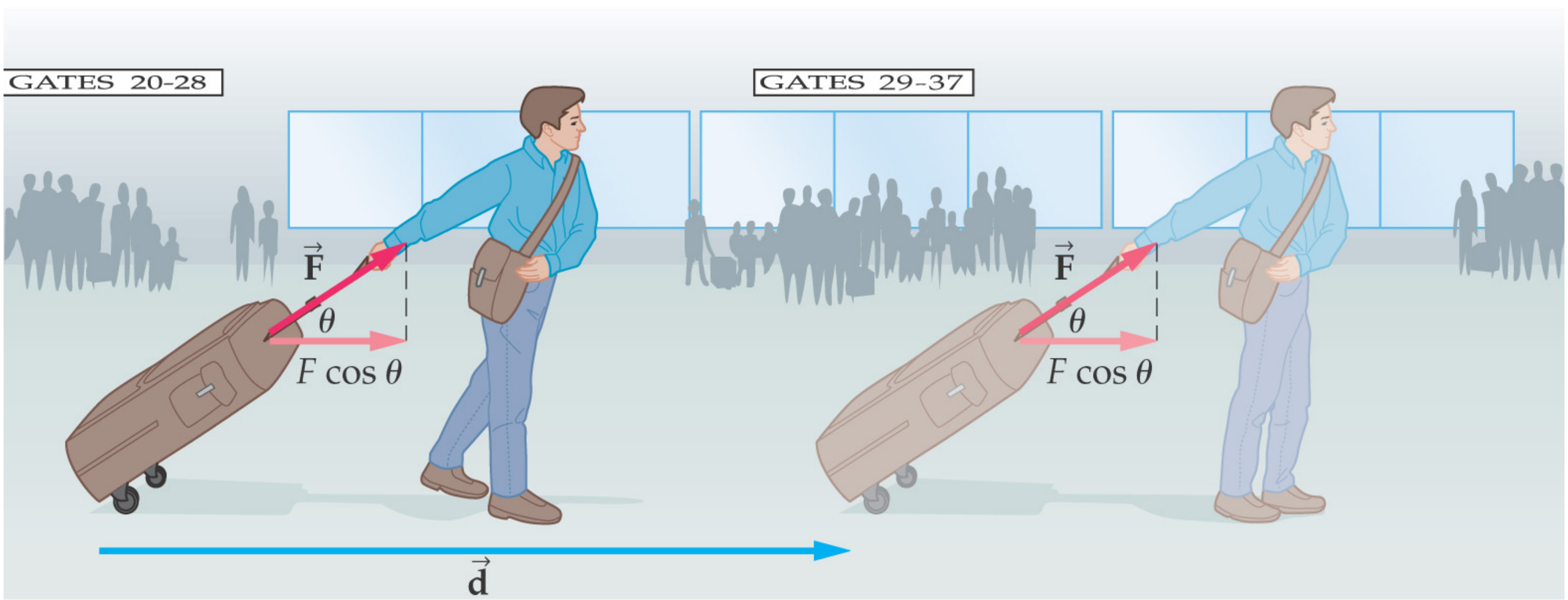
TABLE 7–1 Typical Values of Work

Activity	Equivalent work (J)
Annual U. S. energy use	8×10^{19}
Mt. St. Helens eruption	10^{18}
Burning one gallon of gas	10^{8}
Human food intake/day	10^{7}
Melting an ice cube	10^{4}
Lighting a 100-W bulb for 1 minute	6000
Heartbeat	0.5
Turning page of a book	10^{-3}
Hop of a flea	10^{-7}
Breaking a bond in DNA	10^{-20}

7-1 Work done by a Constant Force

- If the force is at an angle to the displacement:

