# Freeze-out prescription for critical fluctuations

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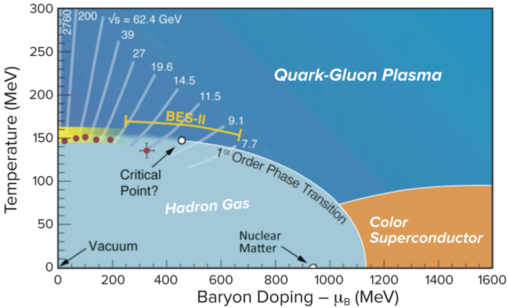
UH-UIUC Journal Club,29th June 2020

# Dynamics of QGP near the critical point




- HIC: Strongly interacting QGP formed
- Large $\sqrt{s_{NN}}$ :QGP is well described by hydro and hadrons by thermal distribution
- Signatures of CP (static): $\kappa_2 \sim \xi^2$ , $\kappa_3 \sim \xi^{9/2}$ , $\kappa_4 \sim \xi^7$ Stephanov, 08'
- Near CP, hydro breaks down due to emergence of new slow modes

# Hydro+ fluctuations before freeze-out

- Critical slowing down : Fluctuations lag behind their equilibrium values as the fluid expands and cools Berdnikov, Rajagopal , 99'
- Near QCD CP : Hydro+ identifies the slowest set of modes Stephanov,Yin, 2017

$$\phi_Q(x) \quad \sim \quad \int e^{iQ\Delta x} \left\langle \delta \frac{s}{n}(x_+) \delta \frac{s}{n}(x_-) \right\rangle , \, x_\pm = x \pm \frac{\Delta x}{2}$$

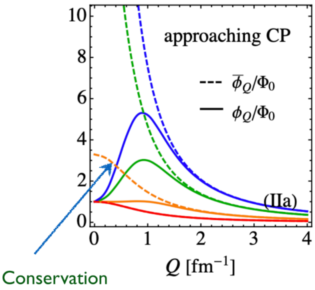
- Hydro+ provides a framework for simultaneous evolution of average conserved densities and set of slow mode

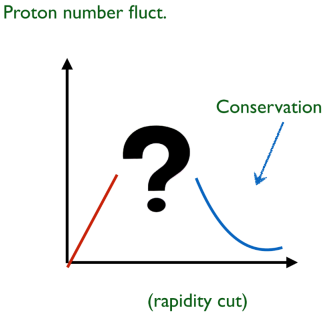
$$\partial_\mu T^{\mu\nu} = 0 \, , \quad \partial_\mu J^\mu = 0 \, , \quad u \cdot \partial \phi_Q = -\Gamma(Q\xi) \left( \phi_Q - \bar{\phi}_Q \right)$$

- Hydro+ simulations in simplified settings Rajagopal et al., 2019, Du et al., 2020
- Next step: To freeze-out these fluctuations

# Overview of the talk

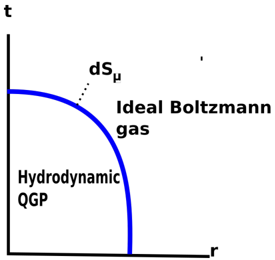
- Motivate the prescription we use to freeze-out hydro+ fluctuations
- Develop explicit example with the Rajagopal et al., 19 set up
- Qualitative predictions for experiment
- Summarize this work in progress

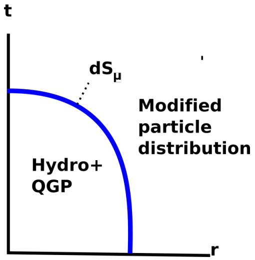



Borrowed from Yi Yin's talk at the INT workshop , 2020

# Cooper-Frye freeze out




$$\langle f_A(x,p)\rangle = e^{-\frac{E_A(x,p)-\mu}{T}}$$

$$f_A(x,p) = \langle f_A(x,p)\rangle + \underbrace{\delta f_A(x,p)}_{\text{critical fluctuations}}$$

$$N_A = \int dS_\mu \int Dp\, p^\mu\, f_A(x,p)$$

# Critical fluctuations in particle multiplicity

- We incorporate the effects of critical fluctuations via the modification of particle masses due to their interaction with the critical sigma field

$$\delta m_A \approx g_A \sigma$$

- Modified particle distribution function:

