Cosmology
Lecture 18

1 Introduction

In the last 20 years or so, physics has made a connection between the very large and the very small, *ie* a connection between particle physics and the evolution of the universe, cosmology. Newton formulated his laws of mechanics in reference frames that moved with constant velocity with respect to each other and to the "fixed" stars. The implication is not only that there is a special reference frame, but this frame is tied to a static universe. We now know that the universe is not fixed but has, and is, evolving.

With the exploration of detailed astronomical observations, discovery of the Cosmic Microwave Background (CMB), and the development of general relativity, a standard cosmological model has developed. This model is surpisingly simple, and while not without some open questions, explains the present stage of our universe and its evolution from a creation approximately 13.7×10^9 years ago.

The fundamental equations that governing this evoluition are solutions to Einstein's formulation of general relatvity. General relativity is a highly non-linear field theory and cannot be developed in this class. However, we attempt to understand some consequences of its solutions as they apply to cosmological observations.

2 Additional review of special relativity and electromagnetism

We began the class with a review of special relativity. Here, I want to remind you of a few features and to review 4-vector and tensor notation. This will help in the discussion of some aspects of general relativity. Remember that special relativity is a consequence of the fact that there is a limiting speed for the propagation of information, and this speed is equal to the speed of light in vacuum. The constancy of this velocity connects space-time through the Lorentz transformation. This is written for velocity in the 1 direction below.

$$x_1 = \gamma[x_1' - \beta\, ct']$$

$$x_2 = x_2'$$

$$x_3 = x_3'$$

$$ct = \gamma[ct_1' - \beta\, x_1']$$

If space time is considered a 4-vector $(\vec{x}, ct)$ then the Lorentz transformation is a matrix, $\mathcal{A}$ such that;

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ ct \end{pmatrix} = (\mathcal{A}) \begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \\ ct' \end{pmatrix}$$

The differential form for the square of a length element in the 4-vector space in tensor notation is (summation convention used);

$$ds^2 = g_{ij} dx^i dx^j$$

Here g_{ij} is the metric tensor of the space. The Lorentz transformation is

$$x_i = g_{ij} x'^j$$

This results in the preservation of length in 4-space *ie* the constancy of the velocity of light, $x^2 - (ct)^2 =$ constant. This is written as the tensor contraction;

$$x_i x^i = (g_{ij} x'^j)(g^{ik} x'_k) = x'_i x'^i$$

The Minkowski metric in tensor form is then;

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Finally note that the stress-energy tensor of electrodynamics is a covarient generalization of the Hamiltonial density.

$$T^{\alpha\beta} = \frac{\partial \mathcal{L}_{em}}{\partial(\partial_\alpha A^\lambda)} \partial^\beta A^\lambda - g^{\alpha\beta} \mathcal{L}_{em}$$

where $\mathcal{L}_{em}$ is the electromagnetic Lagrangian,

$\mathcal{L}_{em} = (-1/16\pi) F^{\mu\nu} F^{\mu\nu}$ and the EM field tensor is defined by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu a_\mu$$

$$\partial_\mu = (\vec{\nabla}, \frac{\partial}{\partial\, ct})$$

$$A_\nu = (\vec{A}, c\phi)$$

Then elements of the field tensor can be identified as

$$\text{Total Energy; } T^{00} = (1/8\pi)(E^2 + B^2) + (1/4\pi)\vec{\nabla} \cdot (\phi\vec{E})$$

$$\text{Total Momentum; } T^{0i} = (1/4\pi)(\vec{E} \times \vec{B}) + (1/4\pi)\vec{\nabla} \cdot (A_i\vec{E})$$

3 Event horizon

Suppose a projection of the time evolution of events onto the 1 + 1 dimensions of space-time as shown in figure 1.

