

POTENTIAL ENERGY AND CONSERVATION OF ENERGY

Chapter 8

Units of Chapter 8

- Conservative and Nonconservative Forces
- Potential Energy and the Work Done by Conservative Forces
- Conservation of Mechanical Energy
- Work Done by Nonconservative Forces
- Potential Energy Curves and Equipotentials

8-1 Conservative and Nonconservative Forces

- **Conservative force**: the work it does is stored in the form of energy that can be released at a later time

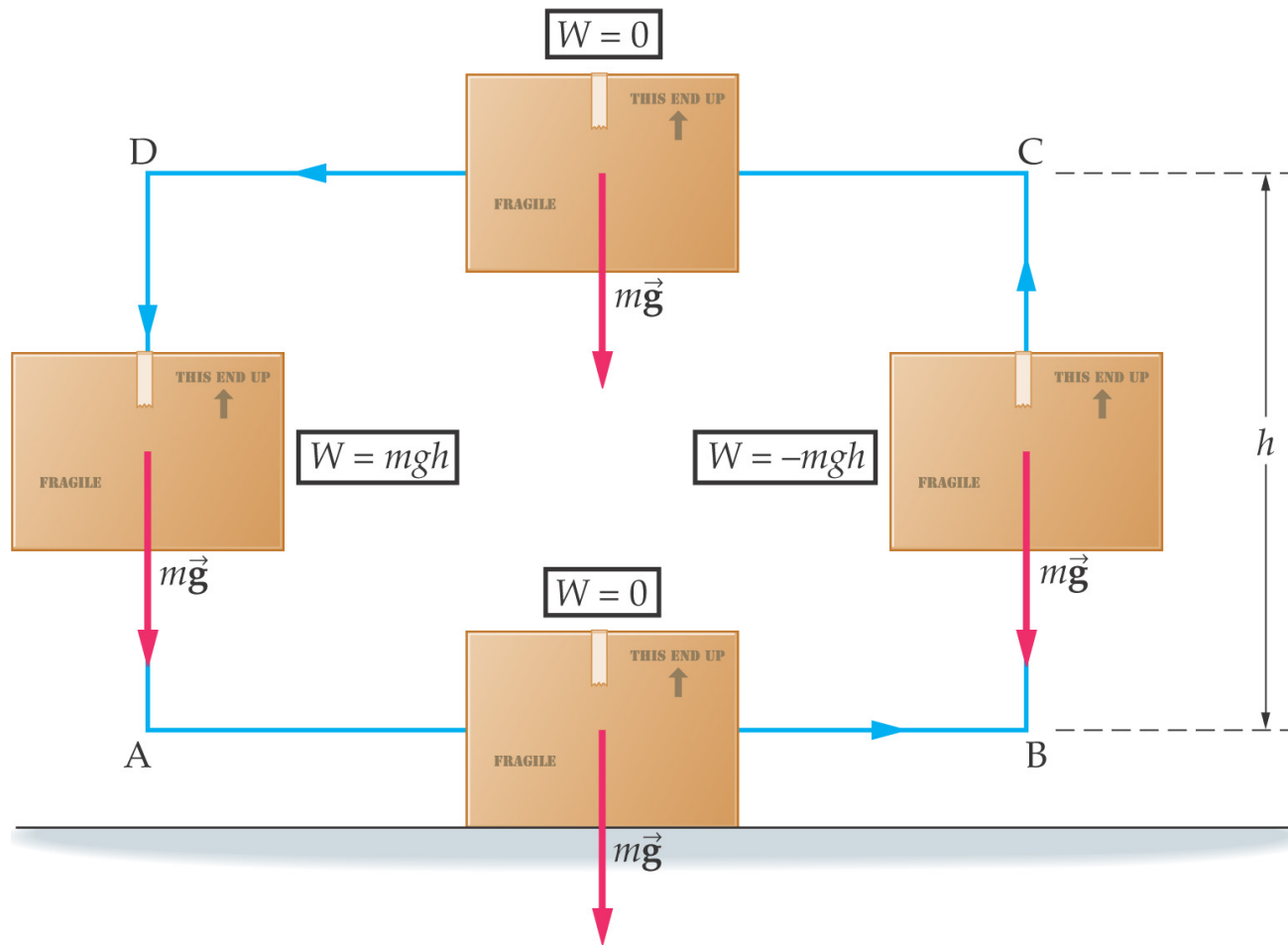
Example of conservative forces: gravity, force of a spring

- **Nonconservative force**: the work it does cannot be recovered later as kinetic energy

Example of a nonconservative force: friction

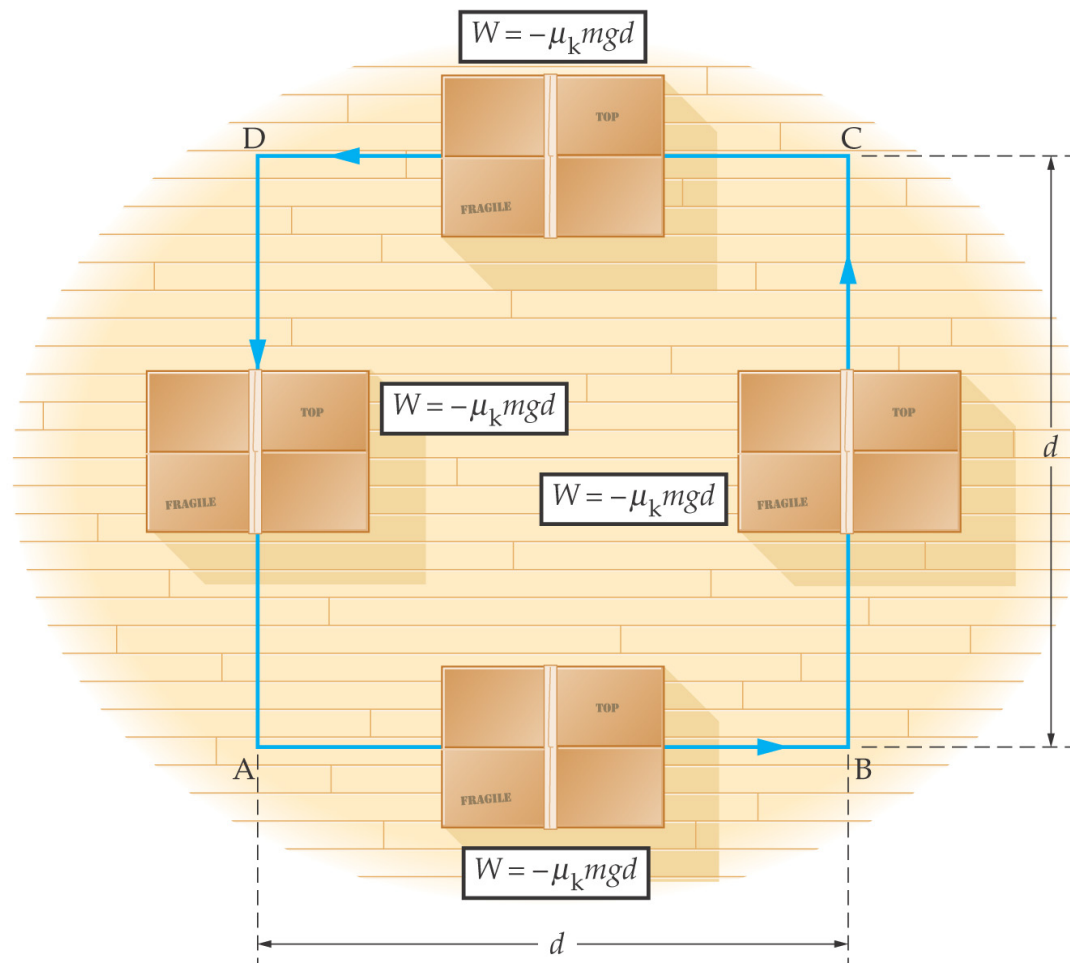
8-1 Conservative and Nonconservative Forces

