9.2)

The function to test is \( f(z, t) = A \sin(kz) \cos(ckt) \)

It is easily shown that this is a solution to the wave equation;

\[
\frac{\partial^2 f}{\partial z^2} - \frac{(1/c)^2}{\partial t^2} \frac{\partial^2 f}{\partial t^2}
\]

by substitution. The equation can be expressed as a standing wave by superimposing waves moving in opposite directions. Thus

\[
f = \frac{A}{4i} [e^{ikz}e^{-ikt}][e^{ikt} + e^{-ikt}]
\]

9.4)

Apply a Fourier transform to the solution \( f(x, t) \)

\[
\int \frac{1}{\sqrt{1/2\pi}} f(k, t) e^{-ikx} \]

\[
f(z, t) = \frac{1}{\sqrt{1/2\pi}} \int dt f(k, t) e^{ikz}
\]

Separation of variables allows a product solution a function of \( x \) and a function of \( k \). If the above is a solution to the wave equation then the time component must have the form \( e^{i\omega t} \). Therefore

\[
f(z, t) = \frac{1}{\sqrt{1/2\pi}} \int dt f(k, t) e^{i(kx - \omega t)}
\]
9.11) This was worked in class. See notes on waves.

9.13) This was worked in class. See notes on waves incident on a boundary.