The horizontal axis defined by t = 0 represents the present time, negative time represents the past, and positive time represents the future. Events which occcur within the light cone defined by $x = \pm\, ct$ can be connected to the present point at $x = 0$, $t = 0$, by a light ray. If an event can be observed, it is spacelike. Events outside the cone are too distant from the present point to be connected by a light ray, so they are unobservable, and are called timelike. The curved path in the figure, called a world line, represents the evolution

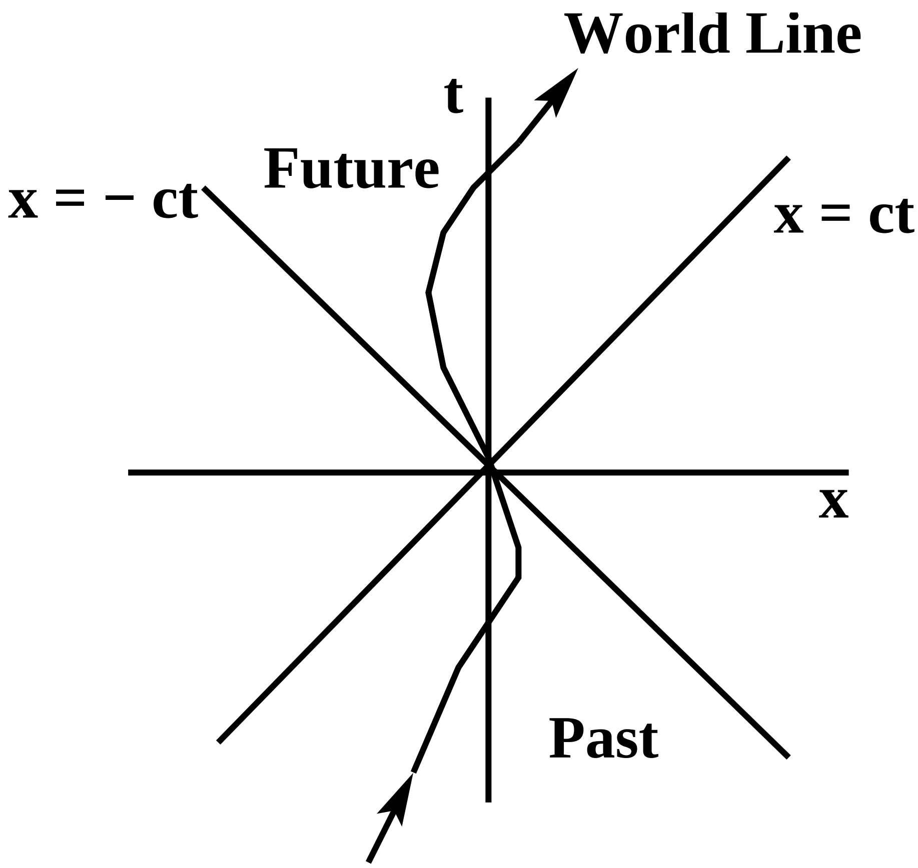

Figure 1: The light cone and world line showing the time evolution of events

of a spacetime point. There is obviously an event "horizon" defined by the light cone beyond which no information is available to an observer for $t \geq 0$.

4 Basic Assumptions of the Standard Cosmological Model, ΛCDM

Present cosmology assumes;

- The evolution of the universe is described by Einstein's general relativity
- Space time is isotropic and homogeneous out to the event horizon
- Physical laws are constant across the universe

One then applies the observations obtained from astronomy, measurements of the CMB, and particle physics to select the solution to the equations of general relativity in order to determine the initial conditions at the beginning of time.

5 The expanding universe

In 1924 Edwin Hubble began measurements of distant galaxies whose redshifts had been previously measured. Remember that the frequency of an electromagnetic wave is changed by the Doppler shift due to the relative velocity of the source with respect to the observer.

$$\omega' = \gamma\omega(1 - \beta\, cos(\theta))$$

When $\theta = 0$ $\omega' = \omega\,\dfrac{1-\beta}{1+\beta}$ which can be approximated for small β as $\omega' = \omega\,(1-\beta)$

In this equation θ is the angle between the velocity and the line between the source and the observer. Hubble determined that a direct correlation existed between the distance to a galaxy and the frequency shift of the lines in the atomic emission spectra from the galaxy. This connected the frequency shift to the relative velocity of the galaxy.

$$V = H_0\, d$$

Here H_0 is the Hubble constant, V is the velocity, and d the distance to the galaxy. One finds $H_0 = 70.1$ km/s/Mpc, figure 2.