- Work done by gravity on a closed path is zero:



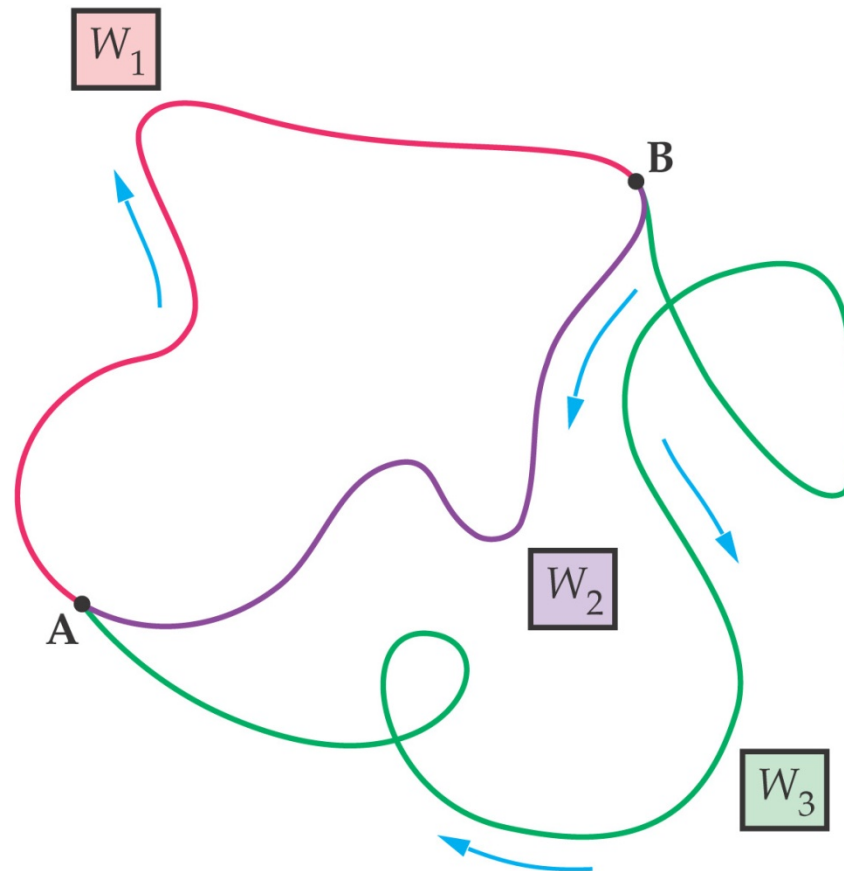
8-1 Conservative and Nonconservative Forces

- Work done by friction on a closed path is not zero:



8-1 Conservative and Nonconservative Forces

- The work done by a conservative force moving an object around any closed path is zero; this is not true for a nonconservative force



8-2 The work done by conservative forces

- If we pick up a ball and put it on the shelf, we have done work on the ball. We can get that energy back if the ball falls back off the shelf; in the meantime, we say the energy is stored as **potential energy** (energy associated with an object based on its location relative to a reference point).

Definition of Potential Energy, U

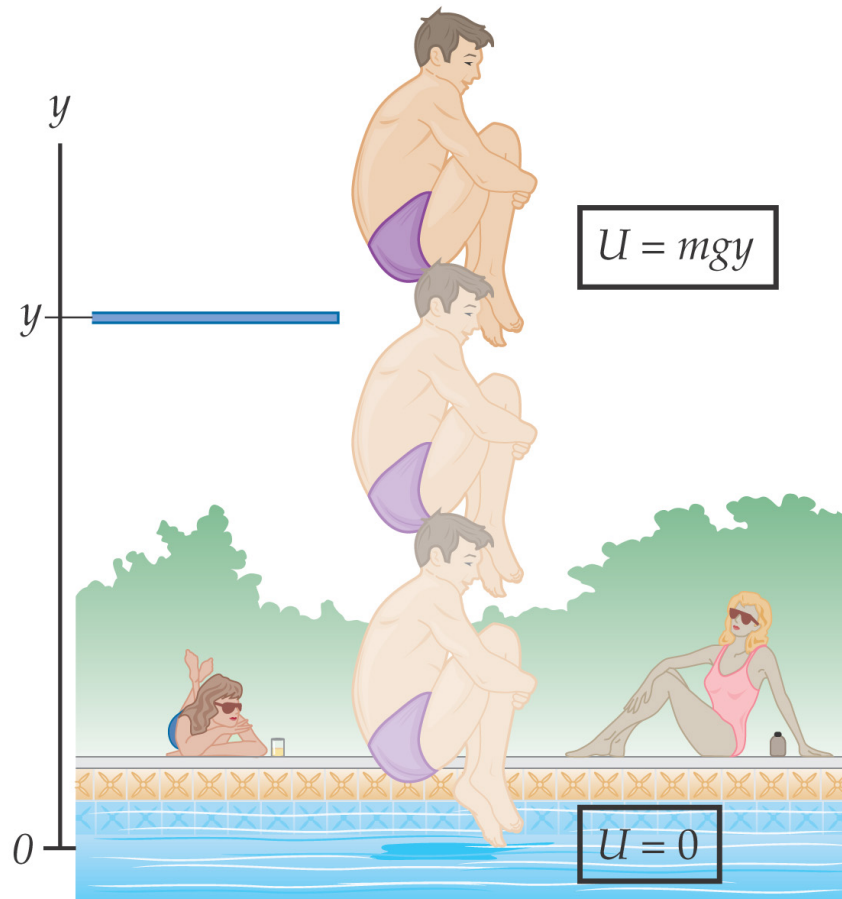
$$W_c = U_i - U_f = -(U_f - U_i) = -\Delta U$$

