

Solutions to practice exam solved on the blackboard in class.

1. The car slows down all the time because the slope of the x vs. t graph is diminishing as time goes on. Remember that the slope of x vs. t is the velocity! At large t , the value of the position x does not change, indicating that the car must be at rest.

Answer: B.

2.

Displacement = final position – initial position → Since the runner runs in a circle, the displacement after one lap is =0.

The distance is the length of the circle of radius $r=60$ m: $L=2\pi r = 377$ m.

Answer: A.

3.

One dimensional motion with constant acceleration.

I use the formula: $x=x_0+v_0t+1/2a*t^2$

$x-x_0$ is the displacement.

$$x-x_0=(5.0\text{m/s})*(6.0\text{s})+1/2*(2\text{m/s}^2)*(6.0\text{s})^2=66\text{m}.$$

Answer: C.

4.

One-dimensional motion in the vertical direction: the rock is subject to an acceleration = -g (I choose the positive y direction pointing upwards and the origin at the point from where the rock starts).

I use the formula: $v^2=v_0^2-2g\Delta y$.

The maximum height is the one at which the velocity of the rock is =0.

I replace $v=0$ in the formula and find:

$$0= v_0^2-2g\Delta y \rightarrow \Delta y=v_0^2/2g=30.6 \text{ m}$$

Answer: D.

5.

I use the formula: $x=x_0+v_0t+1/2a*t^2$.

v_0 is zero because the bird starts from rest.

I solve the equation for the acceleration and I get:

$$a = 2(x - x_0)/t^2.$$

$x - x_0$ is the displacement, which is =28 m.

I therefore get: $a = 0.46 \text{ m/s}^2$.

Answer B.

6.

The motion of the stream of water is the motion of a projectile with a launch angle of 30° above horizontal.

We choose the coordinate system with the origin at the firehose.

We get: $v_{0x} = v_0 \cos 30^\circ = 34.6 \text{ m/s}$; $v_{0y} = v_0 \sin 30^\circ = 20.0 \text{ m/s}$

The motion in the horizontal direction is a motion with constant speed.

The time it takes the water to hit the building is the time it takes the water to cover the horizontal distance between the firehose and the building:

$$t = x/v_{0x} = 50\text{m}/(34.6\text{m/s}) = 1.44\text{s}.$$

I use this information to determine the vertical position of the water making use of the equation:

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \text{ in which I use: } y_0 = 0; v_{0y} = 20.0 \text{ m/s}; a_y = -9.81 \text{ m/s}^2; t = 1.44\text{s}.$$

$$\text{I solve per } y \text{ and I get: } y = 20.0\text{m/s} \cdot 1.44\text{s} - \frac{1}{2} \cdot 9.81\text{m/s}^2 \cdot (1.44\text{s})^2 = 18.7 \text{ m}$$

Answer: C

7.

To calculate the height of the table, we don't need the information about the horizontal initial velocity.

We use the formula:

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 \text{ in which we use: } y = 0; v_{0y} = 0; a_y = -9.81 \text{ m/s}^2; t = 0.3\text{s}$$

$$\text{We solve for } y_0: y_0 = \frac{1}{2}g \cdot t^2 = 0.44 \text{ m}.$$

Answer: D

8.

Answer: D: it is a typical projectile motion, in which the ball experiences a constant, vertical acceleration $= -g$.

9.

When the wheel leaves the helicopter, its velocity only has a horizontal component, which is the velocity of the helicopter: 40 m/s.

When the wheel hits the ground, its velocity has both horizontal and vertical components. The horizontal one stays constant.

The vertical one can be calculated from the formula (motion with constant acceleration):

$$v_y^2 = v_{0y}^2 - 2g\Delta y.$$

Where $\Delta y = y_f - y_i = -100$ m and $v_{0y} = 0$

We get: $v_y = 44.3$ m/s

From that we find: $v^2 = v_x^2 + v_y^2 \quad \rightarrow \quad v = 60$ m/s

Answer: C.

