Phys 4322 Exam 1 - Solutions
Feb. 10, 2010

You may NOT use the text book or notes to complete this exam. You and may not receive any aid from anyone other that the instructor. You will have 1.5 hours to finish. DO YOUR OWN WORK. Express your answers clearly and concisely so that appropriate credit can be assigned for each problem. Full credit for each problem is 25 points.

1)

A voltage, \( V_0 \), is placed between the conductors of a long coaxial cable. The space between the conductors is filled with a material of dielectric constant, \( \epsilon \), and resistivity, \( \rho \). What current flows through the material between the conductors in a length, \( L \), of the cable?

![Figure 1: The geometry for problem 1.](image)

**Solution**

The surfaces of constant potential are cylinders, concentric with the inner conductor. The current follows the electric field lines in the radial direction. Thus for the resistance of a length of cable, \( L \);

\[
dR = \frac{\rho}{2\pi r L} dr
\]

Integrate;

\[
R = \int_{a}^{b} \frac{\rho}{2\pi r L} dr = \frac{\rho}{2\pi L} \ln(b/a)
\]

\[
I = \frac{V}{R} = \frac{2\pi LV_0}{\rho \ln(b/a)}
\]

2)

A conducting rod of length, \( L \), and mass, \( M \), slides without friction, down conducting rails which have an angle, \( \theta \), with respect to the horizontal, see the figure. The rails are connected through a resistance, \( R \), and placed in a constant, vertical magnetic field, \( B \). Find the terminal velocity (velocity which remains constant in time) if the rod is released from rest in the Earth’s gravitational field.
Solution

The magnetic flux through a loop of length, $s$, is;

$$\phi = BS\cos(\theta)$$

$$EMF = -\frac{d\phi}{dt} = -BL\frac{ds}{dt}\cos(\theta) = -BLV\cos(\theta)$$

In the above, $V$, is the downward velocity. The downward force on the rod is, $F_d = Mg\sin(\theta)$. This force introduces a power $F_dV$ into the motion of the rod. By energy conservation the power lost in the resistor, $I^2R$, balances this power input when equilibrium is reached. Thus the current flow is;

$$I = (BLV\cos(\theta))/R$$

$$\left(\frac{BLV\cos(\theta)}{R}\right)^2 R = MgV\sin(\theta)$$

$$V = \frac{MgR\sin(\theta)}{L^2B^2\cos^2(\theta)}$$

A circuit is made of 2 long wires carrying a current in opposite directions. Assume the circuit is closed at $\pm \infty$. The wire diameter is $b$, and the separation of the wires is $a$. Assume only a surface current. Find the self inductance per unit length of the circuit.
Solution

For long wires use Ampere’s law to get the magnetic field. For one of the wires:

\[ \oint \vec{B} \cdot d\vec{l} = \mu_0 I \]

\[ B = \frac{\mu_0 I}{2\pi r} \]

The flux is obtained by integration. Let \( S \) be a length of the wire;

\[ \phi = S \int_{b/2}^{(a-b)/2} dr \frac{\mu_0 I}{2\pi r} \]

Then the total flux is twice the above to account for the 2\(^{nd}\) wire. The inductance is \( 2\phi/I \);

\[ L = \frac{\mu_0}{\pi} \ln(a - b/b) \]

4)

A parallel plate capacitor of separation, \( s \), and plate area, \( A \), is filled with a dielectric of dielectric constant, \( \epsilon \), and resistivity, \( \rho \). Neglect edge effects. The capacitor is initially charged with a potential, \( V_0 \), and then the battery is removed. As the capacitor discharges a conduction and a displacement current are created. Find the values of;

a) The conduction current

b) The displacement current

c) The magnetic field

![Figure 4: The geometry for problem 4.](image)

Solution

The equal potential surfaces are planes parallel to the end plates. The electric field is perpendicular to these planes.
\[ dR = \frac{\rho dy}{A} \]
\[ R = \frac{\rho S}{A} \]

The charge on the capacitor is \( Q \);
\[ V = \frac{Q}{C} = \frac{QS}{(\epsilon A)} \]

The electric field in the capacitor is;
\[ E = \frac{V}{S} = \frac{Q}{(\epsilon A)} \]

The conduction current is;
\[ I_c = \int \vec{J} \cdot d\vec{a} = JA \]

The displacement current density is:
\[ J_d = \epsilon \frac{\partial E}{\partial t} \]

The time change of \( Q \) is negative \( I_c \) for discharge so;
\[ J_d = \epsilon \left( \frac{1}{\epsilon A} \right) \left( \frac{\partial Q}{\partial t} \right) = -I_c/A \]

Ampere’s law is then;
\[ \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon \frac{\partial \vec{E}}{\partial t} \]
\[ \nabla \times \vec{B} = 0 \]

There is no magnetic field