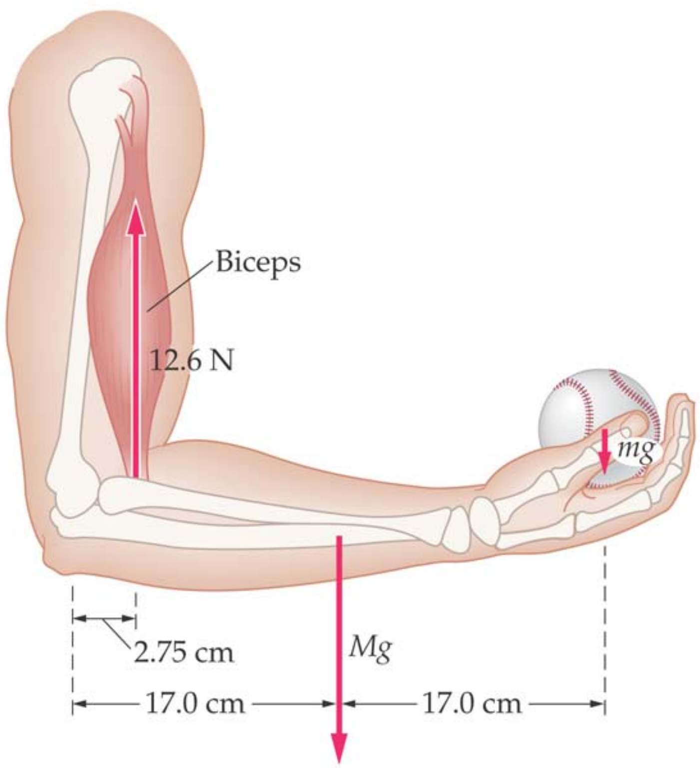
Chapter 11

Rotational Dynamics and Static Equilibrium

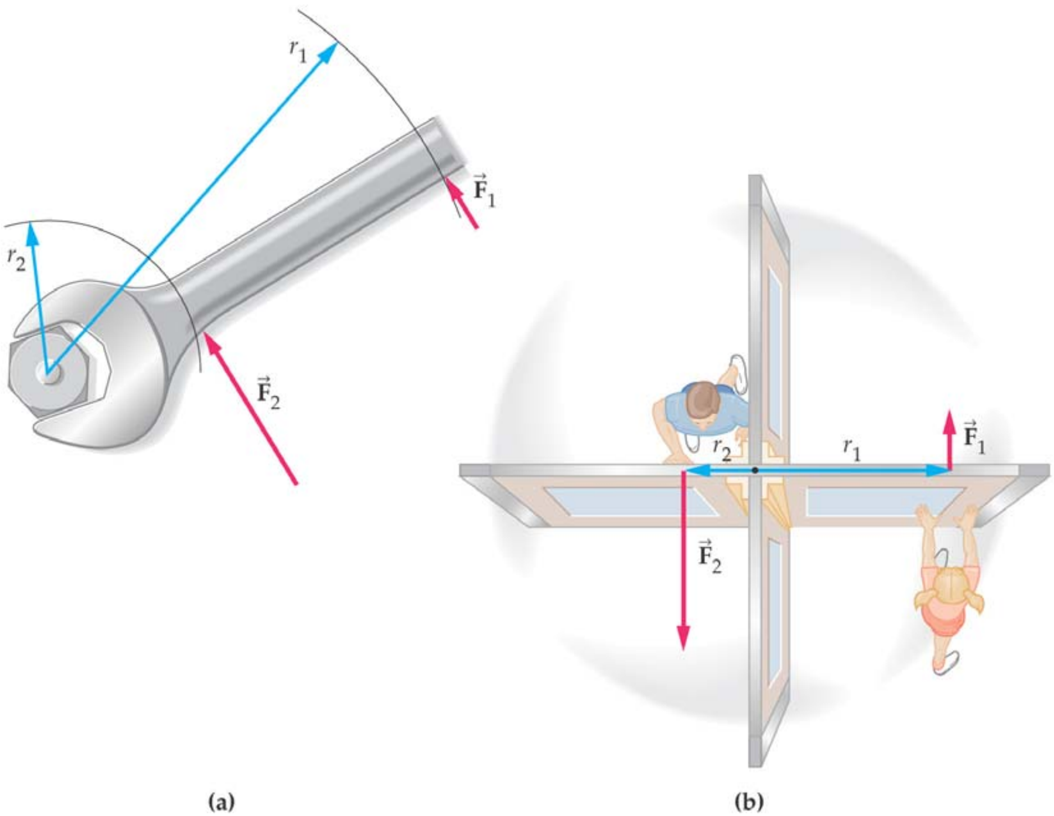


Units of Chapter 11

- Torque
- Torque and Angular Acceleration
- Zero Torque and Static Equilibrium
- Center of Mass and Balance
- Dynamic Applications of Torque
- Angular Momentum
- Conservation of Angular Momentum
- Rotational Work and Power
- The Vector Nature of Rotational Motion

11-1 Torque

From experience, we know that the same force will be much more effective at rotating an object such as a nut or a door if our hand is not too close to the axis.



This is why we have long-handled wrenches, and why doorknobs are not next to hinges.

