8.7)

The magnetic field is $\vec{B} = \mu_0 n I \hat{z}$. The torque is $N = \vec{r} \times \vec{F}$ and $\vec{F} = I' d\vec{l} \times \vec{B}$ where $I'$ is the leakage current through the spoke. We neglect any induced field because the discharge is slow. The force on spoke is:

$$d\vec{F} = -\mu_0 I' n I d\rho \hat{\phi}$$

The torque on the cylinders is then:

$$N = \int_a^R d\rho \times d\vec{F}$$

$$N = (1/2) I' \mu_0 n I (R^2 - a^2) \hat{z}$$

Since $\frac{d\vec{L}}{dt} = N$

$$\vec{L} = \int_0^\infty dt \vec{N} = -(1/2) \mu_0 Q n I (R^2 - a^2) \hat{z}$$

8.10)

The momentum density is $\vec{P} = \vec{S}/c^2$. The magnetic field inside the sphere is:

$$\vec{B} = \begin{bmatrix} (2/3) \mu_0 \vec{M} & \text{inside} \\ (4/3) \pi R^3 \vec{M} & \text{outside} \end{bmatrix}$$

In terms of the magnetic moment:

$$\vec{B} = \begin{bmatrix} (2/3) \mu_0 \vec{M} & \text{inside} \\ \frac{\mu_0}{4\pi r^3} [3(\vec{m} \cot \hat{r}) \hat{r} - \vec{m}] & \text{outside} \end{bmatrix}$$

The electric field in terms of its polarization and dipole moment is;
\[
\vec{E} = \begin{bmatrix}
\frac{1}{4\pi\epsilon_0 r^3}[-(1/\epsilon_0)\vec{P} & \text{inside}] \\
3(\vec{p} \cdot \hat{r})\hat{r} - \vec{p} & \text{outside}
\end{bmatrix}
\]

Substitute to get the momentum density and integrate over the volume to find the momentum in the field. There is a substantial amount of algebra. The result is:

\[
\vec{p} = \begin{bmatrix}
-(8/27)\mu_0\pi R^3(\vec{P} \times \vec{M}) & \text{inside} \\
-(4/27)\mu_0\pi R^3(\vec{P} \times \vec{M}) & \text{outside}
\end{bmatrix}
\]

Adding the two components one obtains

\[
\vec{p} = -(4/9)\mu_0\pi R^3(\vec{P} \times \vec{M})
\]

8.11)

The magnetic field due to a rotating charge with density, \(\sigma = e/(4\pi r^2)\) is

\[
\vec{B} = \begin{bmatrix}
(2/3)\mu_0\sigma R \vec{\omega} & \text{inside} \\
2\mu_0\sigma R^4 \cos(\theta)\hat{r} + \mu_0\sigma R^4 \sin(\theta) & \text{outside}
\end{bmatrix}
\]

The electric field is;

\[
\vec{E} = \begin{bmatrix}
0 & \text{inside} \\
\frac{e}{4\pi\epsilon_0 r^2} & \text{outside}
\end{bmatrix}
\]

The energy density is \(W = (1/2\epsilon)E^2 + (\mu_0/2)B^2\) Substitute for the fields and integrate over the volume.

\[
W = \frac{e^2}{8\pi\epsilon_0 R} + \frac{\mu_0 c^2 R \omega^2}{36\pi}
\]

To find the angular momentum, the angular momentum density is; \(\vec{L} = \vec{r} \times \vec{S}/c^2\). Substitute for the fields and integrate. By symmetry the angular momentum is in the \(\hat{z}\) direction.

\[
\vec{L} = \frac{2\mu_0 \omega R}{48} \hat{z}
\]
8.14)

The magnetic field is;

\[ \vec{B} = \begin{cases} \mu_0 NI \hat{z} & \text{inside} \\ 0 & \text{outside} \end{cases} \]

The electroc field is

\[ \vec{E} = \frac{q}{4\pi\epsilon_0}[(x-a)\hat{x} + y\hat{y} + z\hat{z}] \]

Substitute into the value of the momentum density;

\[ \vec{P} = \vec{E} \times \vec{H} / c^2 = \frac{\mu_0 q n I}{4\pi\epsilon_0}[(a-x)\hat{y} + y\hat{x}][(a-x)^2 + y^2 + z^2]^{3/2} \]

Upon integration over the volume the x component cancels by symmetry. Integration for \(-\infty < z < \infty\) yields

\[ p_y = \mu_0 q n I \int_{-R}^{R} dx \int_{-R}^{R} dy \frac{a-x}{(a-x)^2 + y^2} \]

Change to Polar coordinates \(x = \rho \cos(\phi), y = \rho \sin(\phi), dxdy = \rho d\rho d\phi\) and integrate.

\[ p_y = \frac{\mu_0 q n I R^2}{2a} \]

To get the angular momentum;

\[ \vec{L} = \vec{r} \times \vec{P} = \frac{\mu_0 q n I}{4\pi\epsilon_0[(a-x)^2 + y^2 + z^2]^{3/2}}[-x(a-x)\hat{z} - y^2\hat{x} + (x-a)z\hat{x} + yz\hat{y}] \]

Symmetry removes components in \(\hat{x}\) and \(\hat{y}\). Change to polar coordinates and integrate. The result is;

\[ L_z = -\frac{\mu_0 q n I}{2\pi} \int \rho d\rho \int d\phi \frac{s^2 - as \cos(\phi)}{s^2 + a^2 - 2as \cos(\phi)} \]

\[ L_z = 0 \]