SET 2

7.1)

This problem was essentially worked in class. Solve $\nabla^2 V = 0$ in spherical coordinates, then surfaces of constant potential are spherical shells of area $4\pi r^2$. The resistance between two shells a distance dr apart is;

$$dR = \frac{\rho\, dr}{4\pi r^2}$$

Integrate over $a \leq r \leq b$;

$$R = \frac{\rho}{4\pi} \int_a^b dr/r^2 = \frac{\rho}{4\pi}\,[1/a - 1/b]$$

For $b \to \infty$, the current $I = V/R$, is independent of b. Then for 2 spheres assume that each is enclosed by a larger sphere, so conduction goes between the smaller sphere to the larger one and from the larger one to the other smaller one. The potential must be equal to $V_{total} = V_1 - (-V_2) = 2V$.

7.5)

The circuit potential is;

$$V = \mathcal{E} - IR$$

Power expended in the resistor is,

$$P = V^2/R = (\mathcal{E} - IR)^2/R$$

with $I = \mathcal{E}/(r + R)$

Then find ;

$$\frac{dP}{dR} = 0 = \frac{\mathcal{E}^2}{(r+R)^2} - \frac{2\mathcal{E}^2 R}{(r+R)^3}$$

The solution is $r = R$

7.8)

Use Ampere's law to get the magnetic field, $B = \frac{\mu_o I}{2\pi r}$. Then the flux through the loop is

$$\phi = \int \vec{B} \cdot d\vec{a} = \frac{\mu_0 I a}{2\pi} \int_s^{s+a} dr/r$$

$$\phi = \frac{\mu_o I a}{2\pi} ln[\frac{s+a}{s}]$$

The EMF is $-\frac{\partial\phi}{\partial t}$ Let the velocity of the loop be $V = \frac{ds}{dt}$.

$$EMF = \frac{\mu_0 I c}{2\pi} [\frac{1}{s+a} - \frac{1}{a}]V$$

7.18)

To get the direction of the current, use Lenz's law. From problem 7.8;

$$\phi = \frac{\mu_0 a}{2\pi} ln[\frac{s+a}{a}]I$$

$$\phi = \frac{\mu_0 a}{2\pi} ln[\frac{s+a}{a}] \begin{pmatrix} (1-\alpha t)I_0 \ (0 \le t \le 1/\alpha) \\ 0 \ t > 1/\alpha \end{pmatrix}$$

$$EMF = -\frac{\mu_o a}{2\pi} ln[\frac{s+a}{a}]\alpha T_0$$

$$I = EMF/R$$

$$Q = \int_0^{1/\alpha} I\, dt$$