SI unit: joule, J

8-2 The work done by conservative forces

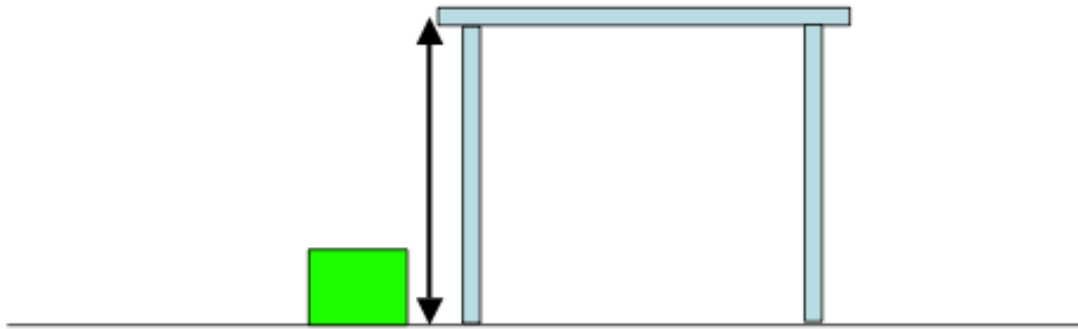
- Gravitational potential energy:

$$-\Delta U = U_i - U_f = W_c = mgy$$



Example

What is the change in gravitational potential energy of the box if it is placed on the table? The table is 1.0 m tall and the mass of the box is 1.0 kg.



First: Choose the reference level at the floor. $U = 0$ here.

$$\begin{aligned}\Delta U_g &= mg\Delta y = mg(y_f - y_i) \\ &= (1.0 \text{ kg})(9.8 \text{ m/s}^2)(1.0 \text{ m} - 0 \text{ m}) = +9.8 \text{ J}\end{aligned}$$

Example continued

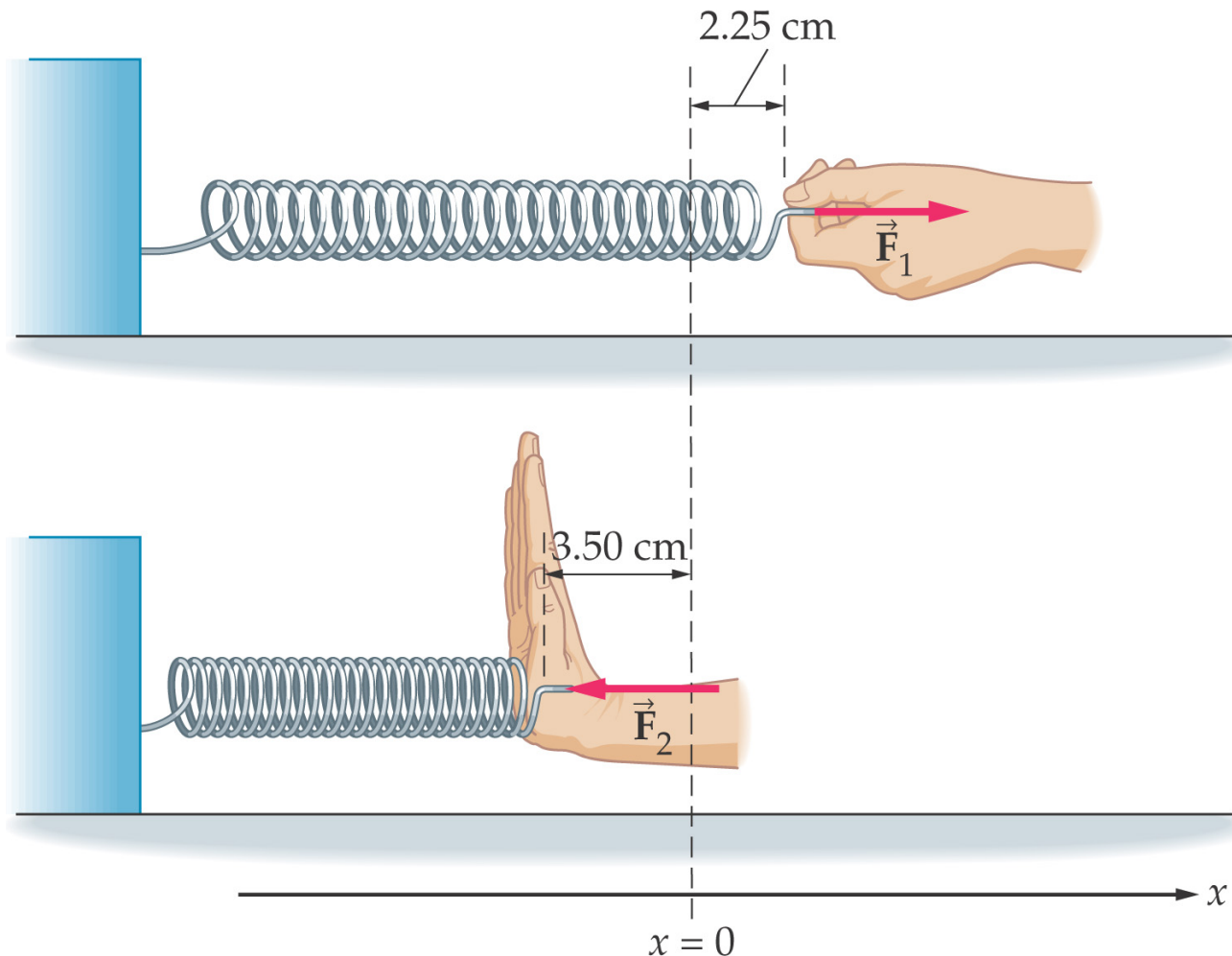
Now take the reference level ($U = 0$) to be on top of the table so that $y_i = -1.0$ m and $y_f = 0.0$ m.

$$\begin{aligned}\Delta U_g &= mg\Delta y = mg(y_f - y_i) \\ &= (1 \text{ kg})(9.8 \text{ m/s}^2)(0.0\text{m} - (-1.0 \text{ m})) = +9.8 \text{ J}\end{aligned}$$

The results do not depend on the location of $U = 0$.

8-2 The work done by conservative forces

- Springs: $W_c = \frac{1}{2} kx^2 = U_i - U_f$



8-3 Conservation of Mechanical Energy

- Definition of mechanical energy:

$$E = U + K$$

- Using this definition and considering only conservative forces, we find:

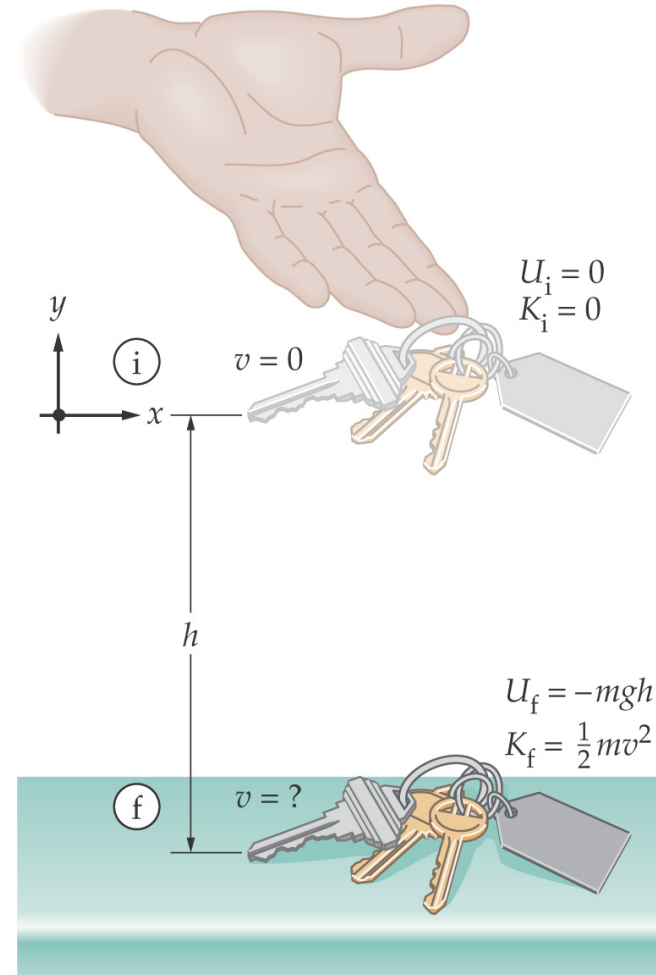
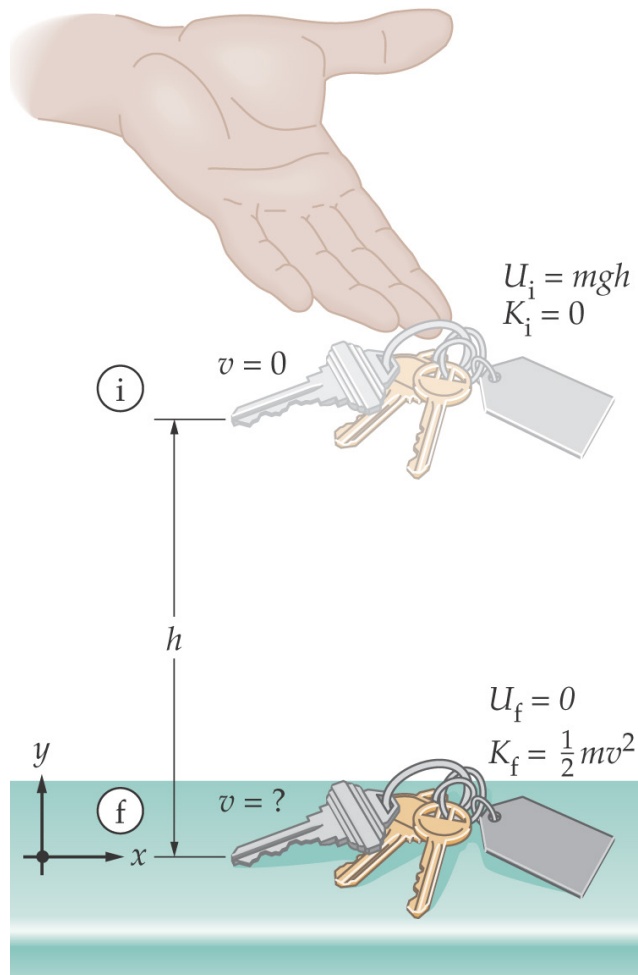
$$E_f = E_i$$

or equivalently:

$$E = \text{constant}$$

8-3 Conservation of Mechanical Energy

- Energy conservation can make kinematics problems much easier to solve:



8-3 Conservation of Mechanical Energy



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8-3 Conservation of Mechanical Energy



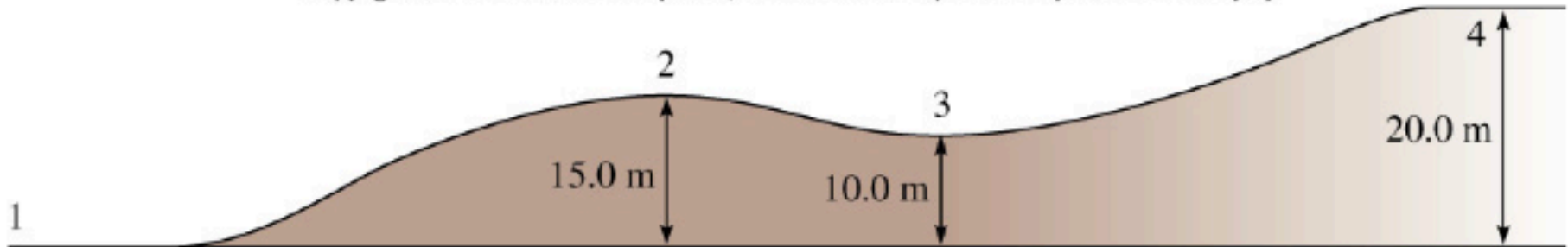
8-3 Conservation of Mechanical Energy



Example

- A cart starts from position 4 with $v = 15.0 \text{ m/s}$ to the left. Find the speed of the cart at positions 1, 2, and 3. Ignore friction.

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$$E_4 = E_3$$

$$U_4 + K_4 = U_3 + K_3$$

$$mgy_4 + \frac{1}{2}mv_4^2 = mgy_3 + \frac{1}{2}mv_3^2$$

$$v_3 = \sqrt{v_4^2 + 2g(y_4 - y_3)} = 20.5 \text{ m/s}$$

Example continued

$$E_4 = E_2$$

$$U_4 + K_4 = U_2 + K_2$$

$$mgy_4 + \frac{1}{2}mv_4^2 = mgy_2 + \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{v_4^2 + 2g(y_4 - y_2)} = 18.0 \text{ m/s}$$

