**Cosmology (Cont.)**
Lecture 21

# 1 Critical Density

The mass density of the universe determines whether the universe is closed or open. The baryonic density component has been experimentally determined to be $\rho(t_p) = 2 \times 10^{-29}\, g/cm^3$. So that the Robertson-Walker metric with $k \neq 0$ when inserted into Einstein's field equations, results in the Friedmann equation which was introduced in the last lecture. The critical density as a function of time is;

$$\rho_c(t) = \frac{3}{8\pi G}\,(\frac{\dot{\Lambda}}{\Lambda})^2 = \frac{3}{8\pi G}\,h^2(t)$$

Now suppose we write $\Omega(t) = \rho(t)/\rho_c(t)$, and substitute into the Friedmann equation;

$$\dot{\Lambda}^2 - \frac{8\pi G\rho}{3}\,\Lambda^2 = -kc^2$$

To obtain the expression;

$$\Omega(t) - 1 = \frac{kc^2}{\dot{\Lambda}^2}$$

When $\Omega(t) = 1$ then $k = 0$. In this case space is flat as may be seen from the equations in the last lecture. There it was pointed out that the curviture of space is related to the current ratio of the density to the critical density. The Cosmological Constant, $\Lambda$, in the Einstein field equation is called Dark Energy. The matter required to make $\Omega = 1$ is called Dark Matter. The universe appears to be spatially flat and homogeneous. Thus experimentally $\Omega \approx 1$. From the Friedmann equation;

$$\Omega(t) - 1 = \frac{kc^2}{\dot{\Lambda}^2} \approx \frac{9kc^2(t - t_0)^{2/3}}{4\gamma^{2/3}}$$

Note that $\Omega(t)$ diverges as $t \to \infty$. To get $\Omega(t) \approx 1$, all parameters must be fine tuned in the distant past. Inflation stretches space and partially solves this problem, making $\Omega - 1 \to 0$

# 2 Dark Mater

The energy budget in the universe is shown in the pie graph. Only about 4% of the total energy in the universe is directly observed, figure 1. About 22% is composed of dark matter and 74% is dark energy. As previously seen, the CMB requires a dark matter component in

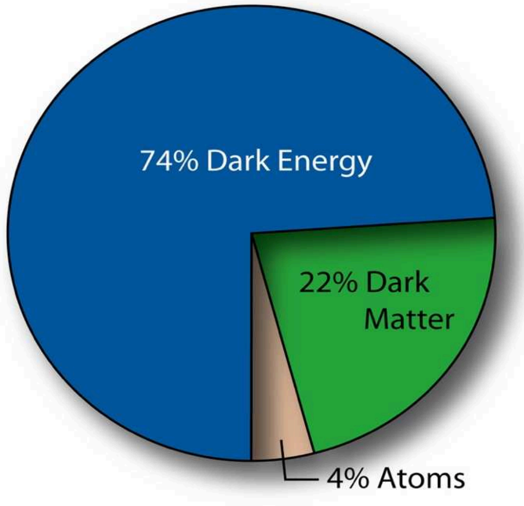



Figure 1: The energy budget in the universe

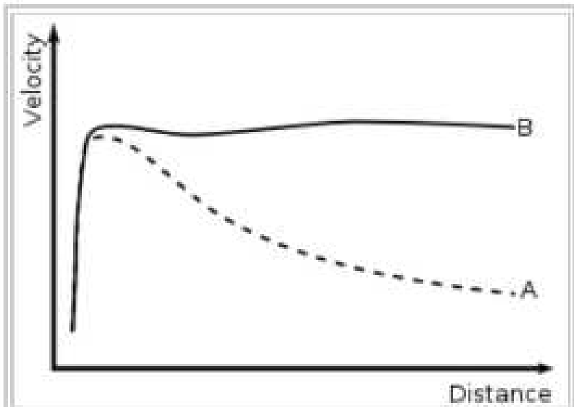
order to fit the anisotropy observations. However, there are other observations which require additional an mass which does not interact electromagnetically.