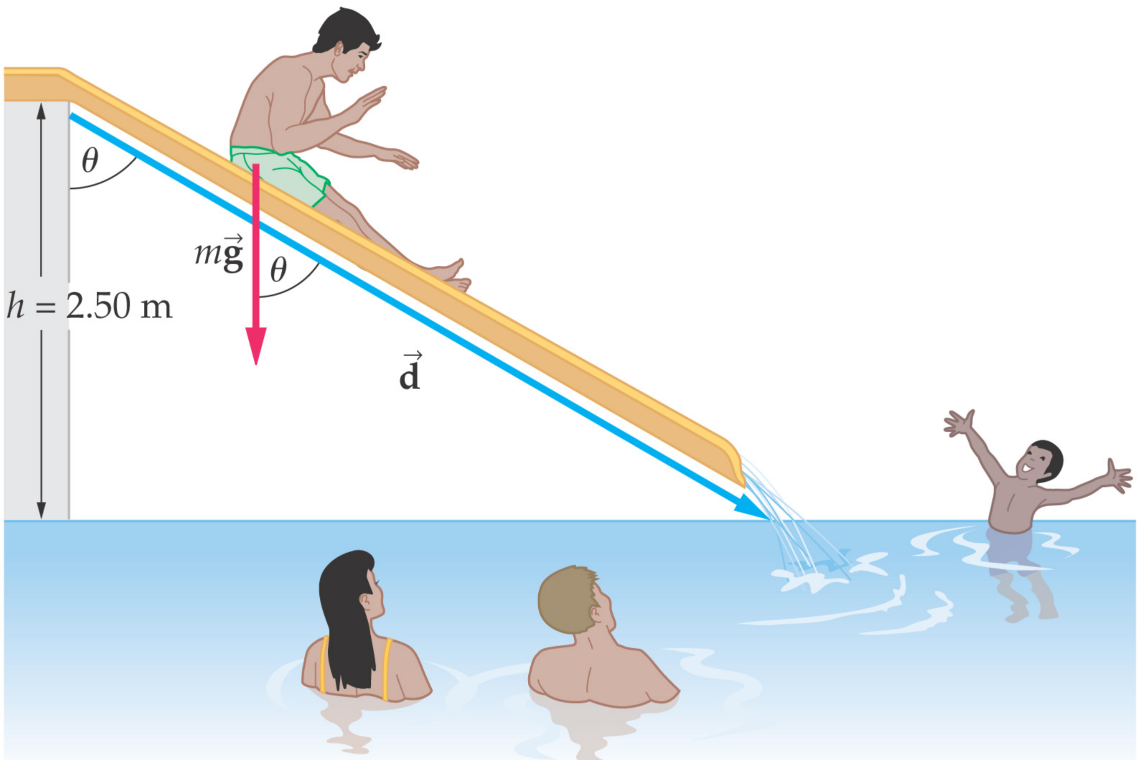
$$W = (F\cos\theta)d = Fd\cos\theta$$



7-1 Work done by a Constant Force

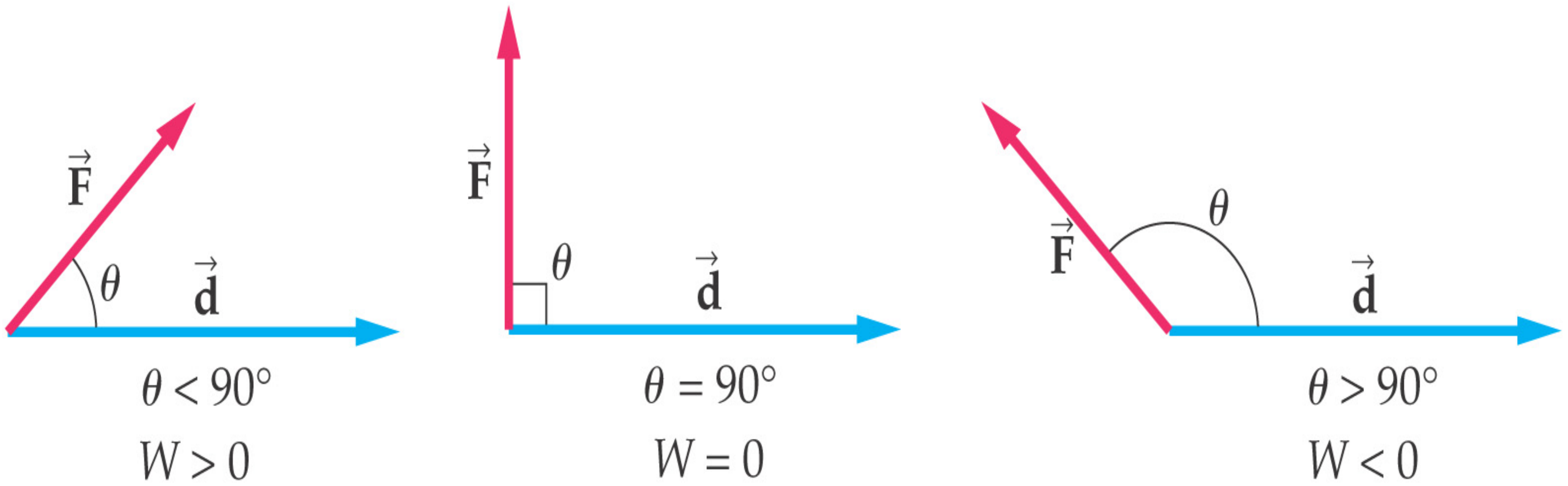
- The work can also be written as the dot product of the force and the displacement:

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = Fd\cos\theta$$



7-1 Work done by a Constant Force

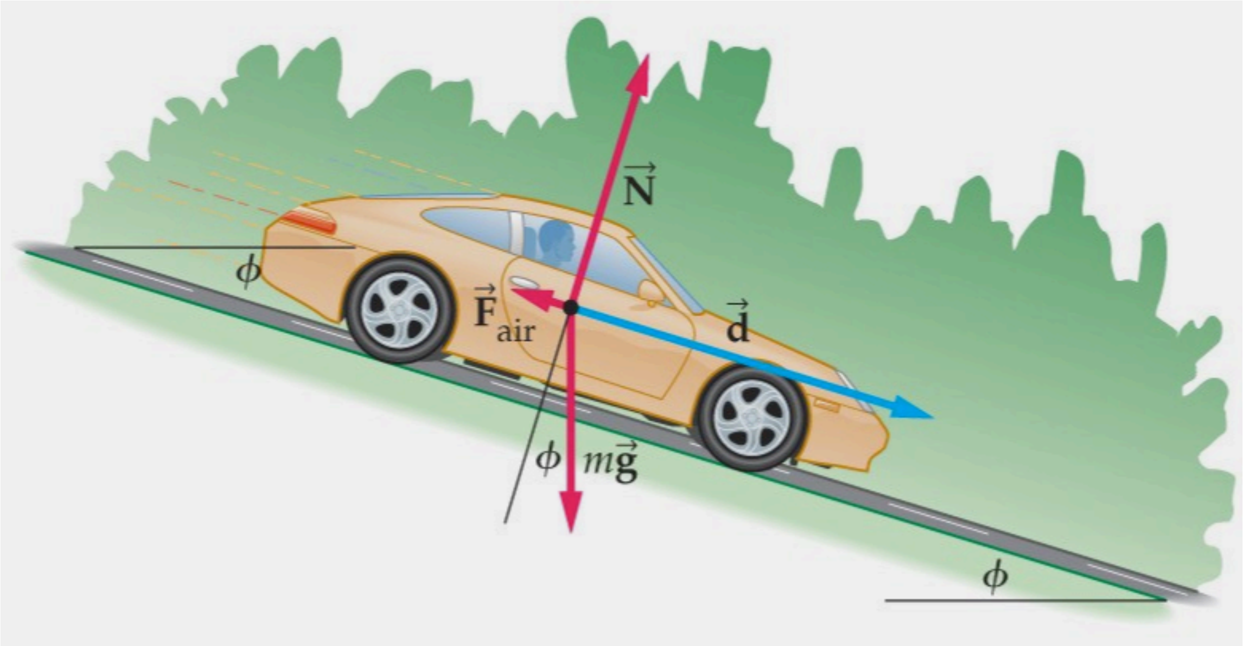
- The work done may be positive, zero, or negative, depending on the angle between the force and the displacement:



7-1 Work done by a Constant Force

- If there is more than one force acting on an object, we can find the work done by each force, and also the work done by the net force:

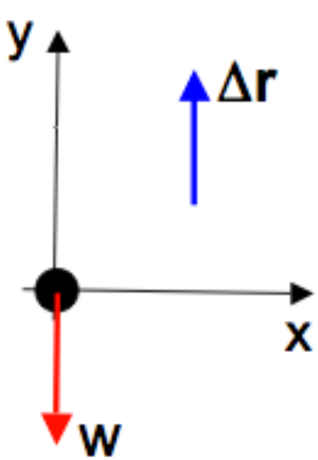
$$W_{\text{total}} = (F_{\text{total}} \cos \theta)d = F_{\text{total}} d \cos \theta$$



Example

- A ball is tossed straight up. What is the work done by the force of gravity on the ball as it rises?

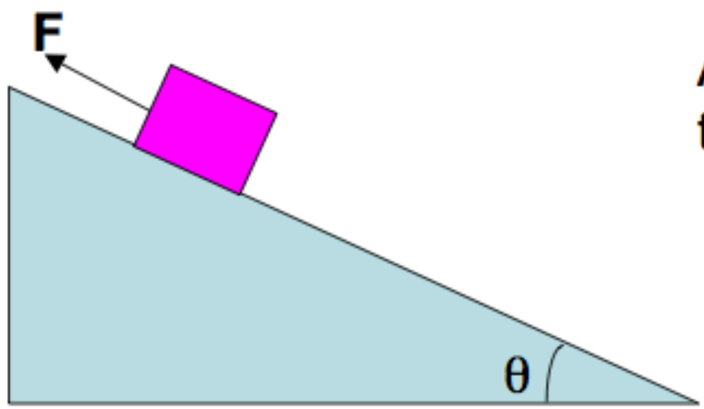
FBD for rising ball:



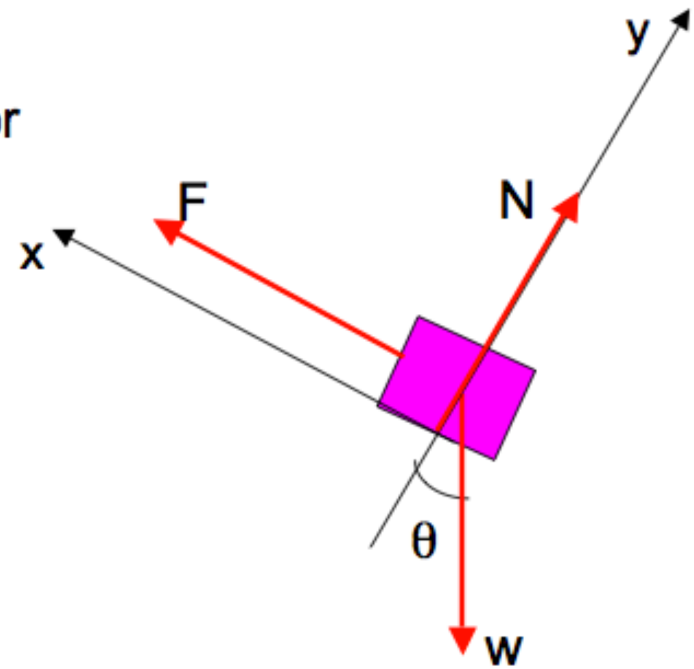
$$W_g = w\Delta y \cos 180°$$
$$= -mg\Delta y$$

Example

- A box of mass **m** is towed up a frictionless incline at constant speed. The applied force **F** is **parallel** to the incline. What is the net work done on the box?



An FBD for the box:



Apply Newton's 2nd Law:

$$\sum F_x = F - w\sin\theta = 0$$

$$\sum F_y = N - w\cos\theta = 0$$

Example continued

The magnitude of F is: $F = mg\sin\theta$

If the box travels along the ramp a distance of Δx the work by the force F is

$$W_F = F\Delta x\cos 0° = mg\Delta x\sin\theta$$

The work by gravity is

$$W_g = w\Delta x\cos(\theta + 90°) = -mg\Delta x\sin\theta$$

Example

- What is the net work done on the box in the previous example if the box is **not** pulled at constant speed?

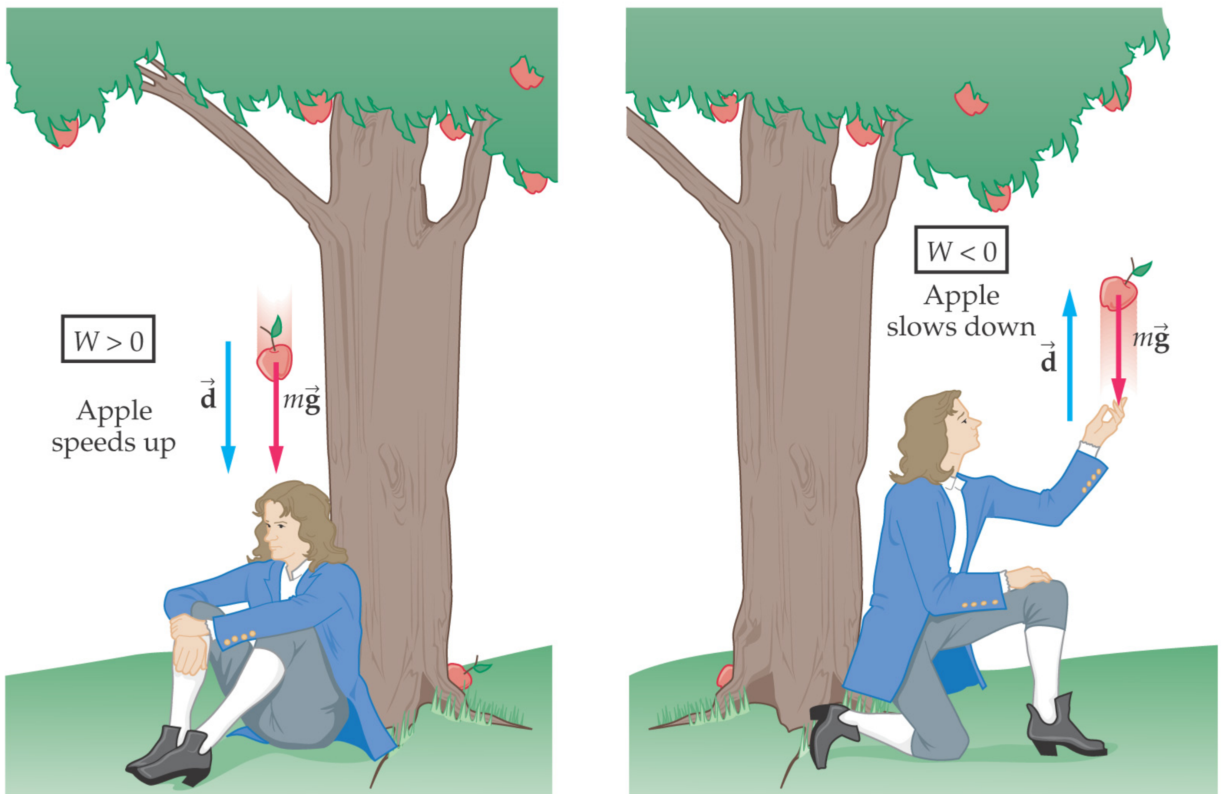
$$\sum F_x = F - w\sin\theta = ma$$
$$F = ma + w\sin\theta$$

Proceeding as before:

$$\begin{aligned} W_{\text{net}} &= W_F + W_g + W_N \\ &= (ma + mg\sin\theta)\Delta x - mg\Delta x\sin\theta + 0 \\ &= (ma)\Delta x = F_{\text{net}}\Delta x \end{aligned}$$

7-2 Kinetic Energy and the Work-Energy Theorem

- When positive work is done on an object, its speed increases; when negative work is done, its speed decreases.



