Rotating Dense Matter)

Lecture 24

1 Neutron Star Summary

Neutron stars are the remnants from a supernova explosion. When the radiative pressure which balances the gravitational force vanishes the nucleons begin to free fall reaching high velocities before impacting the star core. The compression upon impact can produce Temperatures of some 10^{12}K, but this temperature quickly cools through neutrino emission. The neutrinos are initially emitted in free neutron decay $n \rightarrow p + e^{-} + \bar{\nu}_e$. When the temperature drops below 10^9 K and there is an excess of neutrons, the catalytic process $n + n \rightarrow n + p + e^{-} + \bar{\nu}$ and $n + p + e^{-} \rightarrow n + n + \nu$ dissipates heat as T^8. This continues to cooling the star and extinguishes neutrino emission. "Standard cooling" of a neutron star begins with neutron decay, and then by the catalytic process described above. After these processes the star cools by neutrino pair bremsstrahlung, and finally by thermal photon emission. Thus a 1,000 year old neutron star (the Crab pulsar) still has a surface temperature of a few million degrees Kelvin.

The density at the center of a neutron star can be several times nuclear density. At these densities strange matter, pion condensates, lambda hyperons, delta isobars, or free quark matter might form. The star has little convection and the constituents can be layered in terms of a core, an intermediate structure, a crust, and an "atmosphere", figure 1. Neutrons may exist as a superfluid, and protons may form a superconductor in the interior, so detailed astronomical observation, including x-ray emission is needed. The layers are composed as follows.

- The outer crust with densities 10^6 tp 10^{11} g/cm^3 composed of solid heavy nuclei and unboud electrons

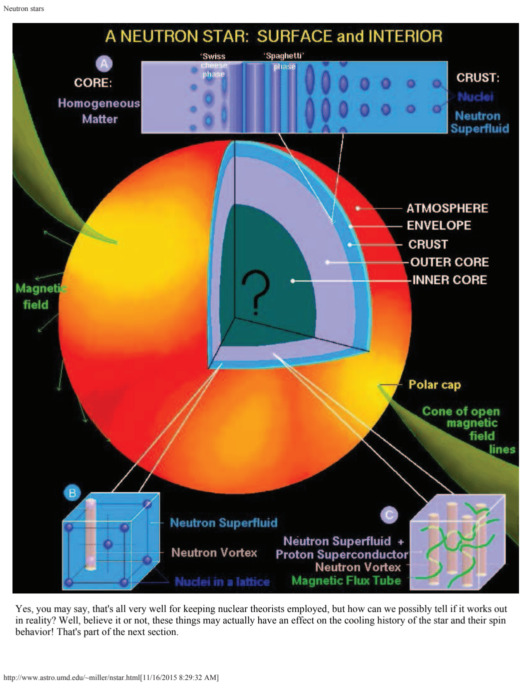
- The inner crust with densities 10^{11} to 10^{14} g/cm^3 composed of very heavy nuclei and freely moving neutrons in a superfluid state

- The outer core with densities 10^{14} to 10^{16} g/cm^3 composed of a liquid of mostly neutrons with a small fraction of protons and electrons.

- The inner core with densities $> 10^{15\, g/cm^3}$ composed of a liquid of unknown particles (neutrons, hyperons, bosons, etc.

the presence of protons and electrons in the outer core blocks neutron decay and establish beta equilibrium between neutron decay and electron capture on protons. The protons and electrons can produce currents creating magnetic fields, and since most quantum states are occupied, scattering to other states is impeded so resistivity is low. Currents can flow for long times and perhaps the protons form a superconducting medium.

Neutron stars have very strong magnetic fields and large angular momentum, so aside from the timing of the energy pulses, detaled observation of shifts in spectrum lines is complicated due to the influence of the magnetic field on the atomic structure. Therefore, the detailed composition of these stars is based on models and theory calibrated with date from observations in the environment of the Earth and not necesarily relevant to the star environment.


Neutron stars

Yes, you may say, that's all very well for keeping nuclear theorists employed, but how can we possibly tell if it works out in reality? Well, believe it or not, these things may actually have an effect on the cooling history of the star and their spin behavior! That's part of the next section.

http://www.astro.umd.edu/~miller/nstar.html[11/16/2015 8:29:32 AM]