After the Big Bang, ordinary baryonic matter had too high a temperature and too much pressure to form structures. Thus dark matter acted as the framework around which galaxies formed. This is evident from the time evolution figure of the CMB shown in the last lecture. To produce the observed structure, a cold dark matter component is needed which is visible at present only by its gravitational interaction. From the evolution of the CMB, as previously seen, neutrinos - at least the conventional neutrons - cannot account for the required matter.

Non baryonic dark matter is divided into three types;

- Hot - ultra-relativistic
- Warm - relativistic
- Cold - non-relativistic

Hot dark matter is identified with neutrinos as their low mass allows high velocities. These neutrinos remain relativistic until approximately the time when electrons and protons combine to atoms. Warm dark matter may be due to as yet undiscovered heavy neutrinos. Cold dark matter may exist as tiny black holes, axions, very heavy neutrinos, or WIMPS (Weakly Interacting Massive Particles). There is a possible supersymmetric particle, the neutralino, which is stable and sufficiently massive to be a WIMP. Experimental searches have been underway in the last few years in order to observe WIMPS which collide with detector nuclei




Rotation curve of a typical spiral galaxy: predicted (**A**) and observed (**B**). Dark matter can explain the velocity curve having a 'flat' appearance out to a large radius

Figure 2: The square of the velocity of stars predicted from the luminous matter and experiment.

as the earth moves through spacial halo of dark matter.

# 3 Experimental Evidence

An example of evidence for Dark Matter is the motion of stars in the spiral arm of a galaxy. Most stars in spiral galaxies orbit the galactic center with an approximately uniform velocity. This implies that the mass in the galaxy is uniform well beyond the location of the stars, or that Newtonian gravity does not apply on large scales. The figure below shows the observed and predicted velocities of stars as a function of the distance from the radial center of the galaxy, figure 2 .

Very simply, a mass, m, outside a spherically symmetric mass distribution, $M$ , has a centrifugal acceleration, $V^2/r$, which equals the gravitational acceleration, $MG/r^2$. Thus its velocity squared equals $GM/r$. More generally we can apply the virial theorem, to obtain the average velocity of the system.

Suppose there are N particles interacting pairwise. The particles have mass $m_k$ and are a distance, $r_k$, from the galactic center. As they move in orbit, the moment of inertia of the system is;

$$I = \sum_{k=1}^{N} m_k r_k^2$$

Then define a function, $H$ such that;

$$H = \sum \vec{p}_k \cdot \vec{r}_k$$

Here $\vec{p}_k$ is the momentum of the $k^{th}$ particle. Therefore;

$$H = (1/2)\frac{dI}{dt} = \sum m_k \frac{d\vec{r}_k}{dt} \cdot \vec{r}_k$$

Apply the time derivative of $H$;

$$\frac{dH}{dt} = \sum \vec{p}_k \cdot \frac{d\vec{r}_k}{dt} + \sum \frac{d\vec{p}_k}{dt} \cdot \vec{r}_k$$

$$\frac{dH}{dt} = \sum m_k \frac{\vec{r}_k}{dt} \cdot \frac{d\vec{r}_k}{dt} + \sum \vec{F}_k \cdot \vec{r}_k$$

Where $F_k$ is the force on particle $k$. Then;

$$\frac{dH}{dt} = 2T + \sum \vec{F}_k \cdot \vec{r}_k$$

In the above $T$ is the kinetic energy. Now for 2-body forces;

$$\vec{F}_k = \sum_j \vec{F}_{kj}$$

so that;

$$\sum_k \vec{F}_k \cdot \vec{r}_k = \sum_{k\, j \neq k} \vec{F}_{kj} \cdot \vec{r}_k = \sum_{k\, j \neq k} \vec{F}_{kj} \cdot (\vec{r}_k - \vec{r}_j)$$

Since $\vec{F}_{kj} = -\vec{F}_{jk}$ we have;

$$\vec{F}_{kj} = -\vec{\nabla} V = \frac{dV}{dr} (\vec{r}_k - \vec{r}_j)/r_{kj}$$