Define the following units;

- 1 yr $= 3.16 \times 10^7$ s
- 1 light-yr $= 9.46 \times 10^{17}$ cm
- 1 pc $= 3.26$ light-yr
- 1 Mpc $= 10^6$ pc $= 3.08 \times 10^{19} km$
- ρ_c Critical Density $= 1.88 \times 10^{-29}\ h^2\, g\, cm^3$
- Baryon Density $(\rho_b/\rho_c) = 0.462$ m^{-3}

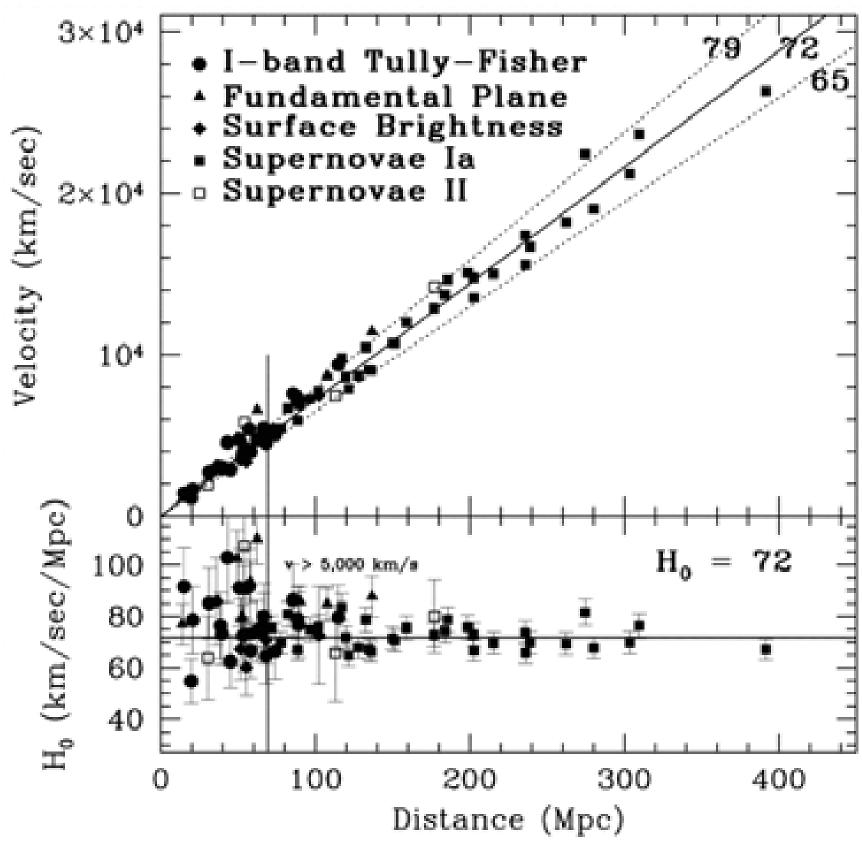

Figure 2: The Hubble relation between the distance to a galaxy and its relative velocity

- Hubble Constant $=$ 70.1 Km/s-Mpc
- Age of the Universe $= 13.73 \times 10^9$ yr

A star is a distance of 1pc from the earth, if its annual parallax is 1 sec of arc (1/3600 deg). The figure 2 shows a Hubble curve, with the lower graph showing the evaluation of the Hubble constant. The experimental data is interpreted as an expanding universe. The most distant galaxies appear to move away with velocity increasing with their distance. This then allows one to infer that if time ran backward, the universe would collapse. Thus the "Big Bang" loosely describes the creation of the universe as expanding from an intital system that was extremely hot, had a high energy density, and was very localized. However, the "Big Bang" is not an explosion driving matter into space. Rather space-time is created as the system expands. The discovery of the CMB validated the general concept of the "Big Bang" creation. The CMB is radiation which has cooled by the expanding remnants of the system.