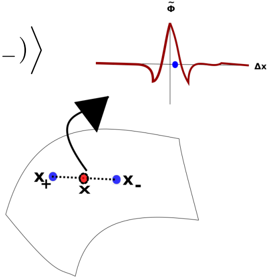
$$f_A = \langle f_A \rangle + g_A \, \sigma \, \frac{\partial \langle f_A \rangle}{\partial m_A}$$

- $\sigma$ field correlations in equilibrium:

$$\langle \sigma \rangle \quad = \quad 0 \, , \quad \langle \sigma(x_+) \sigma(x_-) \rangle = \frac{T e^{-\frac{|\Delta x|}{\xi}}}{4\pi |\Delta x|}$$

# Prescription for freeze-out of critical fluctuations

$$\tilde{\phi}(x,\Delta x) = \int_Q e^{iQ\Delta x}\phi_Q(x) \sim \left\langle \delta\frac{s}{n}(x_+)\delta\frac{s}{n}(x_-)\right\rangle$$




$$\langle\sigma(x_+)\sigma(x_-)\rangle = Z\,\tilde{\phi}(x,\Delta x)$$

Z is chosen such that $\langle\sigma\sigma\rangle$ reduces to the equilibrium expression on the previous slide

$$\left\langle \delta N_A^2\right\rangle_\sigma = g_A^2 Z \int dS_\mu J_A^\mu(x_+) \int dS'_\nu J_A^\nu(x_-)\tilde{\phi}(x,\Delta x)$$

$$J_A^\mu = d_A \int Dp\, p^\mu \,\frac{\partial\langle f_A\rangle}{\partial m_A}$$

Quantity of interest $\Rightarrow$ $\dfrac{\left\langle \delta N_A^2\right\rangle_\sigma}{g_A^2\,\langle N_A\rangle}$

# Project Overview: Freeze-out of two systems near the critical point

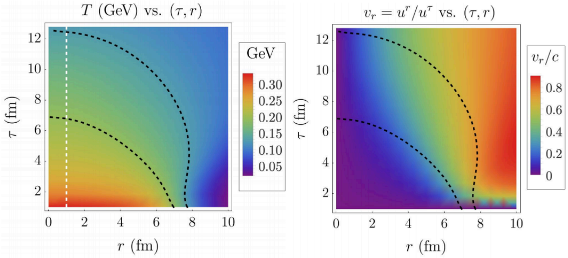
$$u \cdot \partial \phi_Q = -\Gamma(Q)(\phi_Q - \bar{\phi}_Q)$$

| Rajagopal et al., 2019 | A more realistic scenario |
|---|---|
| Order parameter non-conserved | Order parameter conserved |
| Model A | Model H |
| $\Gamma(Q) \propto \xi^{-2} + O(Q^2)$ | $\Gamma(Q) \propto O(Q^2)$ |

# Hydro+ in a simplified setting - Initial conditions

Rajagopal, Ridgway, Weller, Yin, 2019

- Model intitalized at $\tau = \tau_I = 1\,\text{fm}$ with and $T_I = 330\,\text{MeV}$
- $v_r = \Pi^{\mu\nu} = 0$ at $\tau = \tau_I$
- $\epsilon(r)$ at $\tau = \tau_I$ given by Glauber model for $\sqrt{s} = 200\,\text{GeV}$ Au-Au central collision
- $\phi_Q(\tau_I) = \bar{\phi}_Q(T(\tau_I))$



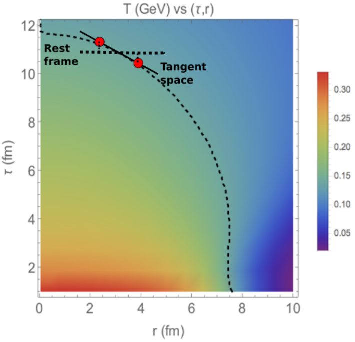
# Hydro+ in a simplified setting

Rajagopal, Ridgway, Weller, Yin, 2019

- Flow: Boost invariant azimuthally symmetric
- Freeze-out condition: $T(x) = 0.14\text{GeV}$ ($T_c \sim 0.16\,\text{GeV}$)

