SET 13

11.1)

The Lorentz condition is;

$$\vec{\nabla} \cdot \vec{A} = -(1/c^2)\, \frac{\partial V}{\partial t}$$

The potentials for electric dipole radiation are;

$$V = \frac{1}{4\pi\epsilon}\, \frac{p\, cos(\theta)}{r}[-(\omega/c)\, sin(\omega[t - r/c]) + (1/r)\, cos(\omega[t - r/c])]$$

$$\vec{A} = -\frac{\mu}{4\pi}\, \frac{p\omega}{r}\, sin(\omega[t - r/c])[\hat{r}\, cos(\theta) - \hat{\theta}\, sin(\theta)]$$

Substitute into the lorentz condition and apply the differential operators. The result follows.

11.4)

Assume radiation from two dipoles $\pi/2$ out of phase. The dipoles are combined to obtain;

$$\vec{p} = p_0[\hat{x}\, cos(\omega t) + \hat{y}\, sin(\omega t)]$$

The general form for the fields is;

$$\vec{E} = -\frac{\mu p_0 \omega^2}{4\pi}(\frac{sin(\theta)}{r})\, cos(\omega[t - r/c])\, \hat{\theta}$$

$$\vec{B} = |E|/c\, \hat{\phi}$$

The term dependent on time for each component is $\pi/2$ out of phase so the term $cos(\omega[t - r/c]) \rightarrow sin(\omega[t - r/c])$ for the field components, and the radiation direction changes.

$$sin(\theta)\, \hat{\theta} = x/r\hat{r} - \hat{x}$$

and for the other dipole;

$$sin(\theta)\,\hat{\theta} \,=\, y/r\hat{r} - \hat{y}$$

Insert these into the equations for the fileds and find the Poynting vector.

$$\langle S\rangle \,=\, (\mu/c)(\frac{mup_0\omega^2}{4\pi r})^2[1-(1/2)\,sin^2(\theta)]$$

11.6)

The power radiated is, where the effective current for the magentic dipole is $I \,=\, m_0/\pi b^2$;

$$\langle P\rangle \,-\, \frac{\mu m_0\omega^4}{4\pi c^3}(\frac{pi^2b^4}{m_0}$$

Then $P \,=\, IR^2$

Solving for R gives;

$$R \,=\, \frac{8\mu c\pi^5b^4}{3\lambda^4}$$

11.9)

In this case a magnetic dipole is formed which is equivalent to two oscillation dipoles as in problem 11.4. The dipole moment in the $\hat{y}$ direction is;

$$p_{0y} \,=\, \int\, sin(\phi)\,\lambda r\,dl \,=\, \pi\lambda b^2$$

Then the moment is ;

$$p \,=\, p_0[cos(\omega t)\,\hat{y} \,-\, sin(\omega t)\hat{x}]$$

Finf the second derivative and insert into the Larmor equation.

$$P \,=\, \frac{\mu\ddot{p}^2}{6\pi c}$$