7-2 Kinetic Energy and the Work-Energy Theorem

- After algebraic manipulations of the equations of motion, we find:

$$W_{\mathrm{total}} = \frac{1}{2}m{v_{\mathrm{f}}}^2 - \frac{1}{2}m{v_{\mathrm{i}}}^2$$

- Therefore, we define the kinetic energy:

$$K = \frac{1}{2}mv^2$$

Example

- The extinction of the dinosaurs and the majority of species on Earth in the Cretaceous Period (65 Myr ago) is thought to have been caused by an asteroid striking the Earth near the Yucatan Peninsula. The resulting ejecta caused widespread global climate change. If the **mass** of the asteroid was 10^{16} kg (diameter in the range of 4-9 miles) and had a **speed** of 30.0 km/sec, what was the asteroid's kinetic energy?

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(10^{16}\text{ kg}\right)\left(30\times10^3\text{ m/s}\right)^2$$

$$= 4.5\times10^{24}\text{ J}$$

This is equivalent to ~10^9 Megatons of TNT.

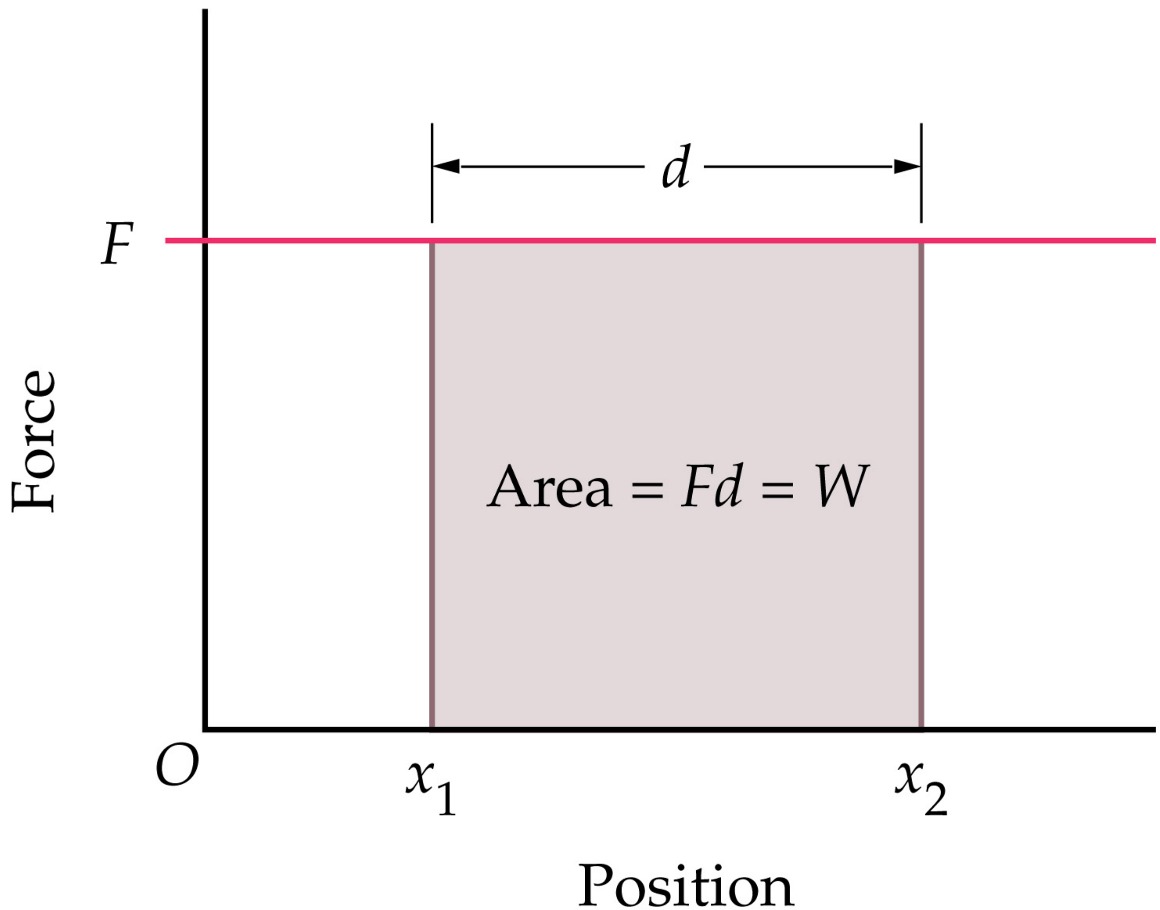
7-2 Kinetic Energy and the Work-Energy Theorem