11-1 Torque

We define a quantity called torque:

Definition of Torque, τ, for a Tangential Force

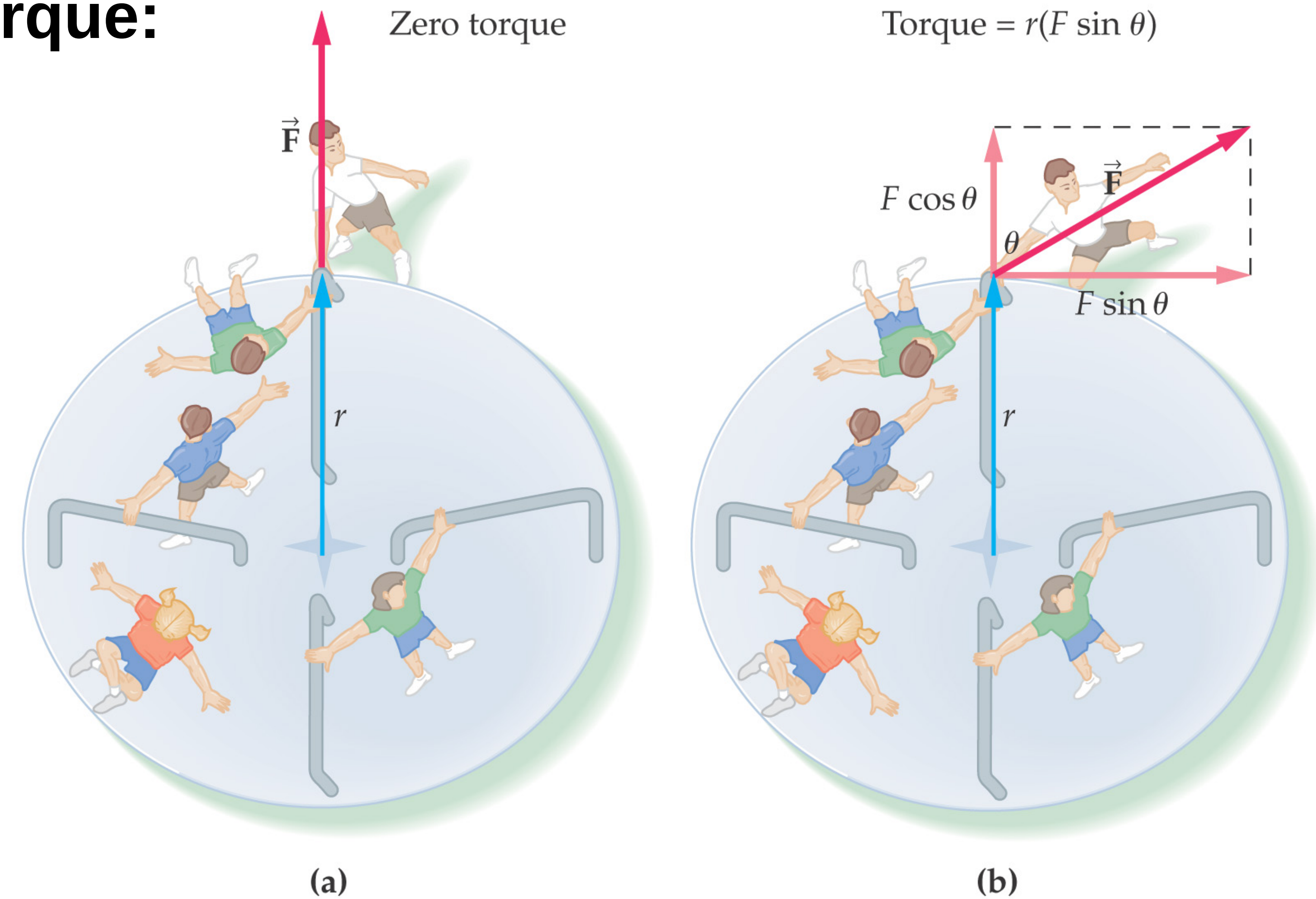
$$\tau = rF$$

SI unit: $\mathrm{N \cdot m}$

The torque increases as the force increases, and also as the distance increases.

11-1 Torque

Only the tangential component of force causes a torque:



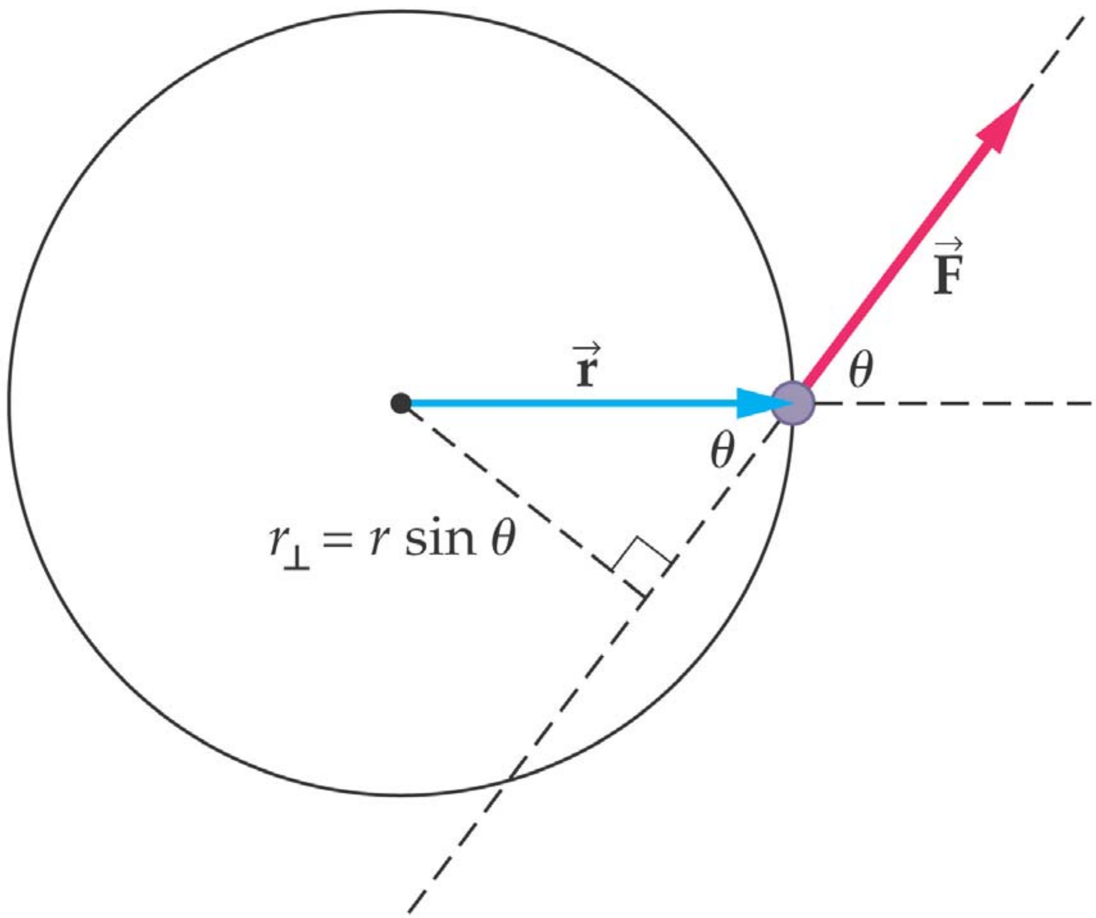
11-1 Torque

This leads to a more general definition of torque:

General Definition of Torque, τ

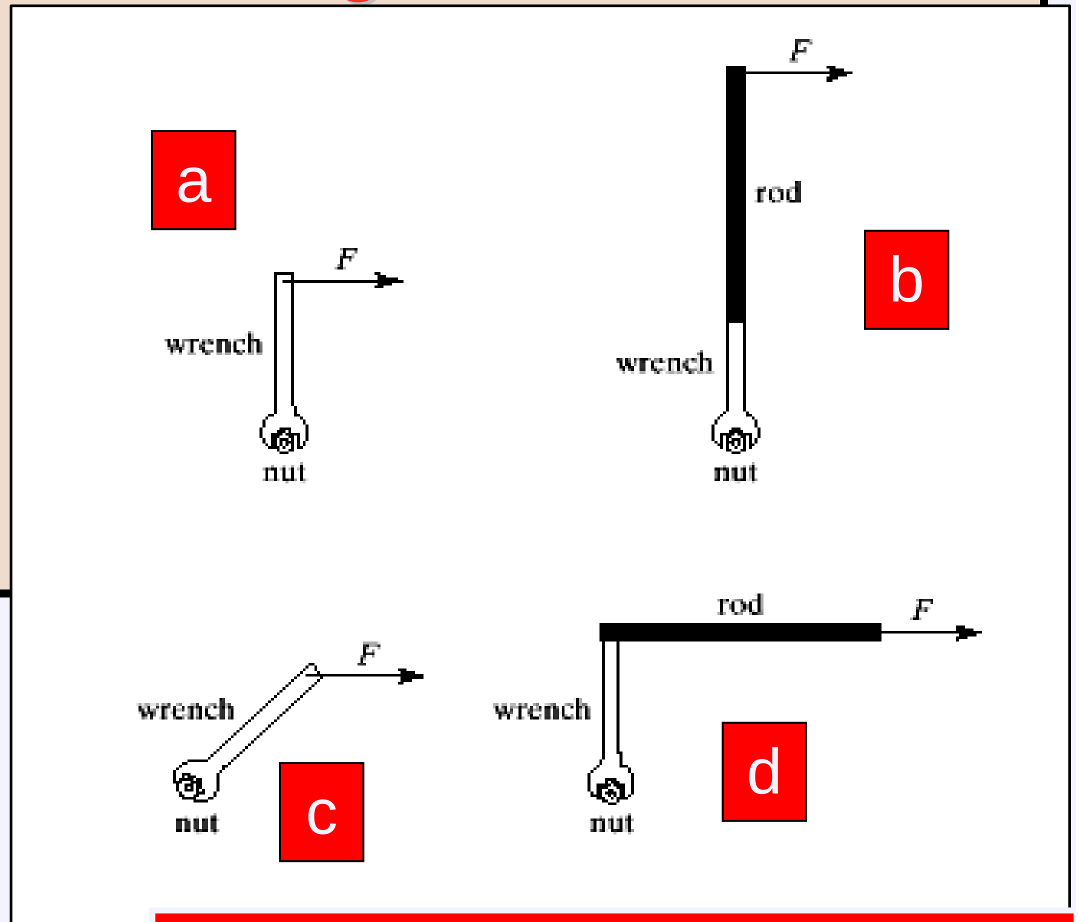
$$\tau = r(F \sin\theta)$$

SI units: $N \cdot m$



Question 11.1 Using a Wrench

You are using a wrench to loosen a rusty nut. Which arrangement will be the most effective in loosening the nut?