Figure 1: A composite of a neutron star showing its layered structure

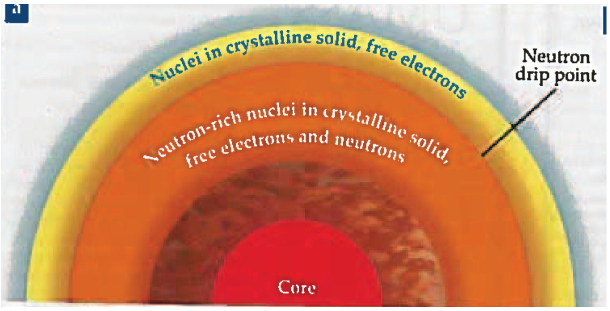


Figure 2: Detailed composition of a neutron star

2 The Kerr metric

Up to this point the Schwarzchild metric has need used. This metric is a true solution of the Einstein general relativisitic equations for the case of a static, fully-symmetric, uncharged system, in the vacuum of space with no gravitational sources. However, as discussed in the last section, most systems of interest have angular momentum. The Kerr metric describes space-time near an uncharged, but rotating black hole. Rotation is due to angular momntum conservation, as these systems obtain momentum fron accretion of matter, and as the star collapses it must spin faster to preserve its angular momentum. The insertion of charge adds additional energy and thus adds a term to the stress energy tensor. The metric for both charged static (Reissner-Nordstrom metric) and rotating (Kerr-Newman metric) systems has also been obtained, but will these not be discussed. The uncharged Kerr metric for a mass, M, and angular momentum, $J = aMc$, has the form;

$$ds^2 = \frac{\Delta - a^2\, sin^2(\theta)}{\rho^2}(c\,dt)^2 + \frac{4GMa}{c\rho^2} r sin^2(\theta)\, d\phi\, dt - (\rho^2/\Delta)\, dr^2 - \rho^2\, d\theta^2 - \frac{A\, sin^2(\theta)}{\phi^2}\, d\phi^2$$

Where;

$$\Delta = r^2 - \frac{2GM}{c^2} r + a^2$$
$$\rho^2 = r^2 + a^2\, sin^2(\theta)$$
$$A = (r^2 + a^2)^2 - a^2 \Delta\, sin^2(\theta)$$

The coordinates are the usual spherical system. When $a = 0$ the Schwarzchild metric is obtained. In order to compact the notation, use; $m = GM/c^2$, $s - c\tau$, and $\tau = ct$. The result is;

$$d\tau^2 = (\Delta/\rho)(dt - a\, sin^2(\theta)\, d\phi)^2 - (sin(\theta)/\rho)^2[(r^2 + a^2)d\phi - a]^2 - \rho^2/\Delta\, dr^2 - \rho^2(d\theta^2)$$

with;

$$\Delta = r^2 - 2mr + a^2$$

The Kerr metric is not diagonal so it is more difficult to use, for example its inverse (change from covarient to contrvarient form) requires some non trivial effort.

3 Frame Dragging

The Kerr metric can be written in terms of the metric components.

$$(c\,d\tau)^2 = (g_{tt} - g_{t\phi}^2/g_{\phi\phi})dt^2 + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}(d\phi + g_{t\phi}/g_{\phi\phi}dt)^2$$

The metric represents a co-rotating reference frame rotating with angular speed, Ω depending on r and θ.

$$\Omega = -g_{t\phi}/g_{\phi\phi}$$

Ω is called the Killing horizon. This frame is pulled to rotate with the central mass and is called frame dragging. Frame dragging is caused by the time dependent warp in space time and can be viewed in analogy to induction in the case of electromagnetism with the caveat that the induced forces act in the opposite direction to the electromagnetic interaction. Light bends in the rotating frame and can orbit the neutron star.

4 Ergosphere

There are 2 surfaces in the Kerr metric which exhibit singularities. The first is similar to the Schwarzschield singularity where the radial metric component, g_{rr}, is infinite. This occurs when Δ in the above vanishes. Thus the postive root is ;

$$R_1 = \frac{r_s + \sqrt{r_s^2 - 4\alpha^2}}{2}$$

The negative root is ignored as it falls with in the event horizon. The second singularity occurs when the rotational velocity exceeds c. This is found where the metric term, g_{tt} changes sign.

$$R_2 = \frac{r_s + \sqrt{r_s^2 - 4\alpha^2\, cos^2(\theta)}}{2}$$

The first surface is spherical, the second is an oblate spheriod. The space between these surfaces is called the ergosphere and is illustrated in figure 3.

The second surface lies outside the surface defined by R_1 and touches this inner surface at the poles. The volume enclosed by these surfaces is called the ergosphere. Within the ergosphere the space is dragged along the with the rotating black hole at the velocity of light. Particles falling into the ergosphere are accerated and gain energy, so they may escape the capture thus removing energy from the black hole. This is called the Penrose process. Matching the Kerr solution to the interior solution of a neutron star for example, has not been successful.

