Antenna Arrays

1 Two linear, in-phase antennas

Consider two linear, half-wave antennas separated by a distance, d, which are driven in phase. Then find the radiation field which is in the far-field $R \gg d$. In the last lecture the equation for the E field of a half wave antenna was developed.

$$E = \frac{\mu q_0 \omega^2}{2\pi} (e^{ikR}/R)[\frac{cos(\pi/2\, cos(\theta))}{sin(\theta)}]$$

Now superimpose the fields of two half-wave antennas assuming that the field magnitudes are the same (Far Field) at the same field point, but the phase may be different. This geometry is shown in Figure 1. The phase is evaluated by approximation for each antenna.

$$R^2 = r^2 + 2r(d/2)\, cos(\eta) \pm (d/2)^2 \approx r^2[1 \pm (d/2r)\, cos(\eta)]$$

Using the above expression for the phase, the equation for the field is;

$$E = \frac{\mu q_0 \omega^2}{2\pi} (e^{ikR}/R)[\frac{cos(\pi/2\, cos(\theta))}{sin(\theta)}][e^{ikd/2\, cos(\eta)} + e^{-ikd/2\, cos(\eta)}]$$

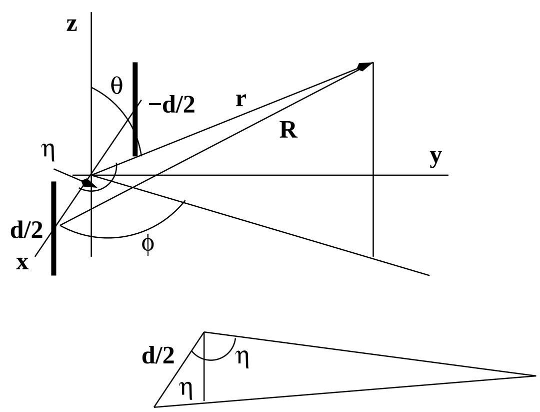

Figure 1: The geometry of 2 half-wave, linear, in-phase antennas separated by a distance d

In the above the phase difference between the antennas is obtained from the exponential term ikR.

$$ikR \rightarrow \pm ikd/2\, cos(\eta)$$

The expression for E when using $\cos(\eta) = sin(\theta)\, cos(\phi)$ is;

$$E = \frac{\mu q_0 \omega^2}{2\pi}(e^{ikR}/R)[\frac{cos(\pi/2\,cos(\theta))}{sin(\theta)}][cos([\pi d/\lambda]\,sin(\theta)\,cos(\phi))]$$

When the antennas are a half wave length apart, $d = \lambda/2$ then in the (x, y) plane $\theta = 0$ and the intensity varies as $cos(\pi/2\,cos(\phi))$. This vanishes at $\phi = 0, \pi$ and is a maximum at $\phi = \pi/2$. There is constructive interference along the y axis and destructive interference along the x axis. The ampliudes in other directions are easily determined and interpreted by analysis of the phase differences of the waves from each of the antennas.

2 Linear quadrupole

An expression for the radiation from a general quadrupole distributon of charge was also obtained in the last lecture. Now consider two dipole antennas arranged so that a linear quadrupole charge distributon is produced. To do this, first cancel the dipole terms, so compose the distribution by two dipoles pointed in opposite directions. In this case there is no net charge (monopole distribution vanishes) and the dipole distribution cancels. Then evaluate the quadrupole moment as illustrated in Figure 2 where point charges are used in the general definition for a quadrupole distribution.