6 Outline of evolution

A time line of the evolution of our universe is shown in figure 3. The universe began, perhaps, as a quantum fluctuation of vacuum energy. At times less than $\hbar c\sqrt{G/\hbar c} = 10^{-43}$s gravity dominated the interactions and the system must be described by a quantum theory of gravity, which does not yet exist. Thus all physical laws are are unknown for times earlier than 10^{-43} s. Based on measurements of the fluctuations in the CMB, the universe has an assumed age of

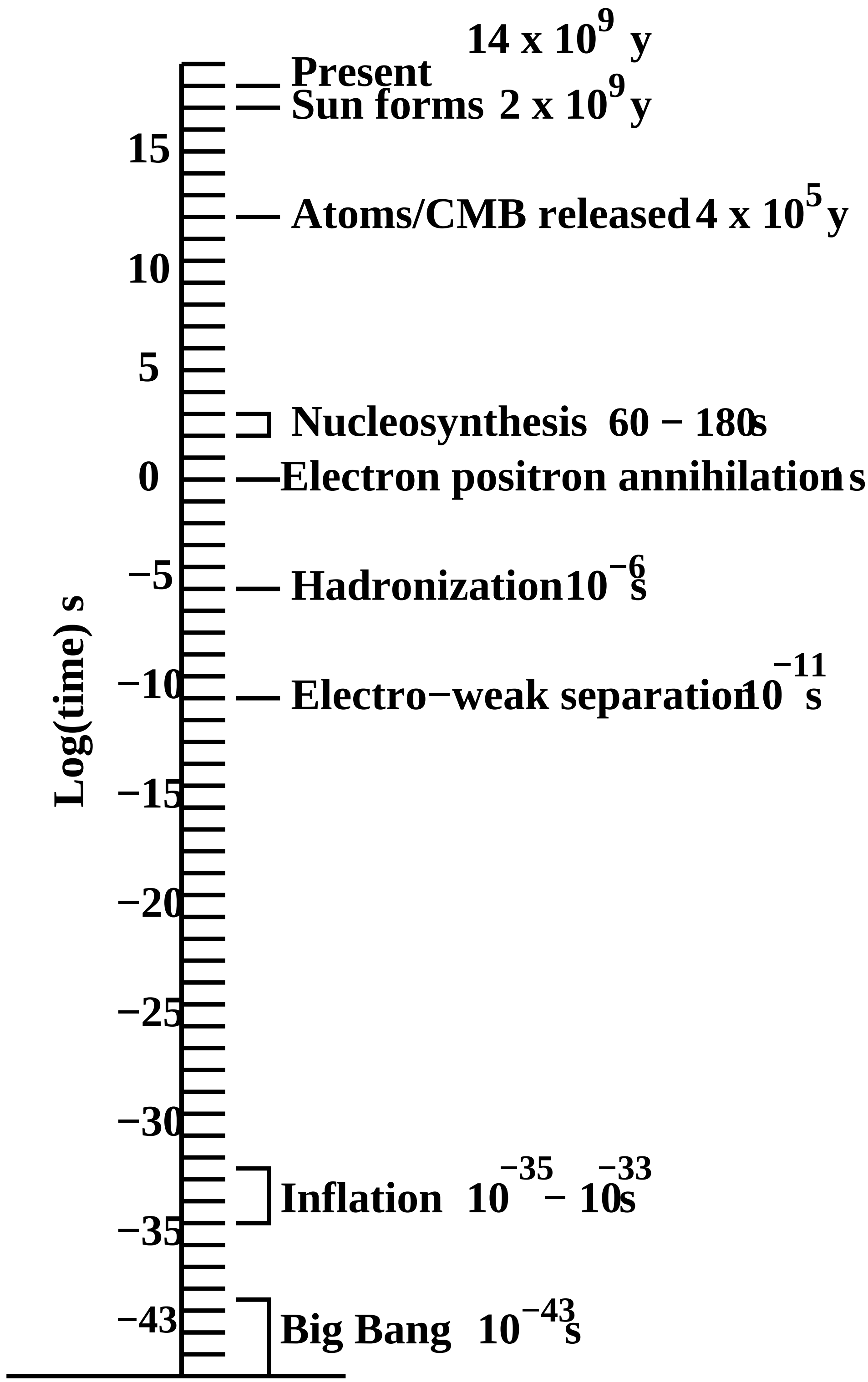

Figure 3: A time line of the universe from creation to the present

13.72×10^9 yr.

In most models, the "Big Bang" consists of an extremely hot expanding system. At 10^{-35}s after creation the expansion undergoes a phase transition which causes an exponential increase of the dimensions of the system. This is called, "Inflation" and is necessary to account for the observation that space-time is isotropic and homogeneous. Inflation will be discussed later, however the physical process of inflation is not understood.