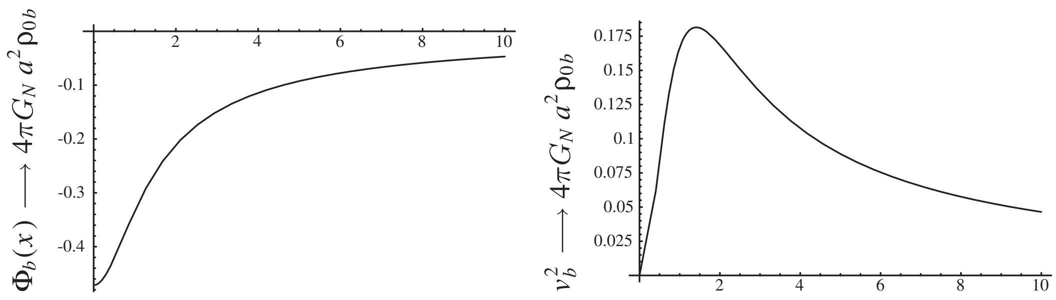
Substitution

$$\sum \vec{F}_k \cdot \vec{r}_k = -\sum_{k\, j \neq k} \frac{dV}{dr} (\vec{r}_k - \vec{r}_j)/r_{kj} = -\sum \frac{dV}{dr} r_{kj}$$

Finally;

$$\frac{dH}{dt} = 2T - \sum \frac{dV}{dr} r_{kj}$$

and if $V(r_{kj}) = \alpha / r_{kj}$ then;




(a) (b)

Figure 3: The mass density and calculated velocity distribution of stars in a galaxy

$$\sum \frac{dV}{dr} r_{kj} = - \sum_{kj \neq k} V(r_{kj}) = V_{TOT}$$

Here $V_{TOT}$ is the total potential energy of the system. For a gravitational field;

$$\frac{dH}{dt} = (1/2) \frac{d^2 I}{dt^2} = 2T + V_{TOT}$$

Average this over time. For a system in equilibrum $\langle \frac{dH}{dt} \rangle = 0$. Then

$$T = (1/2)[(1/2)MV^2] = (1/2)\langle V_{TOT} \rangle$$

The empirical mass density of a galaxy is;

$$\rho x = \rho_0 \frac{\sqrt{2}}{(1 - x^2)^{5/2}}$$

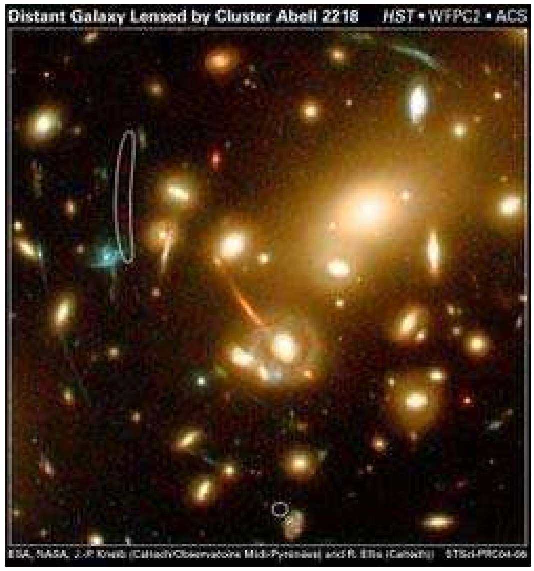
where $x = r/a$ and $a$ is the galactic mass. This can be modeled to give the velocity distribution as shown in figure 3.

Thus the gravitating mass must extend well beyond the star field. However, not all galaxies appear to have formed around dark matter. Our galaxy has a dark matter mass 10 times the luminous matter.

# 4 Gravitational lensing

Recently the observation of distant supernova has observed the effects of light bending around astrophysical objects in the foreground of the image. This bending can be used to obtain mass-to-light ratios. The required dark matter needed by these ratios corresponds to the

**Abell 2218**




Abell 2218. Credit: NASA/ESA

Figure 4: An example of gravitational lensing. The blue arcs in the picture are due to light that has been bent around a galaxy in the foreground

dark matter densities obtained by other observations. An image of gravitational lensing is shown in figure 4.