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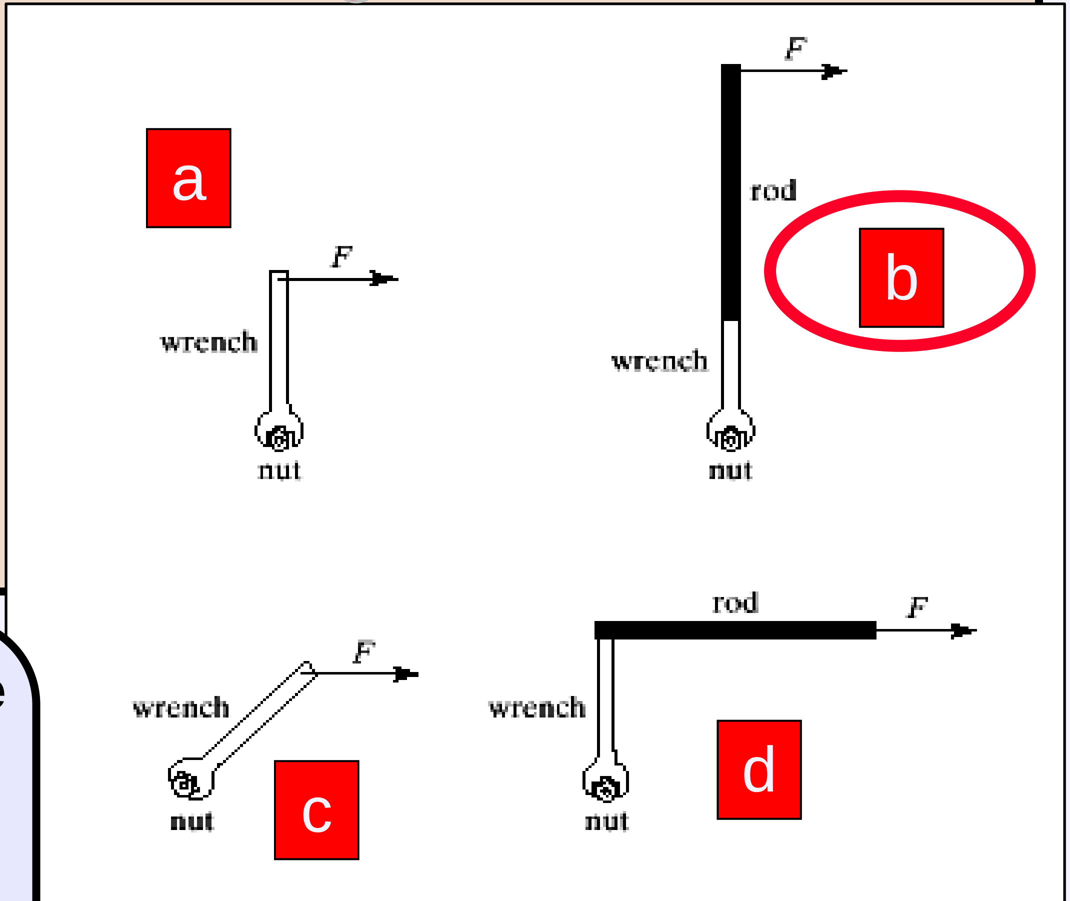

e) all are equally effective

Because the forces are all the same, the only difference is the lever arm. The arrangement with the largest lever arm (case #2) will provide the largest torque.

Follow-up: What is the difference between arrangement 1 and 4?

Question 11.2 Two Forces

Two forces produce the same torque. Does it follow that they have the same magnitude?

a) yes

b) no

c) depends

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b) no

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Because torque is the product of force times distance, two different forces that act at different distances could still give the same torque.

Follow-up: If two torques are identical, does that mean their forces are identical as well?

Question 11.3 Closing a Door

In which of the cases shown below is the torque provided by the applied force about the rotation axis biggest? For all cases the magnitude of the applied force is the same.

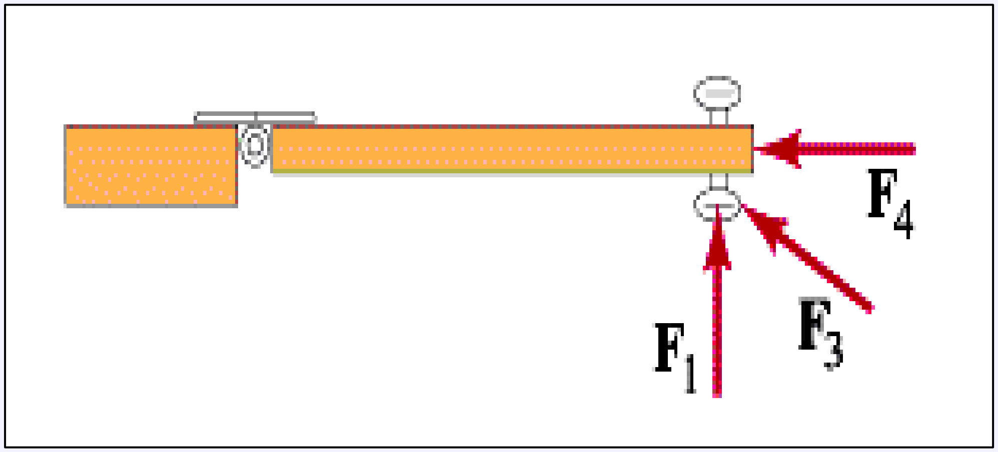
a) F_1

b) F_3

c) F_4

d) all of them

e) none of them



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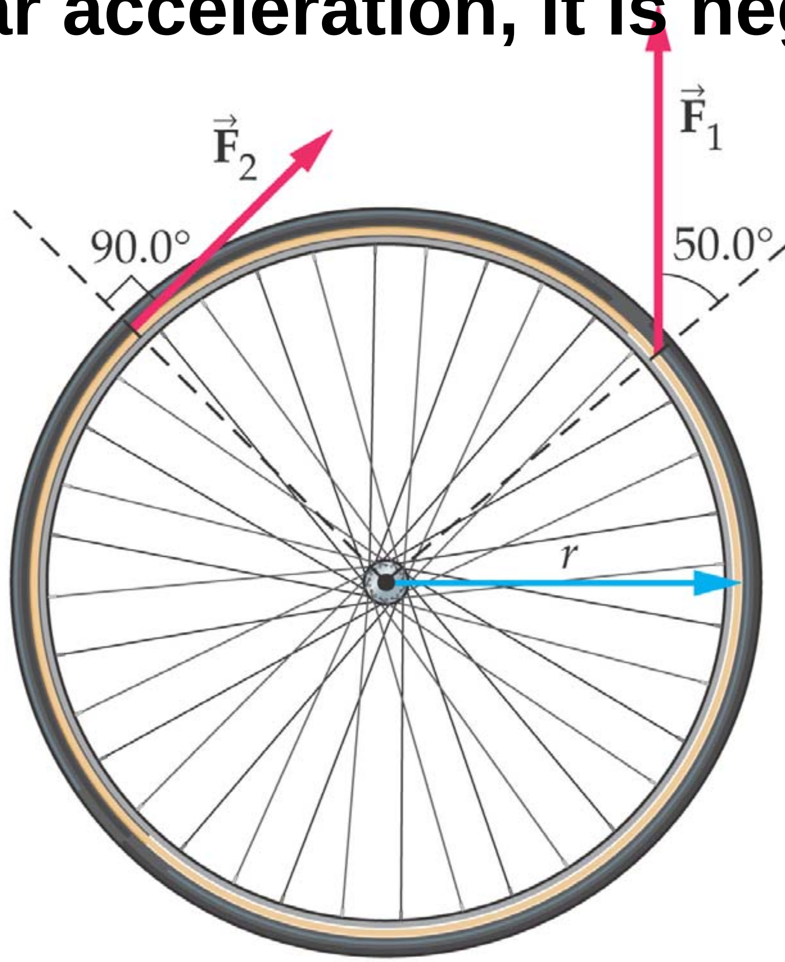
a) F_1
b) F_3
c) F_4
d) all of them
e) none of them

The torque is $\tau = rF\sin\phi$, and so the force that is at 90° to the lever arm is the one that will have the largest torque. Clearly, to close the door, you want to push perpendicularly!!