- **Work-Energy Theorem**: The total work done on an object is equal to its change in kinetic energy.

$$W_{\mathrm{total}} = \Delta K = \frac{1}{2}mv_{\mathrm{f}}^{2} - \frac{1}{2}mv_{\mathrm{i}}^{2}$$

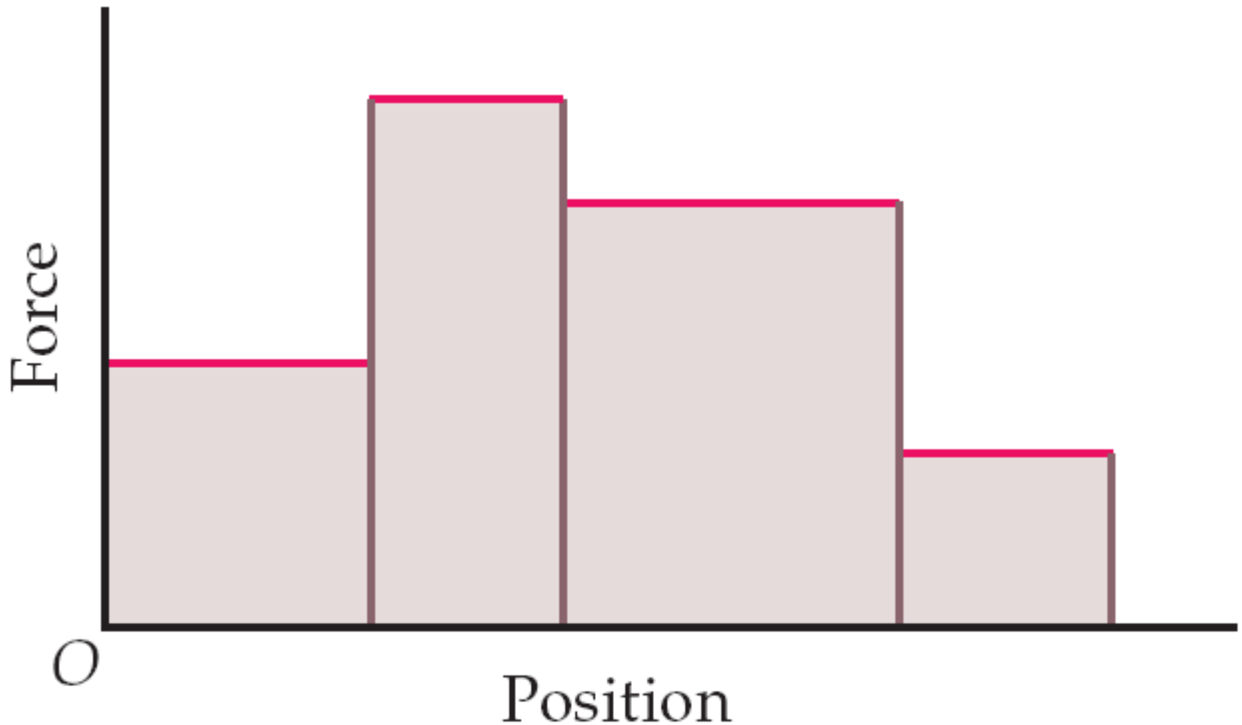
7-3 Work done by a variable force

- If the force is constant, we can interpret the work done graphically:



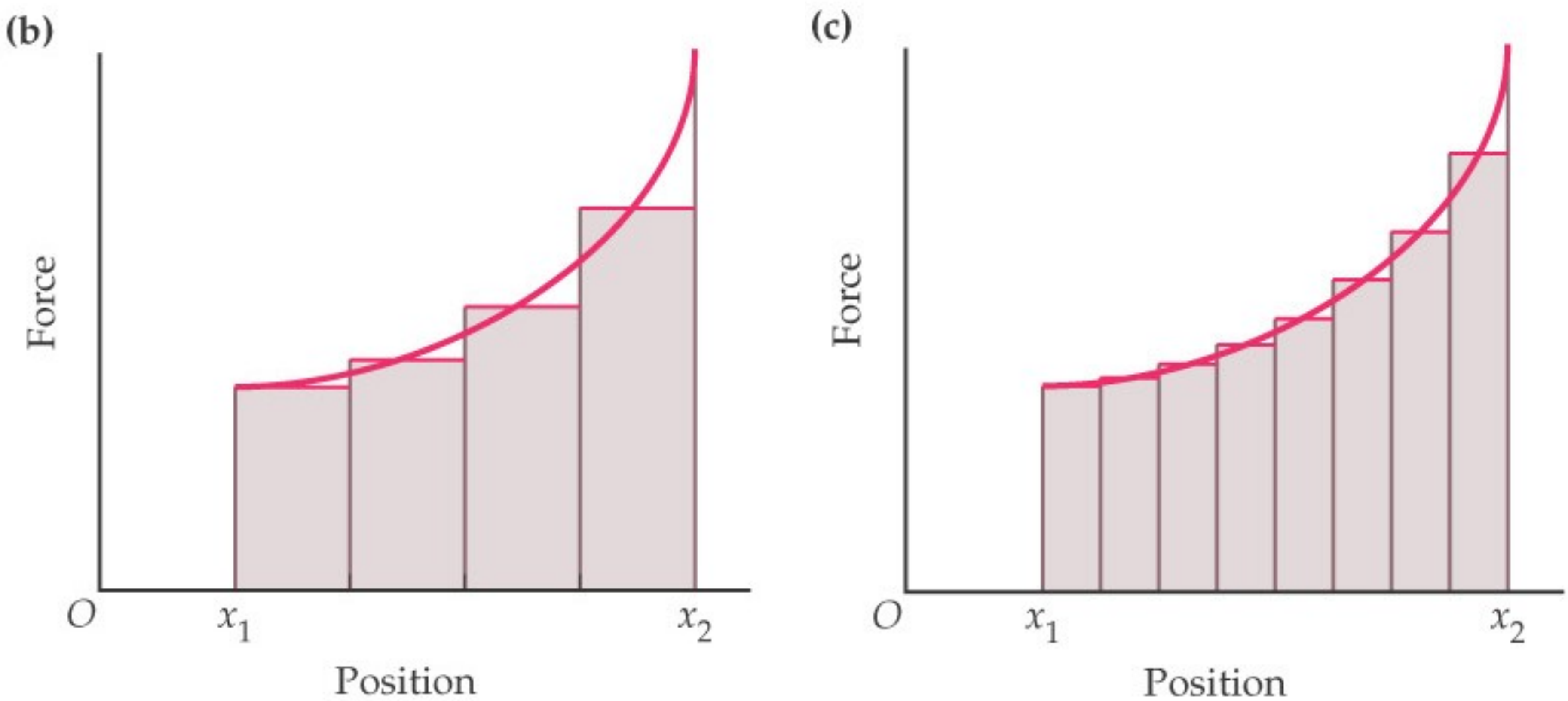
7-3 Work done by a variable force

- If the force takes on several successive constant values:



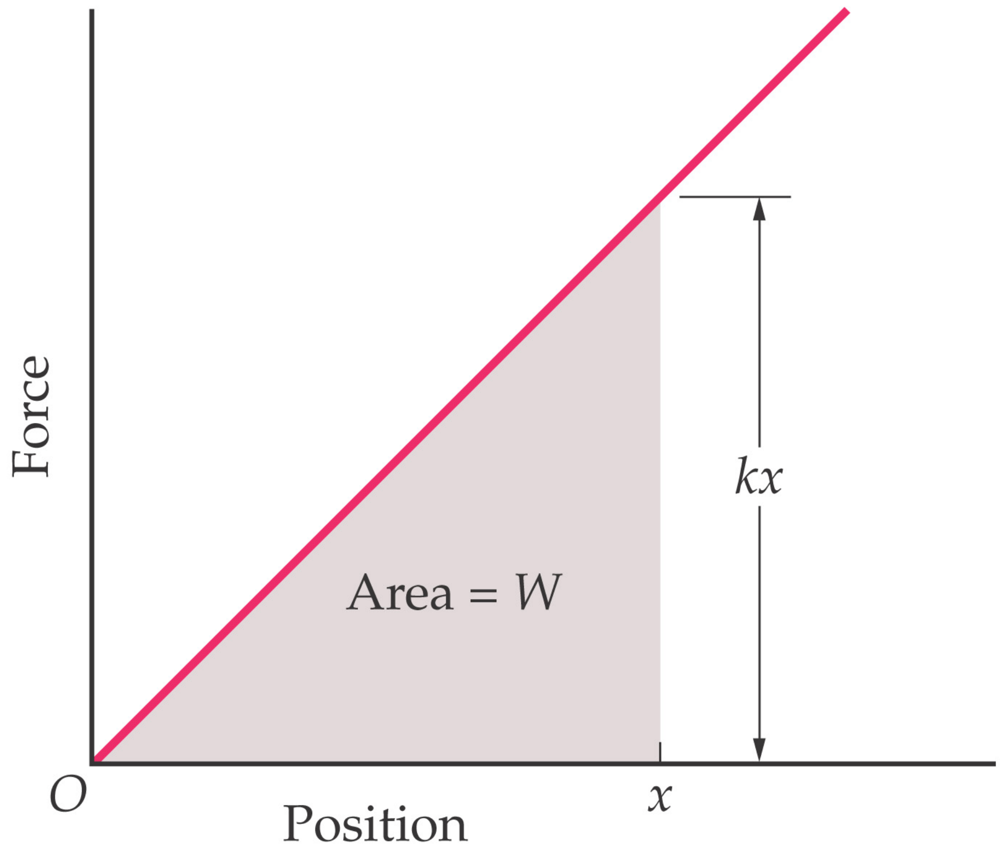
7-3 Work done by a variable force

- We can then approximate a continuously varying force by a succession of constant values.