At the end of inflation the universe consisted of a quark-gluon plasma which is accessible by experimentation today. Temperatures were 10^{28} deg K and particles and radiation were in equilibrum, *ie* the particles, antiparticles, and radiation were created and absorbed without change to their relative numbers. The plasma contiuned to cool until about 10^{-11} sec when the temperature reached 10^{15} deg K (10^{11}ev). At this point the electroweak force falls out of equilibrum. Remember that the reaction rate depends on the temperature through the Boltzman relation, $e^{-E/kT}$.

Sometime during this period a process called baryogenesis occurs which removes antiquarks(anti-baryons) leading to a small excess of quarks over antiquarks - about 1 in 3×10^7. This explains the fact that only matter is observed in the universe. For baryonogenseis to occur, baryon number must be violated, C and CP symmetry is not preserved, and the universe cannot be in thermal equilibrum (Sakaharov conditions). While these conditions can occur in the Standard Model of particle physics, they do not occur at a sufficient rate to explain baryogenesis.

At 10^{-6}s quarks and gluons combine to produce hadrons. Anti-baryons annihilate leaving 1 in 10 of the initial nucleons. At about 1 sec electrons and positrons annihilate leaving only electrons. At this time the temperature is about 10^{10} deg K (10^6 ev) and the energy density is dominated by photons. Several minutes after the Big Bang at a temperature of 10^9 deg K (10^5 ev) and a particle density of 5×10^{19} particles/cm^3 nucleosynthesis begins and lasts for approximately 30 min. Some neutrons and protons combine to form deuterium, although most protons remain unbound.

After 380,000 years the temperature cools suffficiently so that atoms can form. Before this time, radiation and charged particles were in thermal equilibrum. Afterward, because of the lower photon-atom cross sections, the universe becames transparent to electromagnetic radiation. This represents the earliest time that one can observe the conditions of the universe by looking at the emitted radiation which can be observed today (e.g. CMB). At this time the photon spectrum was blackbody with a characteristic temperature of 3000 deg K. This radiation, cooled to 2.7 deg K, is the CMB observed today. From this observation combined with other data we obtain the age of the universe as 13.7×10^9 yr.

Finally the most dense regions of matter, perhaps in the form of dark matter to be discussed later, gravitationally attracted to form gas clouds, then stars, and later galaxies. Stars burned the deutereium and hydrogen to heavier elements, and supernova modified the abundance, spraying the elements into space. These materials then recombine into varions astronomical structures.

7 Energy relations in expansion

Consider an expanding sphere. The gravitational force of a symmetrical distribution of mass acts as if the mass inside the distance from the center of the distribution to the field point is concentrated at the center. That is the potential energy is

$$E_{grav} = -GM/R$$

where G is the gravitational constant, M is the mass, and r is the distance from the center to the field point. If this point moves its kinetic energy is;

$$T = (1/2)MV^2$$

Which gives a total energy per mass point of;

$$E_{tot} = (1/2)V^2 - GM/r$$

This is rewritten as;

$$V^2 = 2GM/r + k$$

Where $k = 2E_{tot}$

There are 3 cases.

1. $k = 0$ In this case the velocity exactly equals the escape velocity. A particle moving away from a mass center would reach an infinite distance with zero velocity.
2. $k < 0$ In this case the particle does not have sufficient energy to reach an infinite distance and fall back toward the gravitating mass after reaching some distance < infinity.
3. $k > 0$ In this case the velocity is sufficient to allow the particle to continue to move toward infinity as it will always have non-zero velocity.

Now suppose the spherical mass represents the universe at some time. The three cases describe the possible curvature of space time, which is shown in the 3 figures below.