Follow-up: How large would the force have to be for F_4?

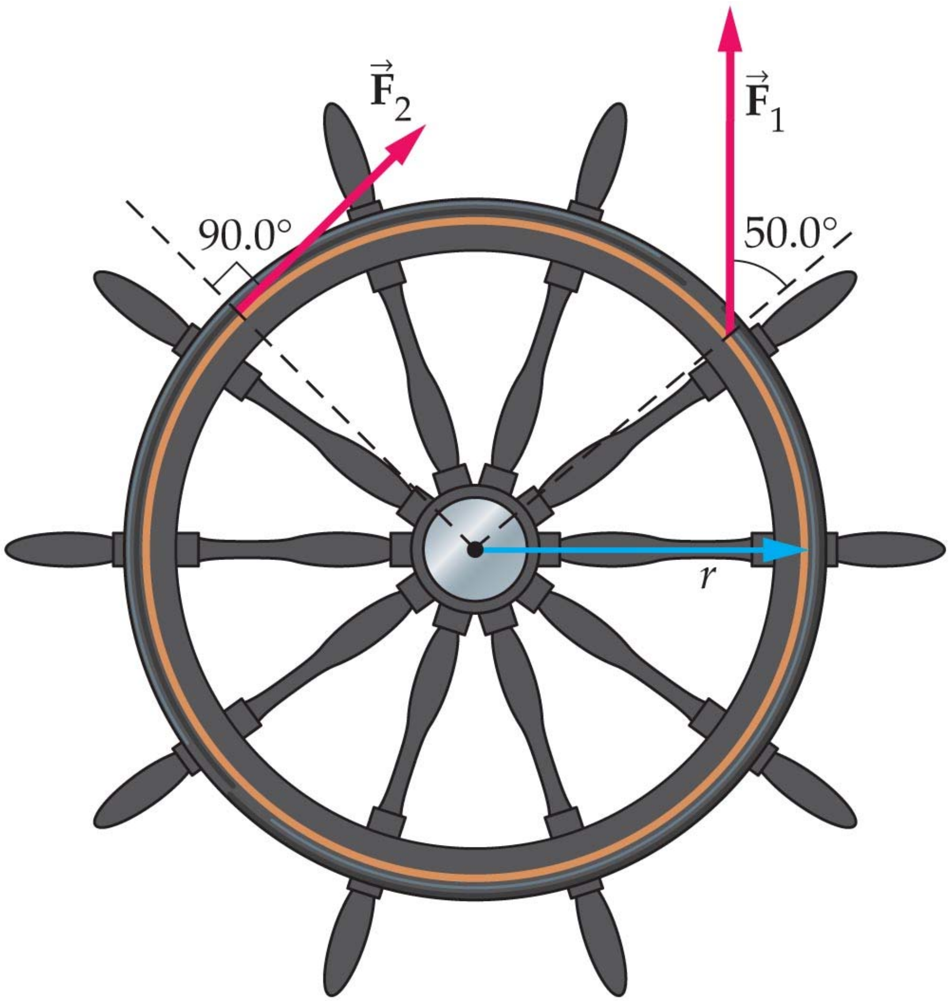
11-1 Torque

If the torque causes a counterclockwise angular acceleration, it is positive; if it causes a clockwise angular acceleration, it is negative.



EXAMPLE 11–1 TORQUES TO THE LEFT AND TORQUES TO THE RIGHT

Two helmsmen, in disagreement about which way to turn a ship, exert the forces shown below on a ship's wheel. The wheel has a radius of 0.74 m, and the two forces have the magnitudes $F_1 = 72$ N and $F_2 = 58$ N. Find **(a)** the torque caused by $\vec{\mathbf{F}}_1$ and **(b)** the torque caused by $\vec{\mathbf{F}}_2$. **(c)** In which direction does the wheel turn as a result of these two forces?



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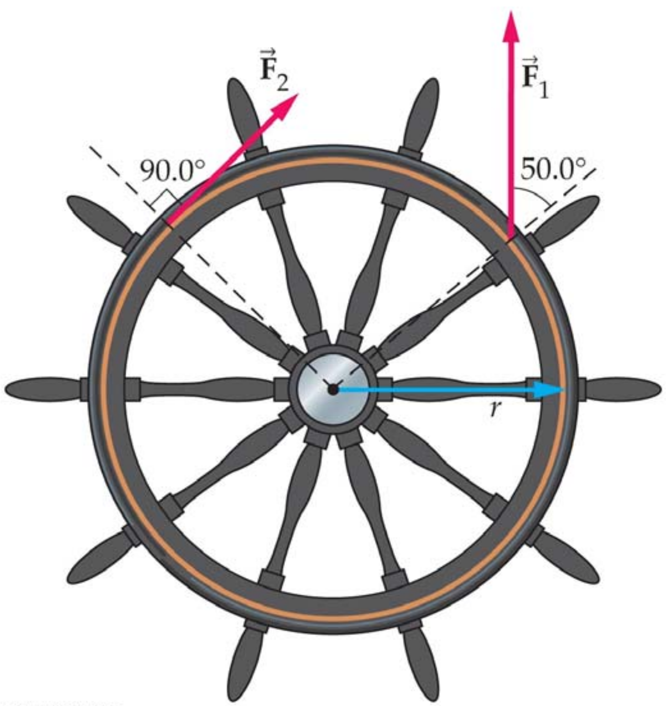
$$\tau_1 = rF_1 \sin 50.0° = (0.74\ \mathrm{m})(72\ \mathrm{N}) \sin 50.0° = 41\ \mathrm{N \cdot m}$$



$$\tau_2 = -rF_2 \sin 90.0° = -(0.74\ \mathrm{m})(58\ \mathrm{N}) = -43\ \mathrm{N \cdot m}$$

$$\tau_{\mathrm{net}} = \tau_1 + \tau_2 = 41\ \mathrm{N \cdot m} - 43\ \mathrm{N \cdot m} = -2\ \mathrm{N \cdot m}$$