# 5 Other explainations of dark matter

It is possible that Newtonian gravity is not correct at large distances, but the effects of lensing and light deflection are difficult to reconcile with alternative theories. Other ideas involve modifications to gravity due to string theory and extra dimensions. However, these are not sufficiently developed for further discussion.

# 6 Dark energy - introduction

The cosmological constant was proposed by Einstein in order to obtain a static solution to his field equations. However, it was found that any local fluctuations would lead to instabilities as the system was in an unstable equilibrum. Thus a fluctuation which caused a contraction would create a continued contraction, while an expansion would lead to a continued expansion. For this reason the cosmological constant was ignored.

In 1970 Guth proposed that negative pressure could drive an inflation, and this was needed to explain cosmological features as previously discussed. However, inflation requires a much larger energy density than presently allowed, and in any event a connection between dark energy and inflation has not been made.

Accelerated expansion, as observed by the redshift of distant supernova, means that an additional mass-energy component of the universe is needed. This placed the cosmological constant on a firmer foundation. We have also seen that the CMB requires dark energy in order to fit the experimental data. Dark energy can be explained by the Cosmological Constant, and the present standard model of cosmology is $\Lambda$-CDM, where the $\Lambda$ is the cosmological constant.

# 7 Models of dark energy

There are two proposed models of dark energy;

- The cosmological constant
- A scalar field that dynamically alters space-time called quintessence

A scalar field can also have a constant term which cannot be distinguished from the effects of the cosmological constant, as it is equivalent to vacuum energy density.

However, the simplest explanation of dark energy is the negative energy expended in the creation of the virtual energy density of space. While field theories in particle physics require a vacuum energy density, this energy density is orders of magnitude larger than would be obtained from a cosmological constant - about $10^{-120}$ to 1. In order to make this explanation viable, extreme fine tuning of parameters is required to cancel terms. In some supersymmetric theories the cosmological constant is exactly zero. Philosophically, one has the somewhat unsatisfactory answer that although such fine tuning has a small probability of occurrence, it is not zero and it only needs to happen only once. Finally there are problems with singularities at low matter density which effect the time scales at early times, so perhaps the interpretation of the supernova data is incorrect.

Quintessence models assume the acceleration is due a dynamically changing potential energy, and so it can vary in space and time. It assumes a scalar potential which is at an unstable point at the time of the Big Bang. This scalar form moves toward a stable equilibrum releasing energy. This is illustrated in figure 5 and is comparable to spontaneous symmetry breaking. The Standard Model and String Model of particle physics predict scalar

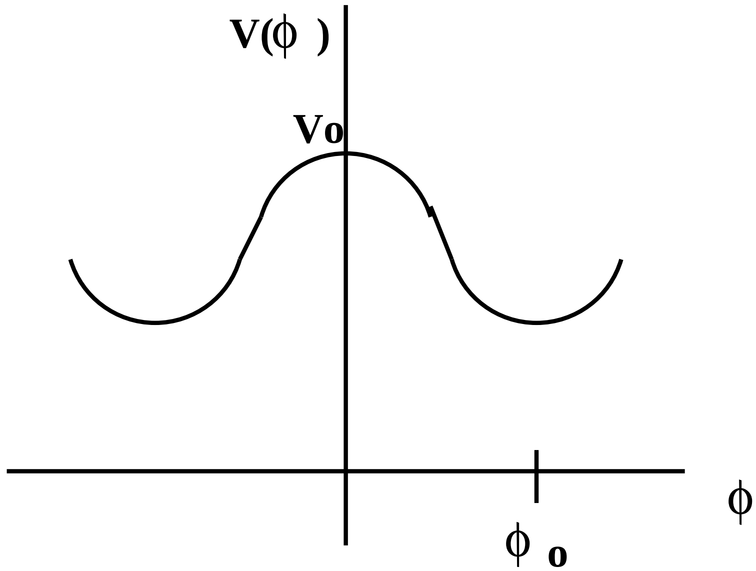



Figure 5: An example of gravitational lensing. The blue arcs in the picture are due to light that has been bent around a galaxy in the foreground

fields, but these fields have masses that are much too large. The scalar field must be coupled to the radiation density so that it becomes more important at later times in the evolution.