Or use
 $E_3 = E_2$

$$E_4 = E_1$$

$$U_4 + K_4 = U_1 + K_1$$

$$mgy_4 + \frac{1}{2}mv_4^2 = mgy_1 + \frac{1}{2}mv_1^2$$

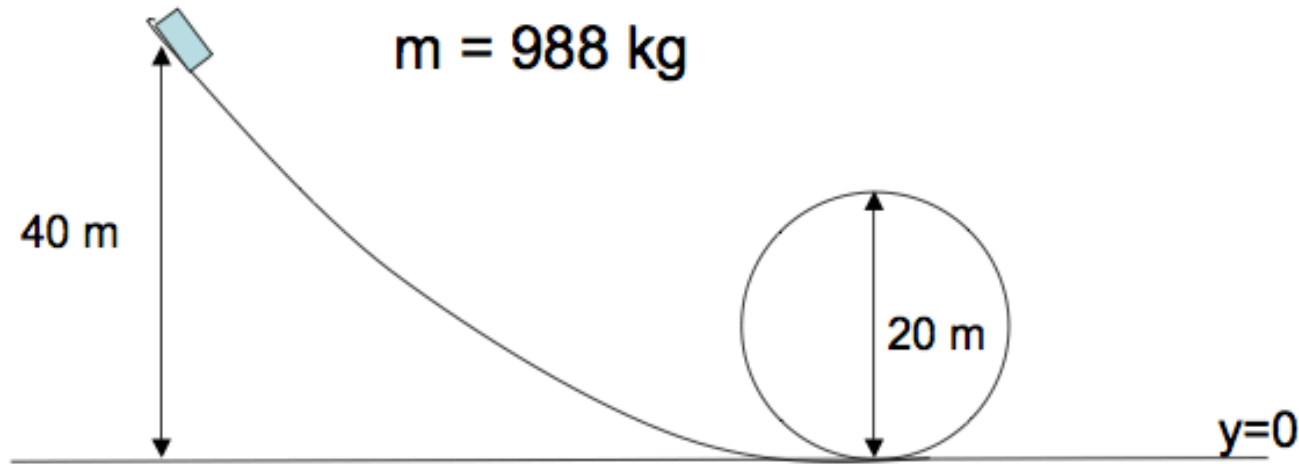
$$v_1 = \sqrt{v_4^2 + 2g(y_4 - y_1)} = 24.8 \text{ m/s}$$

Or use
 $E_3 = E_1$

$E_2 = E_1$

Example

- A roller coaster car is about to roll down a track. Ignore friction and air resistance.



(a) At what speed does the car reach the top of the loop?

$$E_i = E_f$$

$$U_i + K_i = U_f + K_f$$

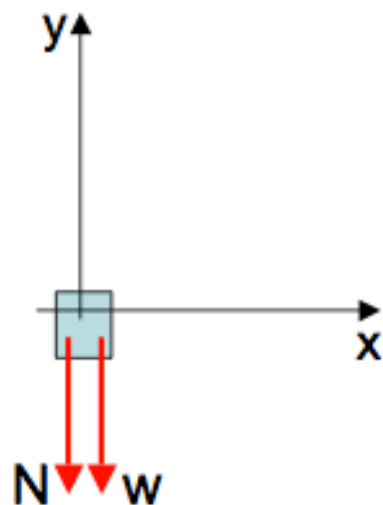
$$mgy_i + 0 = mgy_f + \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{2g(y_i - y_f)} = 19.8 \text{ m/s}$$

Example continued

(b) What is the force exerted on the car by the track at the top of the loop?

FBD for the car:



Apply Newton's Second Law:

$$\sum F_y = -N - w = -ma_r = -m \frac{v^2}{r}$$

$$N + w = m \frac{v^2}{r}$$

$$N = m \frac{v^2}{r} - mg = 2.9 \times 10^4 \text{ N}$$

Example continued

(c) From what minimum height above the bottom of the track can the car be released so that it does not lose contact with the track at the top of the loop?

Using conservation of mechanical energy:

$$E_i = E_f$$

$$U_i + K_i = U_f + K_f$$

$$mgy_i + 0 = mgy_f + \frac{1}{2}mv_{\min}^2$$

Solve for the starting height

$$y_i = y_f + \frac{v_{\min}^2}{2g}$$

Example continued

What is $v_{\min}$? $v = v_{\min}$ when $N = 0$. This means that

$$N + w = m \frac{v^2}{r}$$

$$w = mg = m \frac{v_{\min}^2}{r}$$

$$v_{\min} = \sqrt{gr}$$

The initial height must be

$$y_i = y_f + \frac{v_{\min}^2}{2g} = 2r + \frac{gr}{2g} = 2.5r = 25.0 \text{ m}$$

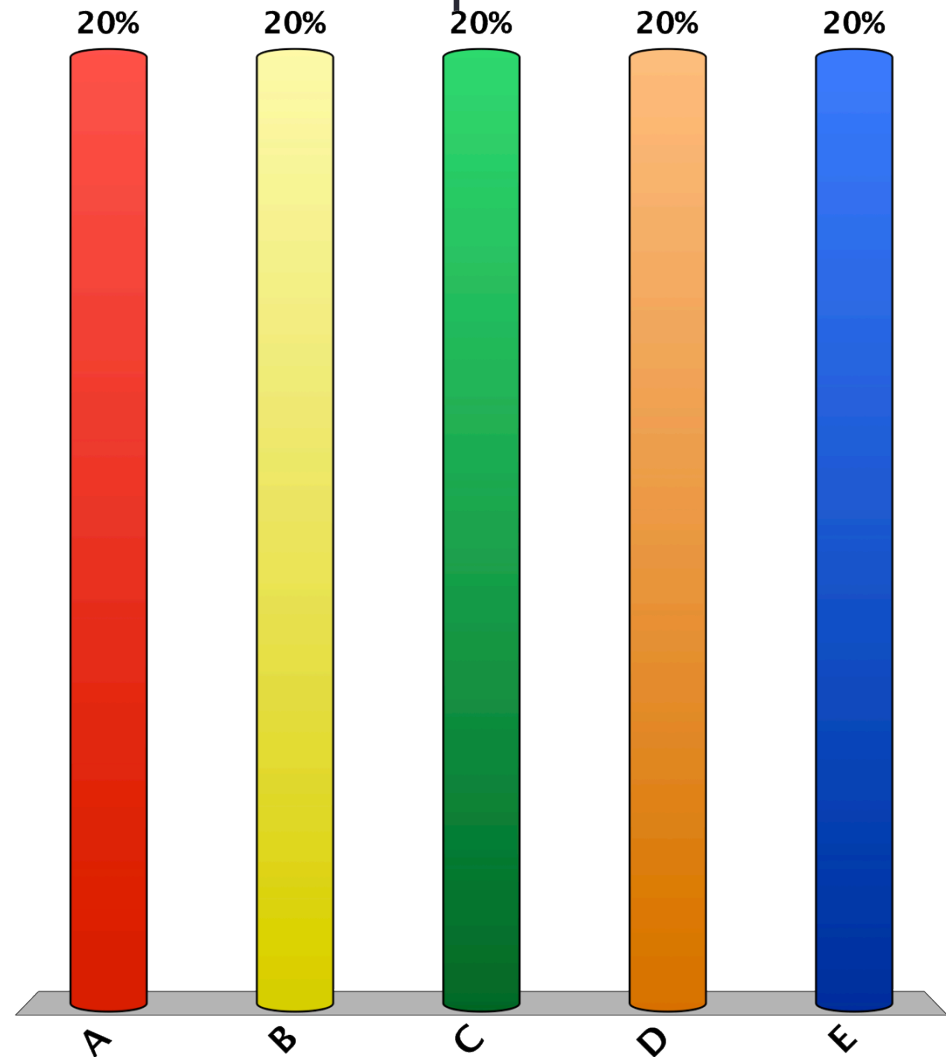
Reset your clickers!

- Channel Number: 72
- Reset your clicker:
- **Ch + 72 + Ch**
or
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A block initially at rest is allowed to slide down a frictionless ramp and attains a speed v at the bottom. To achieve a speed $2v$ at the bottom, how many times as high must a new ramp be?