$$Q_{ij} = \int d\tau\,[3x_i x_j - r^2\delta_{ij}]\rho$$

$$Q_{zz} = 2qs^2$$

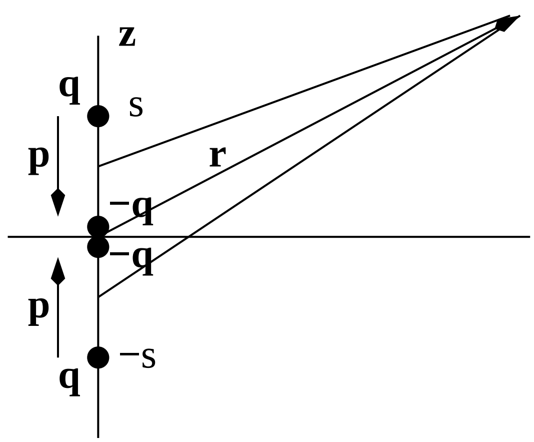

Figure 2: The geometry of a linear quadrupole composed of two in-line electric dipoles

Assume that this distribution is harmonic with time dependence, $e^{i\omega t}$. In this case, return to the results of the last section,

$$\frac{dP}{d\Omega} = [\frac{c^2 z_0}{1152\pi^2}]k^6\,|(\hat{n}\times\vec{Q})\times\hat{n}|^2$$

.

$$Q_j = \sum Q_{ij}\hat{n}_i$$

However, here consider the radiation as due to the superposition of two dipoles. As in the last section, assume that the amplitudes from the dipoles are equal but one must properly account for phase differences. The E field from a dipole has the form;

$$E = -\frac{p_0\, sin(\theta)}{2\epsilon\lambda^2}(e^{ikR}/R)$$

Then superimpose the fields from each dipole taking into account phase differences.

$$E = -\frac{p_0\, sin(\theta)}{2\epsilon\lambda^2}(e^{ikR}/R)[e^{i\pi s/\lambda} - e^{-i\pi s/\lambda}]$$

Expand the exponentials in a power series when $s \ll \lambda$. Collecting terms;

$$E = -\frac{p_0\, sin(\theta)}{4\epsilon\lambda^2}(e^{ikR}/R)\,cos(\theta)$$

The magnetic field in the radiation zone is perpendicular to E and equal to E/c, so the Poynting vector is easily determined.

3 Magnetic quadrupole

As in the previous section, develop the electric field for a magnetic quadrupole by superposition of two magnetic dipoles of opposite directions, separated by a small distance, a, Figure 3. The expression for a magnetic dipole as obtained from the last lecture is;

$$E = -\frac{z_0 k^2 m\, sin(\theta)}{4\pi}(e^{ikR}/R)$$

The phase difference between the two E field amplitudes is $\pm ka/2\,cos(\theta)$. Because the magnetic dipoles are in opposite directions, the amplitudes, including the phase components, are subtracted. The magnitudes of the amplitudes are the same and the phase components take the form;

$$e^{ika/2\,cos(\theta)} - e^{-ika/2\,cos(\theta)} \approx 2ka\,cos(\theta)$$

The expression above is obtained by expansion of the exponential assuming $\lambda \gg a$. The final result is;

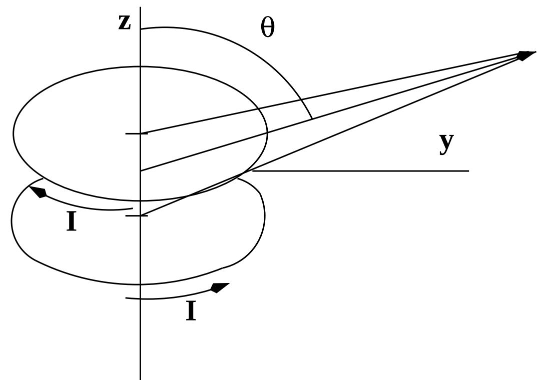

Figure 3: The geometry of a magnetic quadrupole composed of two magnetic dipoles pointed in opposite directions

$$E \;=\; -\frac{z_0 2\pi^2 ma}{\lambda^3}\, sin(\theta)\, cos(\theta)$$

Again the Poynting vector is easily obtained using B which is perpendicular to E and $B = E/c$ in the radiation zone.