11-2 Torque and Angular Acceleration

Newton's second law: $a = \frac{F}{m}$

If we consider a mass *m* rotating around an axis a distance *r* away, we can reformat Newton's second law to read:

$$a_t = r\alpha \longrightarrow \alpha = \frac{rF}{mr^2} = \frac{\tau}{I}$$

Or equivalently,

Newton's Second Law for Rotational Motion

$$\tau = I\alpha$$

11-2 Torque and Angular Acceleration

Once again, we have analogies between linear and angular motion:

Linear Quantity	Angular Quantity
m	I
a	α
F	τ

Newton's Second Law for Rotational *Motion*

$$\tau = I\alpha$$

EXERCISE 11–2

A light rope wrapped around a disk-shaped pulley is pulled tangentially with a force of 0.53 N. Find the angular acceleration of the pulley, given that its mass is 1.3 kg and its radius is 0.11 m.

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SOLUTION

The torque applied to the disk is

$$\tau = rF = (0.11\ \mathrm{m})(0.53\ \mathrm{N}) = 5.8 \times 10^{-2}\ \mathrm{N \cdot m}$$

Since the pulley is a disk, its moment of inertia is given by

$$I = \tfrac{1}{2}mr^2 = \tfrac{1}{2}(1.3\ \mathrm{kg})(0.11\ \mathrm{m})^2 = 7.9 \times 10^{-3}\ \mathrm{kg \cdot m^2}$$

Thus, the angular acceleration of the pulley is

$$\alpha = \frac{\tau}{I} = \frac{5.8 \times 10^{-2}\ \mathrm{N \cdot m}}{7.9 \times 10^{-3}\ \mathrm{kg \cdot m^2}} = 7.3\ \mathrm{rad/s^2}$$

Summary of Chapter 11

- A force applied so as to cause an angular acceleration is said to exert a torque.
- Torque due to a tangential force: $\tau = rF$
- Torque in general: $\tau = rF \sin\theta$
- Newton's second law for rotation: $\tau = I\alpha$

11-3 Zero Torque and Static Equilibrium

Static equilibrium occurs when an object is at rest – neither rotating nor translating.

Conditions for Static Equilibrium

For an extended object to be in static equilibrium, the following two conditions must be met:

(i) The net force acting on the object must be zero,

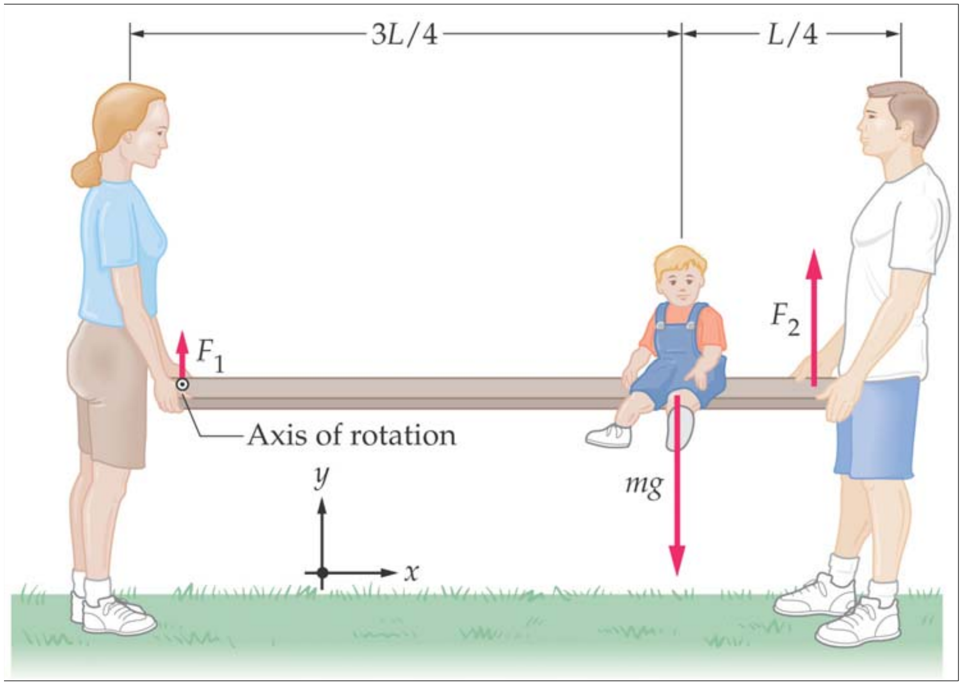
$$\sum F_x = 0, \sum F_y = 0 \qquad \textbf{11–5}$$

