10.12)

Use the Jefimenko equations. For $B$;

$$\vec{B}(\vec{r}, t) = \frac{\mu}{4\pi} \int d\tau' \left[ \frac{\vec{J}(\vec{r}', t_{\tau t}) \times \vec{R}}{R^4} + \frac{\dot{\vec{J}}(\vec{r}', t_{\tau t}) \times \vec{R}}{R^2c} \right]$$

Where $\vec{R} = \vec{r} - \vec{r}'$ and $t_{\tau t} = t - R/c$. Use an expansion;

$$\dot{\vec{J}}(\vec{r}, t_{\tau}) = \vec{J}(\vec{r}, t_{\tau}) + (t_{\tau} - t) \dot{\vec{J}}(\vec{r}, t)$$

Insert this into the expression for $\vec{B}$ and assume that to 1st order one can write $\dot{\vec{J}}(\vec{r}, t_{\tau}) \approx \dot{\vec{J}}(\vec{r}, t)$. Cancel terms to obtain the result.

$$\vec{B}(\vec{r}, t) = \frac{\mu}{4\pi} \int d\tau' \frac{\vec{J}(\vec{r}', t) \times \vec{R}}{R^4}$$

10.15)

For a particle in hyperbolic motion along the $\hat{x}$ axis look at the 2-D plot of position vs time which forms the hyperbola;

$$x = \sqrt{b^2 + (ct)^2}$$

A light ray moves in this 2-D space as a line with slope, $c$. The graphical solution is shown in the figure.

The light becomes visible when a right ray from a retarded point on the trajectory passes through the point $x_0$. 

1
9.19)

The field of a point charge is;

\[ \vec{E} = \frac{1}{4\pi\epsilon} \frac{1 - \beta(2)}{[1 - \beta^2 \sin^2(\theta)]^{3/2}} \frac{\hat{r}}{r^2} \]

For a continuum of point charges, let \( q \rightarrow \lambda \, dx \).

\( \sin(\theta) = d/r \)

\( dz/r^2 = d\theta/d \)

Substitute into the integral and project the field perpendicular to the wire as the horizontal component cancels. There will be an additional \( \sin(\theta) \) in the numerator.

\[ \vec{E} = \frac{1}{4\pi\epsilon} \lambda [1 - \beta^2] \int_0^\pi d\theta \frac{\sin(\theta)}{[1 - \beta^2 \sin^2(\theta)]^{3/2}} \]

Change variable to \( u^2 = 1 - \sin^2(\theta) \) and integrate to obtain;

\[ E_\perp = \frac{1}{4\pi\epsilon} \frac{2\lambda}{d} \]

The magnetic field is a constant due to the constant current on the wire.

\[ \vec{B} = \frac{I\mu}{2\pi a} \hat{\phi} \]

2
The Lorentz gauge is \( \vec{\nabla} \cdot \vec{A} = 1/c^2 \frac{\partial V}{\partial t} \).

The potentials due of a point charge are;

\[
V = \frac{1}{4\pi\epsilon} \frac{q}{r - \beta \cdot r}
\]

\[
\vec{A} = \vec{\beta}/cV
\]

In the problem \( \vec{\nabla} \cdot \vec{\beta} = 0 \) substitute into the differential operators. there is much algebra, but the result is forthcoming.