$$\left\langle \delta N_A^2 \right\rangle_\sigma = g_A^2\, Z \int d^3x_+ \int d^3x_-\, I_A(x,\Delta x)\,\tilde{\phi}(x,\Delta x)$$

$$\approx g_A^2\, Z \int d^3x \int d^3\Delta x\, I_A(x,\Delta x)\,\tilde{\phi}(x,\Delta\tilde{x})$$




$$\mathbf{I_A(x,\Delta x) = n(x_+)\cdot J_A(x_+)\, n(x_-)\cdot J_A(x_-)}$$

$$\mathbf{n\cdot J_A = d_A \int Dp\, \frac{\partial \left\langle f_A \right\rangle}{\partial m_A}\, n\cdot p}$$

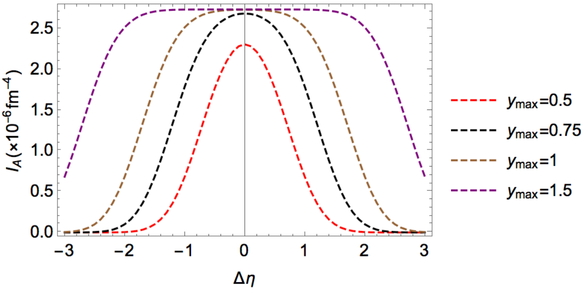
$\Delta\tilde{x}$ is the spatial projection of $\Delta x$ on the local rest frame at $x$.

# $\Delta x$ dependence of $I_A$ $\boxed{\int d^3\Delta x\, I_A(x,\Delta x)\,\tilde{\phi}(x,\Delta\tilde{x})}$

- No spatial dependence when integrated over full phase space

$$n\cdot J_A = \frac{d_A m_A}{T_f}\int Dp\, e^{-\frac{u\cdot p}{T}}\,\frac{n\cdot p}{u\cdot p}$$

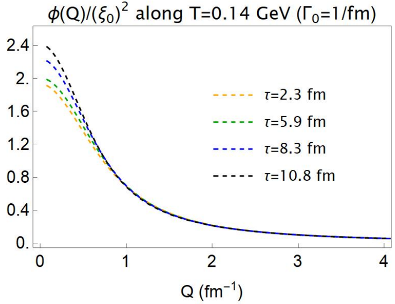
$$\left\langle \delta n_A^2\right\rangle = g_A^2\, Z\, I_A(x)\,\phi_0(x)$$

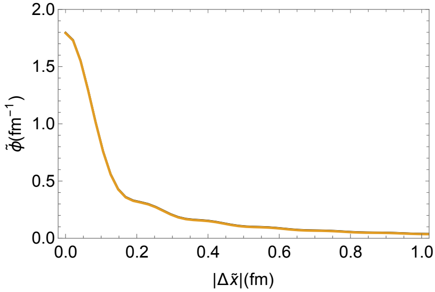



$$\Delta\eta \sim 4\sqrt{T/m_A}\, y_{\rm max}$$

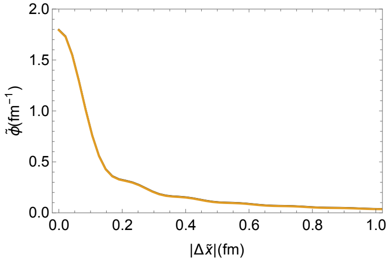
Rapidity cuts in the lab frame $\Leftrightarrow$ Cuts in spatial rapidity on the FHS