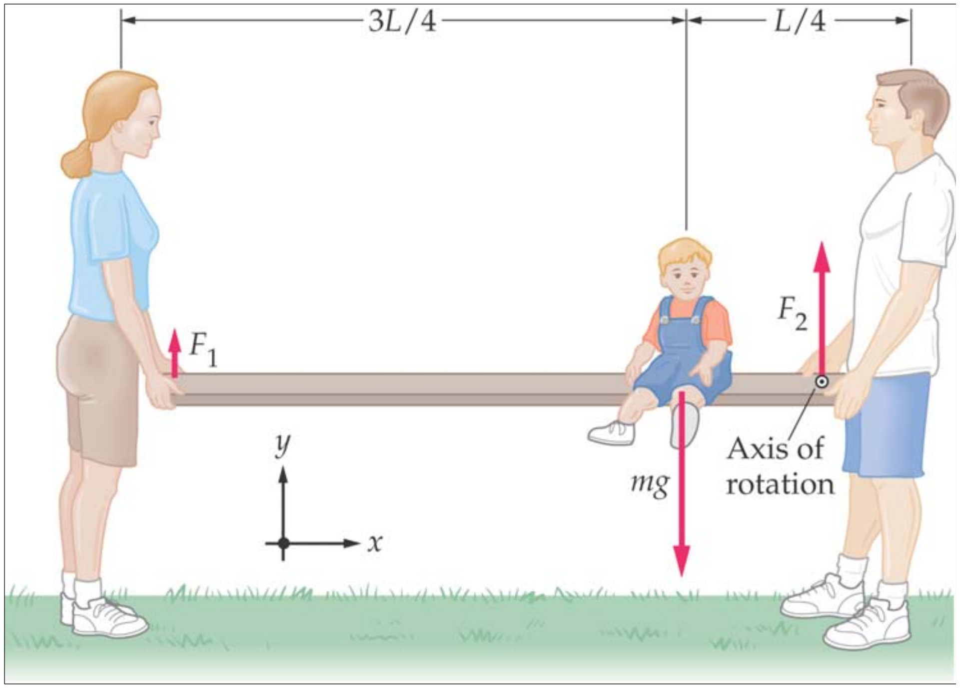
(ii) The net torque acting on the object must be zero,

$$\sum \tau = 0 \qquad \textbf{11–6}$$

11-3 Zero Torque and Static Equilibrium

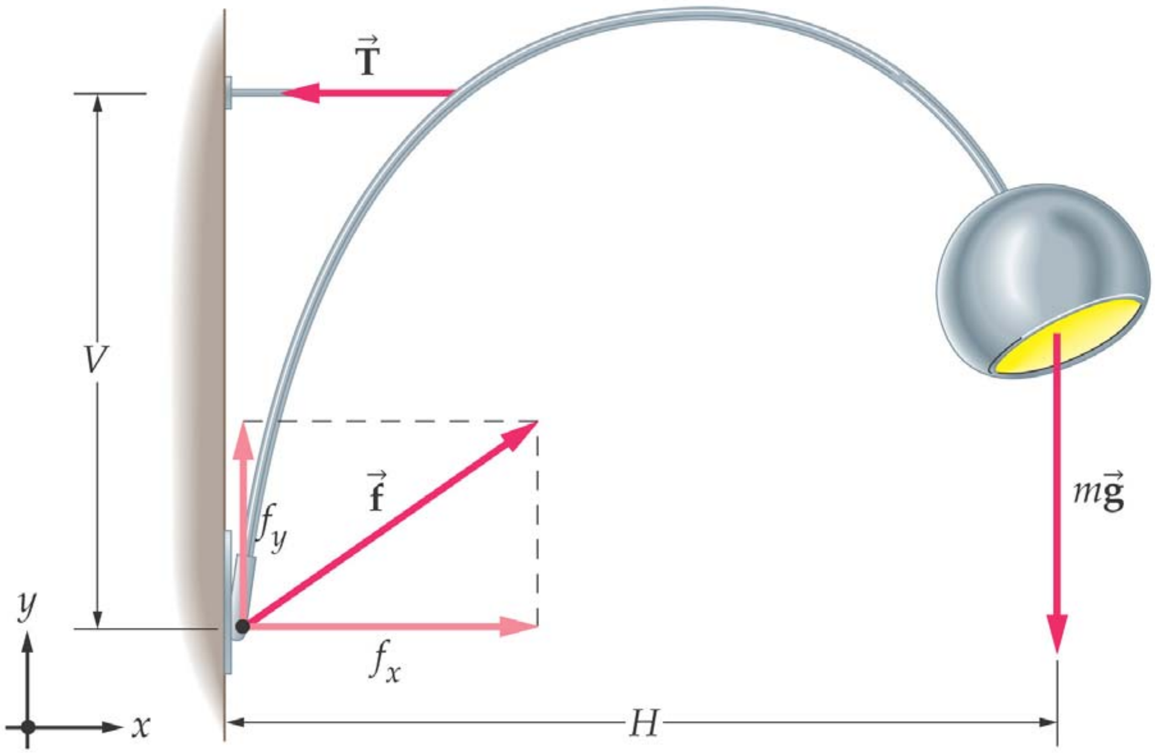
If the net torque is zero, it doesn't matter which axis we consider rotation to be around; we are free to choose the one that makes our calculations easiest.