10.

The initial velocity of the stone has both horizontal and vertical components.

$$v_{0x} = v_0 \cos 53^\circ \quad v_{0y} = v_0 \sin 53^\circ$$

The horizontal one stays constant throughout the trajectory.

The vertical component, as the stone hits the ground, is given by:

$$v_y^2 = v_{0y}^2 - 2g\Delta y$$

Where $\Delta y = y_f - y_i = -35$ m (I set the origin of the axes on the ground at the bottom of the building).

We get: $v_y = 28.8$ m/s

From which we can get the speed (magnitude of the velocity):

$$v^2 = v_x^2 + v_y^2 \quad \rightarrow \quad v = 30$$
 m/s

Answer: C.

11.

Since the horizontal motion has constant velocity, the horizontal displacement of the box is simply given by:

$$x = v_{0x}t.$$

I need to find the time t , which is the time that the box takes to reach the ground.

We can calculate it with the formula:

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

where:

$$y_0 = 500 \text{ m}; \quad y = 0; \quad v_{0y} = 0; \quad a_y = -g = -9.81 \text{ m/s}^2.$$

The equation reduces to:

$$0 = 500 \text{ m} - 4.9 \text{ m/s}^2 t^2 \rightarrow t = 10.1 \text{ s}.$$

$$\text{From } x = v_{0x}t \text{ we get: } x = 2.02 \cdot 10^3 \text{ m}.$$

Answer: A.

12.

I choose to have \mathbf{F}_1 along x and \mathbf{F}_2 along y (the two forces have a right angle between them, the most convenient coordinate system choice is to have them along the x and y axes).

From Newton's law we have:

$$\sum \mathbf{F} = m\mathbf{a} \rightarrow \mathbf{a} = \mathbf{F}_1/m + \mathbf{F}_2/m = (25\text{N}/40\text{kg})\mathbf{x} + (40\text{N}/40\text{kg})\mathbf{y}.$$

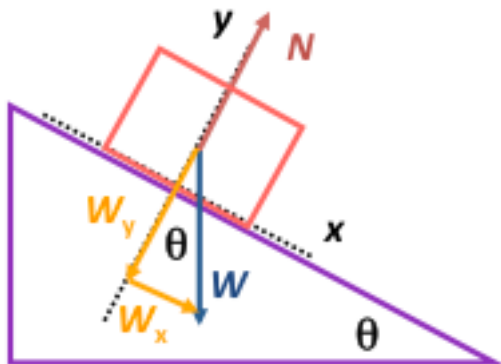
The magnitude of the acceleration is:

$$a^2 = a_x^2 + a_y^2 \rightarrow a = 1.2 \text{ m/s}^2$$

Answer: A.

13.

We draw the free body diagram.



We choose the coordinate system with the x axis parallel to the slope and the y axis perpendicular to the slope.

We apply Newton's law ($\theta=30^\circ$):

$$\Sigma F_x = ma_x \rightarrow mg \sin \theta = ma_x \rightarrow a_x = g \sin \theta = 4.9 \text{ m/s}^2.$$

The skier moves with constant acceleration parallel to the slope.

To calculate the distance, we use the formula:

$$v_x^2 = v_{0x}^2 + 2a_x \Delta x$$

$$\text{Where } v_x = 56 \text{ m/s, } v_{0x} = 0, \quad a_x = 4.9 \text{ m/s}^2$$

We get: $\Delta x = 320 \text{ m}$.

Answer: C.

14.

The system is in equilibrium: all horizontal and vertical components of the forces balance each other.

In the vertical direction I have:

$$\Sigma F_y = 0 \rightarrow -F_1 + F_3 \sin 30^\circ = 0 \rightarrow F_3 \sin 30^\circ = F_1 \rightarrow F_3 = F_1 / \sin 30^\circ = 160 \text{ N}.$$

Answer: A.