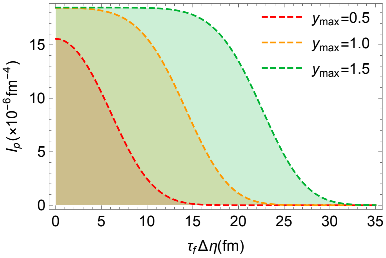
# Output from Hydro+ simulation

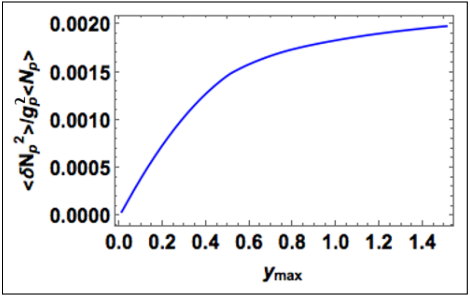


# Critical proton multiplicity fluctuations in Model A



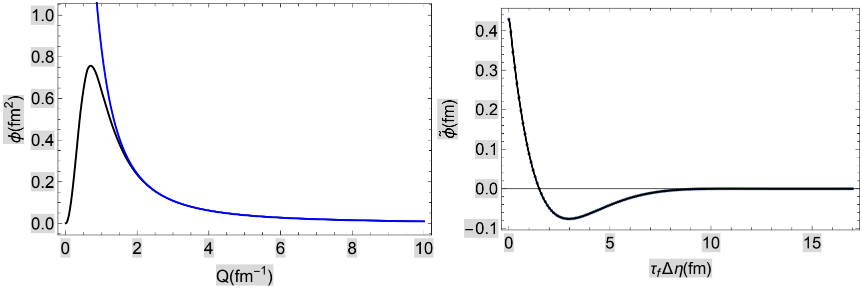




$$\left\langle \delta N_A^2 \right\rangle g_A^{-2} \approx Z \int I_A(x, \Delta x)\, \tilde{\phi}(x, \Delta\tilde{x})$$

Model H calculation for the same set up is in progress

# Model H dynamics in an analytically solvable Bjorken model

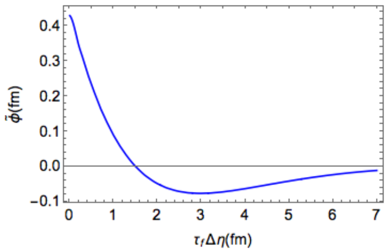


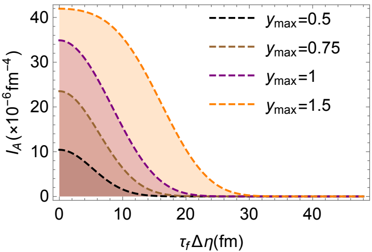

$$\frac{d\phi_Q}{d\tau} = -\Gamma(Q)(\phi_Q - \bar{\phi}_Q)\,; \quad \phi_Q(\tau_0) = 0$$

$$\Gamma(Q) = \Gamma_0 \left(\frac{\xi_0}{\xi}\right)^3 K(Q\xi)\,, \quad \Gamma_0 = 1\,\mathrm{fm}^{-1}\,, \quad \xi_0 = 1\mathrm{fm}$$

$$\bar{\phi}_Q = \frac{\bar{\phi}_0}{1 + (Q\xi)^2}$$

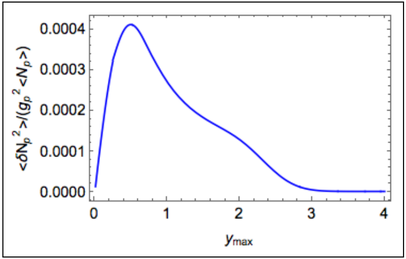
# Model H: Effects of conservation-1




$$\frac{\left\langle \delta N_A^2 \right\rangle}{g_A^2 \left\langle N_A \right\rangle} = \frac{\tau_f \int d\eta \int d\Delta\eta \, I_A(\eta, \Delta\eta)\, \tilde{\phi}(\eta, \Delta\eta)}{\int d\eta \int Dp\, f(\eta, p)}$$

- At $y_{\max} << 1$, $\dfrac{\left\langle \delta N_A^2 \right\rangle}{\left\langle N_A \right\rangle} \propto y_{\max}$
- As $y_{\max} \to \infty$, the effect of charge conservation takes over $\left\langle \delta N_A^2 \right\rangle \approx 0$



# Summary and ongoing work

- Demonstrated the freeze-out of Hydro+ fluctuations in a simplified setting Rajagopal et. al, 19.
- The procedure could be extended to more realistic scenarios
- Prediction for a non-monotonic behavior of $\left\langle \delta N_p^2 \right\rangle / (g_p^2 \left\langle N_p \right\rangle)$ with acceptance
  - Robustness of the location of the peak to changes in $\Gamma_0$, size of the critical region and initial conditions to be studied
- Numerical simulation for Hydro+ with Model H dynamics in progress
- The contribution of resonances to the fluctuations of observed particles needs to be investigated
- The procedure should be extended for higher cumulants

Thank you for your attention