- A. 1
- B. 2
- ✓ C. 4
- D. 5
- E. 6



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- A. 1
- B. 2
-  C. 4
- D. 5
- E. 6

Use energy conservation:

➤ initial energy: $E_i = PE_g = mgH$

➤ final energy: $E_f = KE = 1/2 mv^2$

Conservation of Energy:

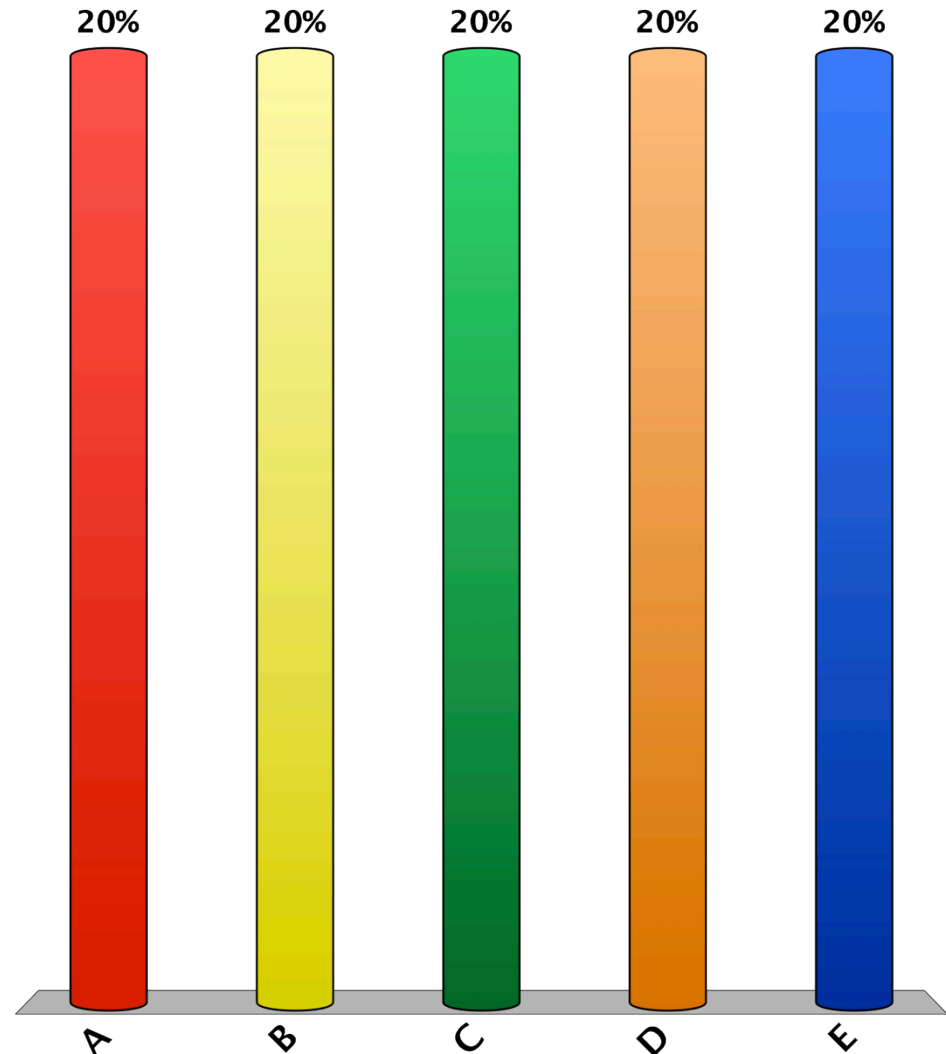
$$E_i = mgH = E_f = 1/2 mv^2$$

therefore: $gH = 1/2 v^2$

So if v doubles, H quadruples!

A box sliding on a frictionless flat surface runs into a fixed spring, which compresses a distance x to stop the box. If the initial speed of the box were doubled, how much would the spring compress in this case?

- A. Half as much
- B. The same amount
- C. $\sqrt{2}$ times as much
- ✓ D. Twice as much
- E. Four times as much



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Use energy conservation:

➤ initial energy: $E_i = KE = 1/2 mv^2$

➤ final energy: $E_f = PE_s = 1/2 kx^2$

Conservation of Energy:

$$E_i = 1/2 mv^2 = E_f = 1/2 kx^2$$

therefore: $mv^2 = kx^2$

So if v doubles, x doubles!

8-4 Work done by nonconservative forces

- In the presence of nonconservative forces, the total mechanical energy is not conserved:

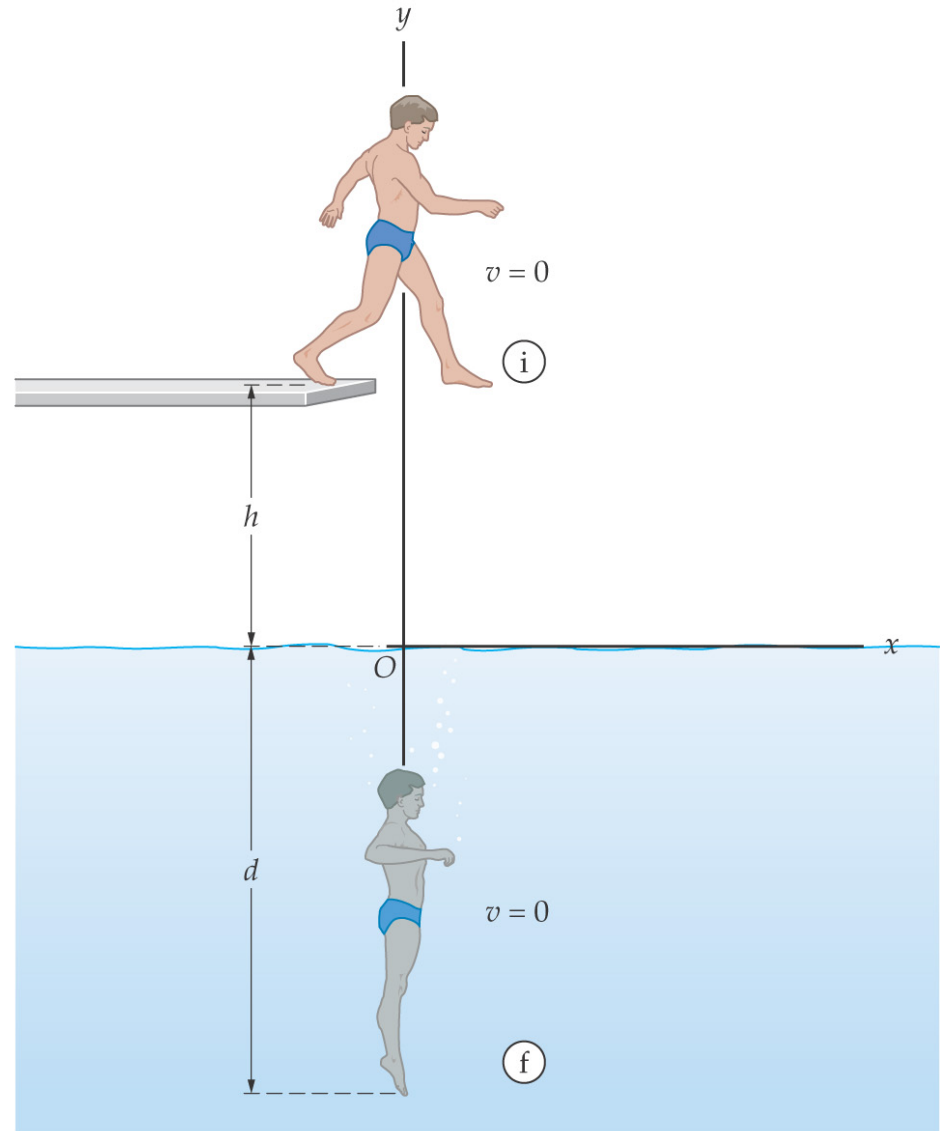
$$\begin{aligned}W_{\text{total}} &= W_{\text{c}} + W_{\text{nc}} \\&= -\Delta U + W_{\text{nc}} = \Delta K\end{aligned}$$