- $k = 0$ Zero curvature - a flat space universe
- $k < 0$ Negative curvature - a closed universe
- $k > 0$ Postive curvature - an open universe

Standard Cosmological Model

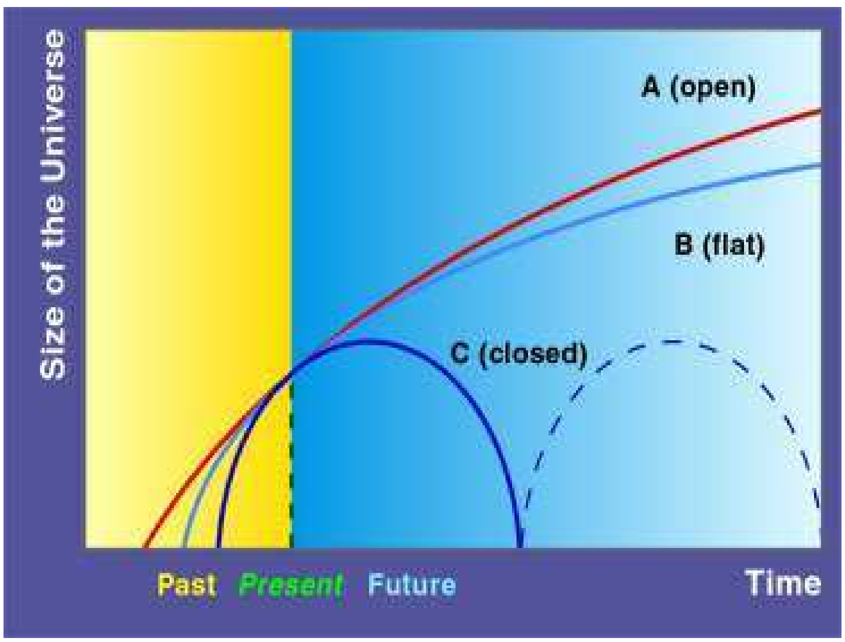

Figure 4: A schematic description of the geometry of space-time

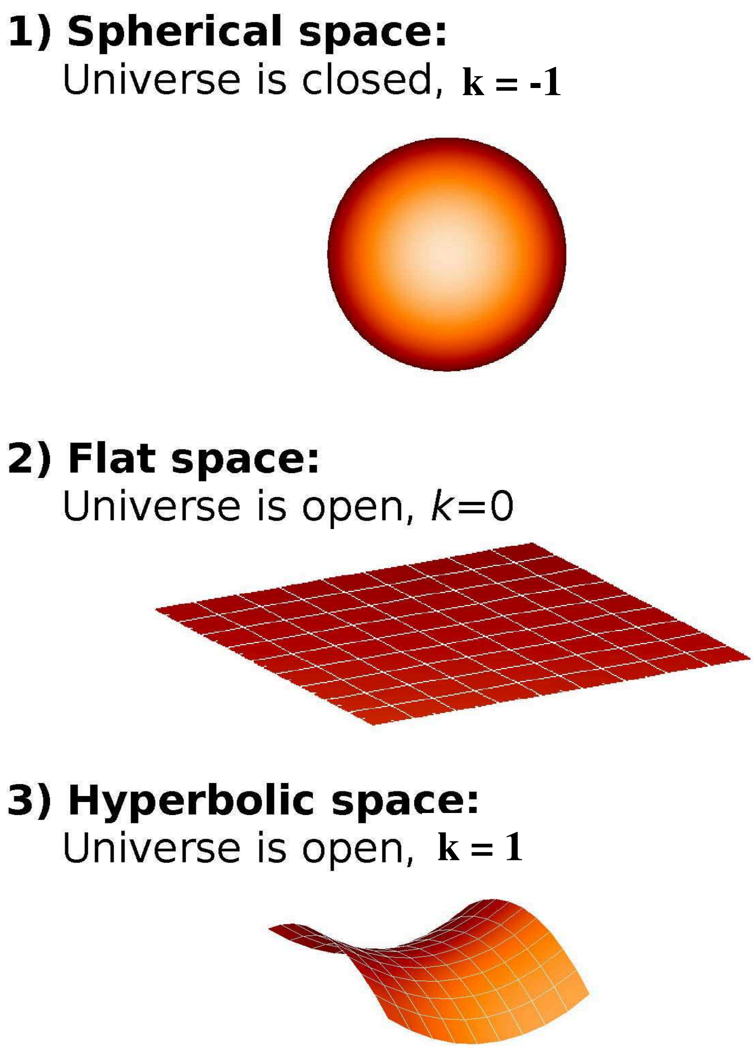

Figure 5: The curvature of space-time