5 Magnetic field

The inferred magnetic fields in neutron stars can be as high as 10^{14} G. These fields appear to last for very long time scales so most likely are “frozen” in rather than generated by some mechanism. However the magnetic interaction, although strong, still is insufficient to effect the stellar structure. The simplest assumption is that there is a single, conducting fluid creating and interacting with the magnetic field. The collapse which occurs during formation leaves a convective, differntially rotating proto-star which settles to equilibrium

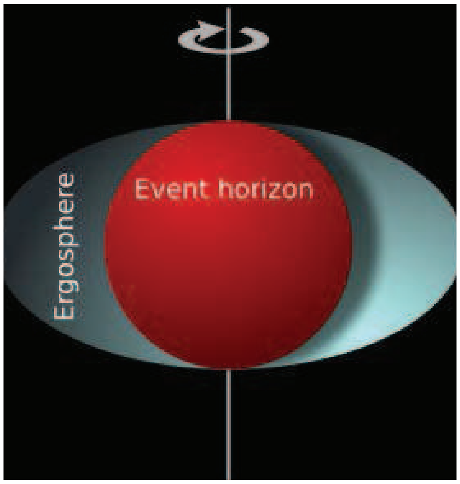


Figure 3: The singular surfaces on a rotating neutron star showing the event horizon and the ergosphere

with magneto-hydrodynamic magnetic fields. The crust becomes a solid, the neutrons a superfluid and the protons superconducting.

6 Pulsar

Pulsars are radiating neutron stars with precise emissions of X and gamma radiation with respect to time. They can be classified as;

- Rotation-powered pulsars - rotation energy provided the power for emission
- Accretion-powered pulsars - gravitational energy of accreted matter powers the emission
- Magnetars - the magnetic field provides the power for emission

X-ray pulsars probably are old rotationally powered objects which become visable when the binary companions expand and begin transferring matter to the neutron star, figure 4. This process produces weak power emission and is observed in the milisecond pulsars of globular clusters which stopped neutron star formation 10^9 years ago. Emission is presumed tied to the star's magnetic field and its rotation. An example of this radiation is shown in figure 5.

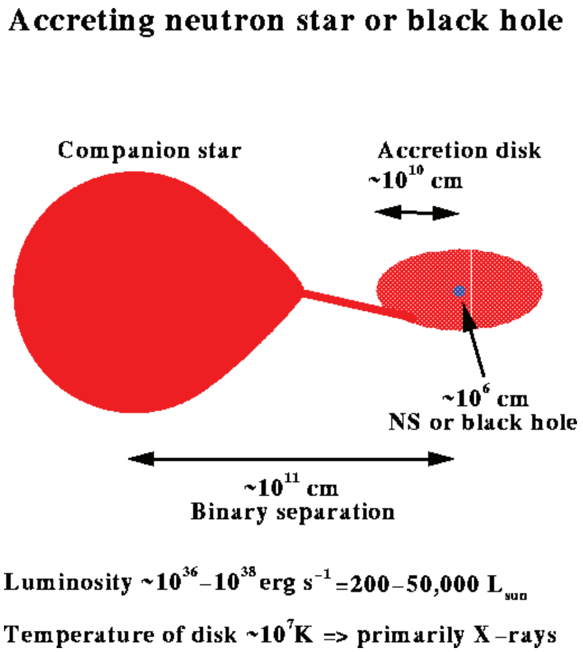


Figure 4: A binary star pair where one star toward the end of its life expands and begins to transfer matter to its companion.

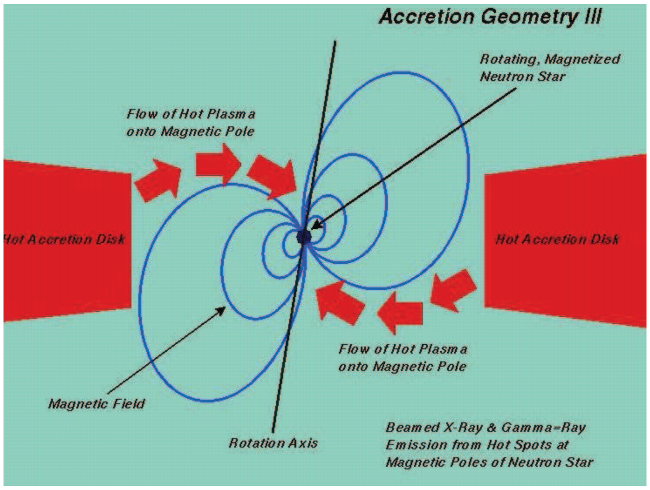


Figure 5: A figure illustrating why the radiation is directionally emitted from a neutron star. The radiated beam is observed when it points toward earth explaining the pulsed nature of the radiation.

7 Magnetar

A magnetar is a neutron star with an extremely high magnetic field. The field is so strong that the vacuum is polarized and atoms are squeezed into cyinders. The magnetic field results from a magneto-hydrodynamic process in the dense conducting fluid at the formation of the neutron star. The fields persist due to current, possibly superconducting current, flow in the proton and electron fluids.