Solving,

$$W_{\text{nc}} = \Delta U + \Delta K = \Delta E$$

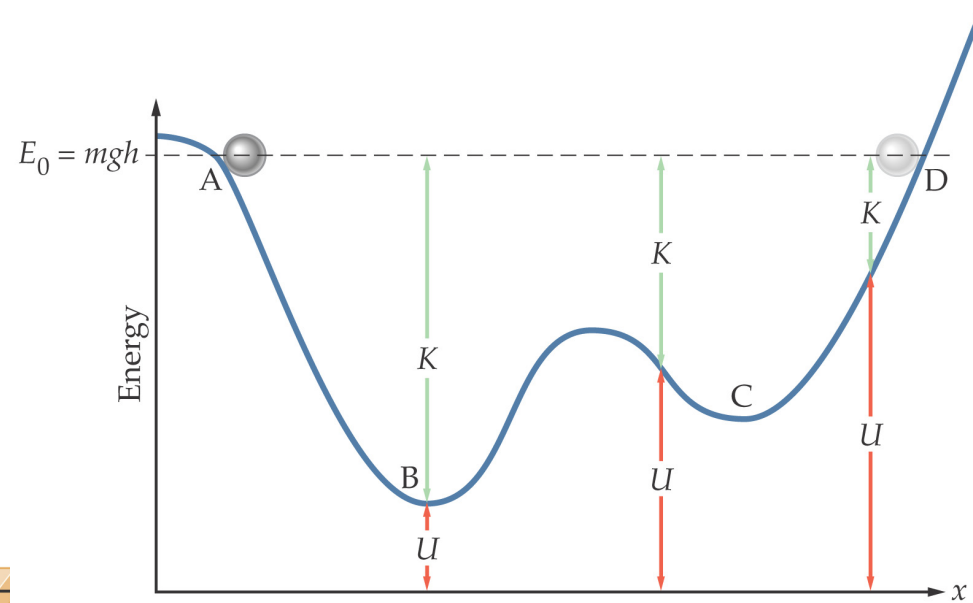
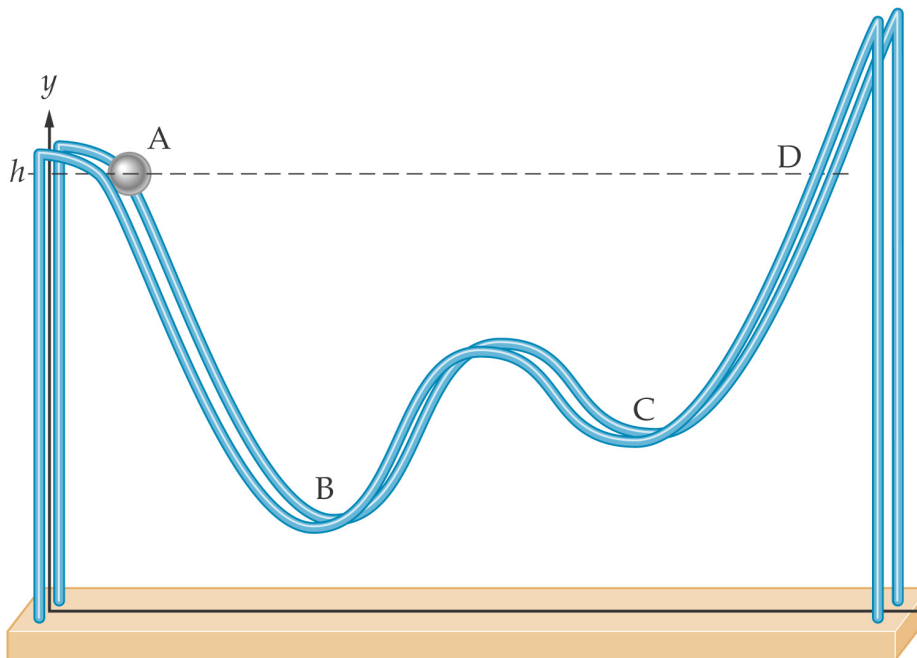
8-4 Work done by nonconservative forces

- In this example, the nonconservative force is water resistance:



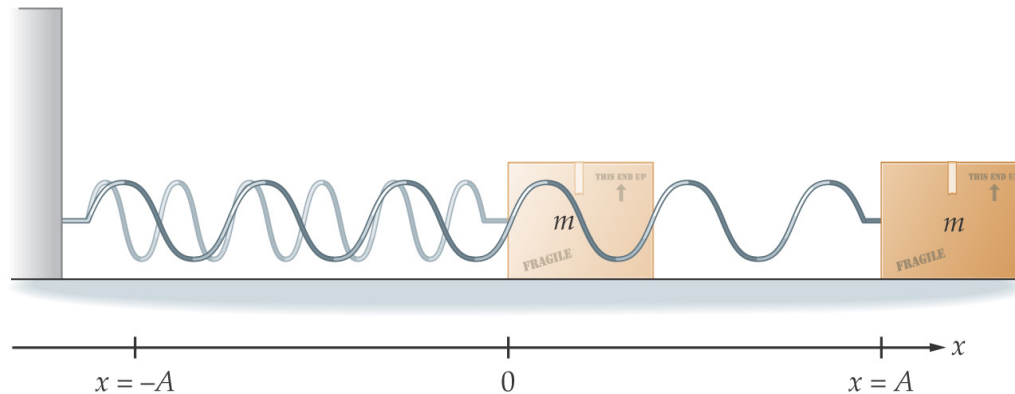
8-5 Potential Energy Curves and Equipotentials

- The curve of a hill or a roller coaster is itself essentially a plot of the gravitational potential energy:

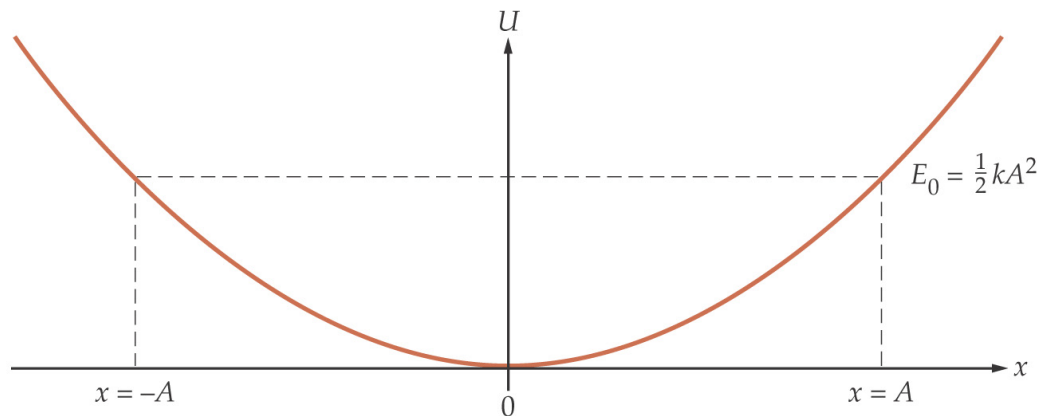


8-5 Potential Energy Curves and Equipotentials

- The potential energy curve for a spring:



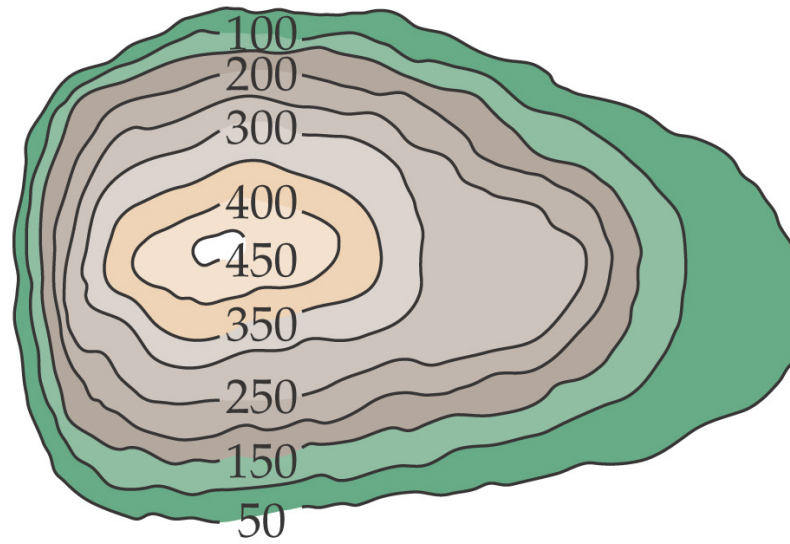
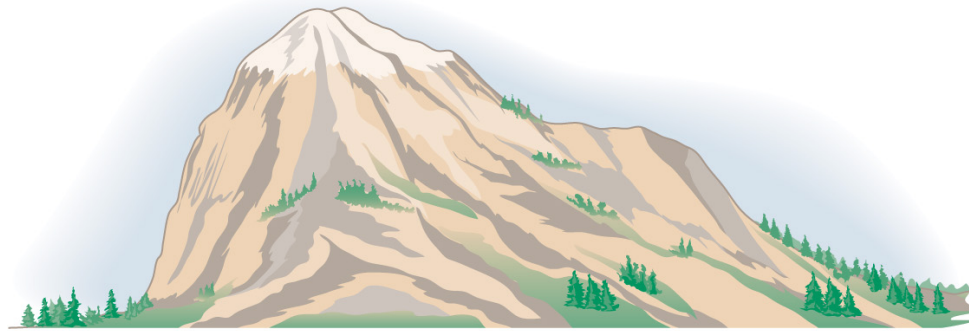
(a)



(b)

8-5 Potential Energy Curves and Equipotentials

- Contour maps are also a form of potential energy curve:



Example

A car of mass 800.0 kg is moving with a speed of 50.0 m/s at the top of a hill of 200 m height. What is the total energy of the car at the top of the hill?

- A) $16.0 \times 10^3 \text{ J}$
- B) $20.2 \times 10^3 \text{ J}$
- C) $2.57 \times 10^6 \text{ J}$
- D) $17.6 \times 10^4 \text{ J}$
- E) $8.80 \times 10^6 \text{ J}$

Example

Under the same conditions, what is the speed of the car after it rolled down to the bottom of the hill?

- A) 66.3 m/s
- B) 20.0 m/s
- C) 46.9 m/s
- D) 80.2 m/s
- E) There is not enough information to solve this problem

Example

A 5.00-kg block is moving along a horizontal frictionless surface toward an ideal massless spring with a spring constant of 45 N/m that is attached to a wall. After the block collides with the spring, the spring is compressed a maximum distance of 0.68 m. How fast was the block?

- A) 1.0 m/s
- B) 2.0 m/s
- C) 3.0 m/s
- D) 4.0 m/s
- E) 5.0 m/s

Example

An object of mass 2.00 kg starts at rest from the top of a rough inclined plane of height 20.0 m . If the work done by the force of friction is -150 J , what is the speed of the object as it reaches the bottom of the inclined plane?

- A) 15.6 m/s
- B) 19.8 m/s
- C) 12.2 m/s
- D) 23.3 m/s
- E) 30.4 m/s

Example

A car of mass 1000.0 kg is moving with an initial speed of 20.0 m/s at the highest point of a 100 m long ramp inclined at 20 degrees to the horizontal. What is the speed of the car when it reaches the bottom of the ramp?

- A) 66.3 m/s
- B) 32.7 m/s
- C) 46.9 m/s
- D) 17.6 m/s
- E) 20.0 m/s

Example

If 3.8 J of work is done in raising a 179 g apple, how far is it lifted?

- A) 2.17 m
- B) 21.3 m
- C) 0.37 m
- D) 37.3 m
- E) 0.68 m

Summary of Chapter 8

- Conservative forces conserve mechanical energy
- Nonconservative forces convert mechanical energy into other forms
- Conservative force does zero work on any closed path
- Work done by a conservative force is independent of path
- Conservative forces: gravity, spring

Summary of Chapter 8

- Work done by nonconservative force on closed path is not zero, and depends on the path
- Nonconservative forces: friction, air resistance, tension
- Energy in the form of potential energy can be converted to kinetic or other forms
- Work done by a conservative force is the negative of the change in the potential energy
- Gravity: $U = mgy$
- Spring: $U = \frac{1}{2} kx^2$

Summary of Chapter 8

- Mechanical energy is the sum of the kinetic and potential energies; it is conserved only in systems with purely conservative forces
- Nonconservative forces change a system's mechanical energy
- Work done by nonconservative forces equals change in a system's mechanical energy
- Potential energy curve: U vs. position