7-3 Work done by a variable force

- The force needed to stretch a spring an amount x is $F = kx$.

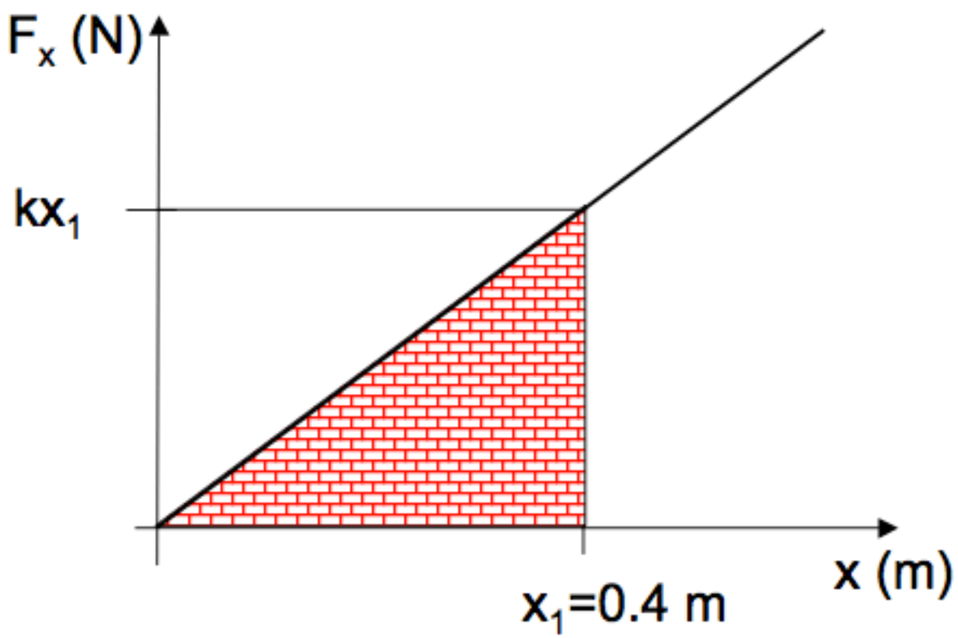


Therefore, the work done in stretching the spring is

$$W = \frac{1}{2}kx^2$$

Example

- An ideal spring has k = 20.0 N/m. What is the amount of work done (by an external agent) to stretch the spring 0.40 m from its relaxed length?



$$W = \text{Area under curve}$$

$$= \frac{1}{2}(kx_1)(x_1) = \frac{1}{2}kx_1^2 = \frac{1}{2}(20.0\ \text{N/m})(0.4\ \text{m})^2 = 1.6\ \text{J}$$

7-4 Power

Power is a measure of the rate at which work is done:

$$P = \frac{W}{t}$$

SI unit: J/s = watt, W

1 horsepower = 1 hp = 746 W

7-4 Power

TABLE 7–3
Typical Values of Power

Source	Approximate power (W)
Hoover Dam	1.34×10^9
Car moving at 40 mph	7×10^4
Home stove	1.2×10^4
Sunlight falling on one square meter	1380
Refrigerator	615
Television	200
Person walking up stairs	150
Human brain	20

7-4 Power

If an object is moving at a constant speed in the face of friction, gravity, air resistance, and so forth, the power exerted by the driving force can be written:

$$P = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv$$

Example

- A race car with a mass of 500.0 kg completes a quarter-mile (402 m) race in a time of 4.2 s starting from rest. The car's final speed is 125 m/s. What is the engine's average power output? Neglect friction and air resistance.

$$P_{\mathrm{av}} = \frac{\Delta E}{\Delta t} = \frac{\Delta U + \Delta K}{\Delta t}$$

$$= \frac{\Delta K}{\Delta t} = \frac{\frac{1}{2} m v_f^2}{\Delta t} = 9.3 \times 10^5 \text{ watts}$$

Summary of Chapter 7

- If the force is constant and parallel to the displacement, work is force times distance
- If the force is not parallel to the displacement,

$$W = (F\cos\theta)d = Fd\cos\theta$$

- The total work is the work done by the net force:

$$W_{\text{total}} = (F_{\text{total}}\cos\theta)d = F_{\text{total}}d\cos\theta$$

Summary of Chapter 7

- SI unit of work: the joule, J
- Total work is equal to the change in kinetic energy:

$$W_{\text{total}} = \Delta K = \tfrac{1}{2}mv_{\text{f}}^2 - \tfrac{1}{2}mv_{\text{i}}^2$$

Where:

$$K = \tfrac{1}{2}mv^2$$

Summary of Chapter 7

- Work done by a spring force:

$$W = \frac{1}{2}kx^2$$

- Power is the rate at which work is done:

$$P = \frac{W}{t}$$

- SI unit of power: the watt, W