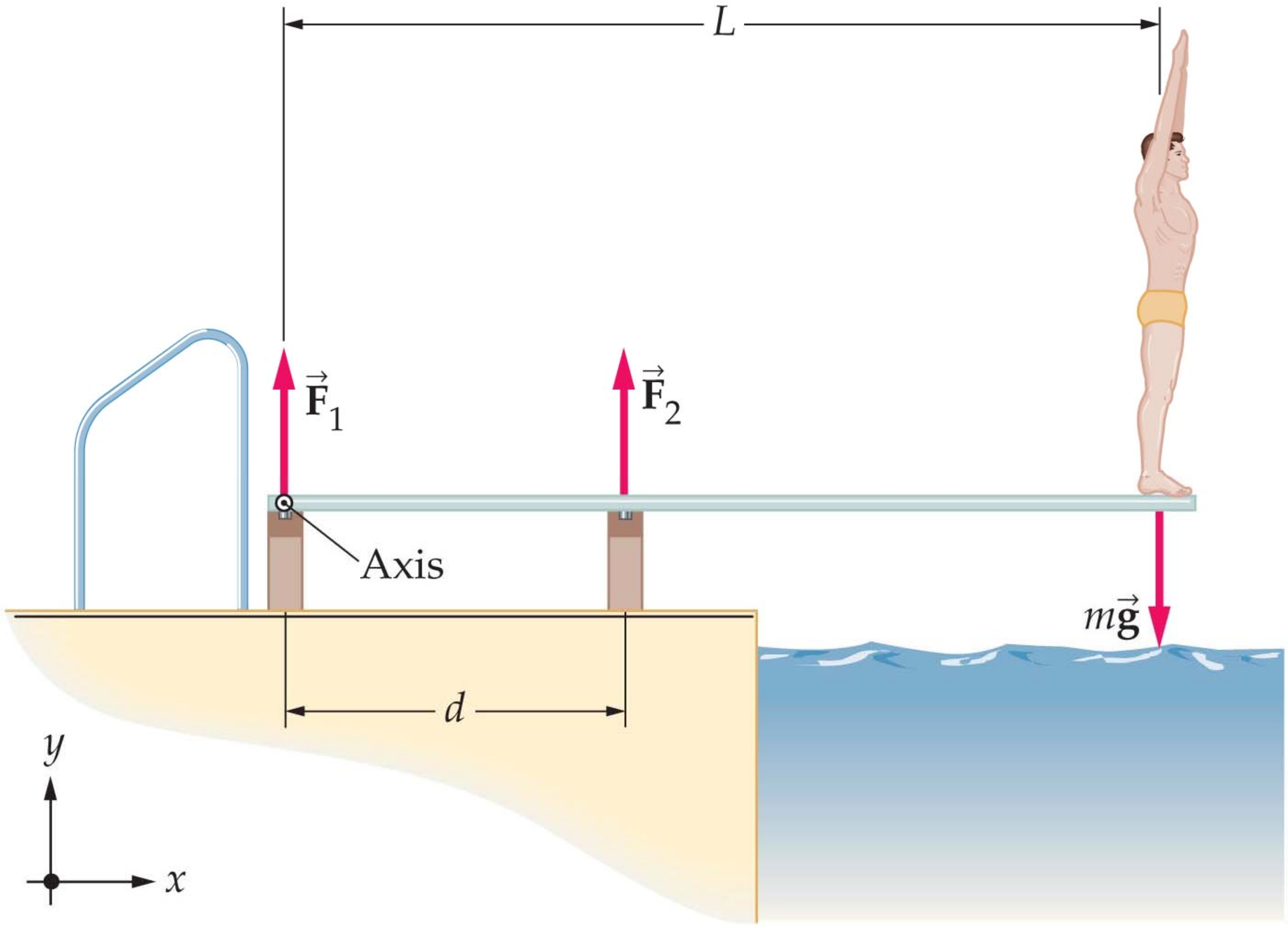
11-3 Zero Torque and Static Equilibrium

When forces have both vertical and horizontal components, in order to be in equilibrium an object must have no net torque, and no net force in either the *x*- or *y*-direction.



EXAMPLE 11–4 TAKING THE PLUNGE

A 5.00-m-long diving board of negligible mass is supported by two pillars. One pillar is at the left end of the diving board, as shown below; the other is 1.50 m away. Find the forces exerted by the pillars when a 90.0-kg diver stands at the far end of the board.



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1. Set the net y component of force acting on the diving board equal to zero:

$$\sum F_y = F_{1,y} + F_{2,y} - mg = 0$$

2. Calculate the torque due to each force, using the left end of the board as the axis of rotation. Note that each force is at right angles to the radius and that $\vec{\mathbf{F}}_1$ goes directly through the axis of rotation:

$$\tau_1 = F_{1,y}(0) = 0$$

$$\tau_2 = F_{2,y}(d)$$

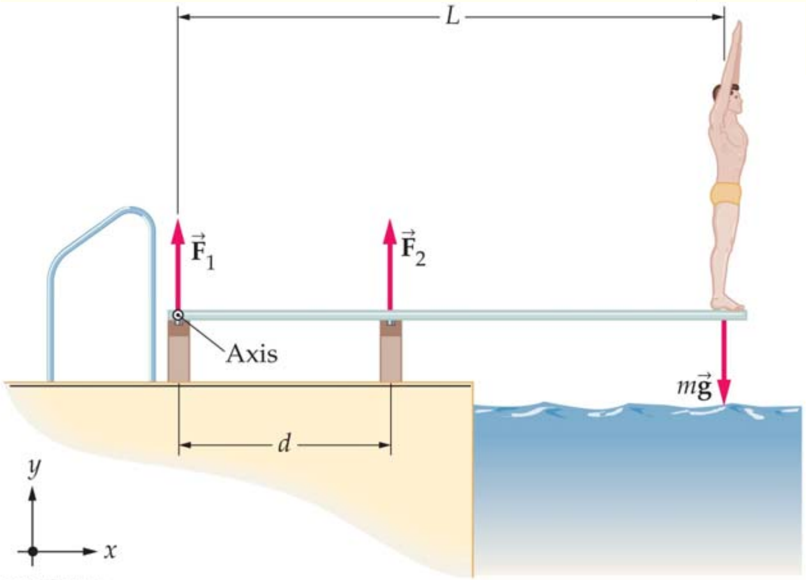
$$\tau_3 = -mg(L)$$

3. Set the net torque acting on the diving board equal to zero:

$$\sum \tau = F_{1,y}(0) + F_{2,y}(d) - mg(L) = 0$$

4. Solve the torque equation for the force $F_{2,y}$:

$$F_{2,y} = mg(L/d)$$
$$= (90.0\ \mathrm{kg})(9.81\ \mathrm{m/s^2})(5.00\ \mathrm{m}/1.50\ \mathrm{m}) = 2940\ \mathrm{N}$$



$$F_{1,y} = mg - F_{2,y}$$
$$= (90.0\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) - 2940\ \mathrm{N} = -2060\ \mathrm{N}$$