SET 6

12.33)

The pion decays into 2 gamma rays which must conserve momentum and energy. Assume that the photons are emitted along the direction of motion on the pion.

Energy Conservation

$$E_{pi} = E_1 + E_2$$

Momentum Conservation

$$p_\pi = p_1 \pm p_2$$

The relatavistic relation between energy and momentum gives ;

$$E_\pi^2 = (p_\pi c)^2 + (m_\pi c^2)^2$$

$$E_{1,2} = p_{1,2} c$$

Given $p_\pi = 3/4\, m_\pi c$ and solving the above equations one finds that

$E_1 = m_\pi c^2$ and $E_2 = m_\pi c^2/4$

12.37)

For uniform acceleration, the force, F, is obtained from, $\frac{d(\beta\gamma mc)}{dt}$. Integrate this to obtain β and then integrate β to obtain the position as a function of time.

$$(x - x_0) = \frac{mc^2}{\mathrm{F}}[\sqrt{1 + (\frac{\mathrm{F}_t}{mc})^2} - 1].$$

The photon travels as $x = ct$. The distance between the positions of the person and the light cone for large t is $\frac{mc^2}{\mathrm{F}}$, so if $x_0 \geq \frac{mc^2}{\mathrm{F}}$, the light will never reach the person.

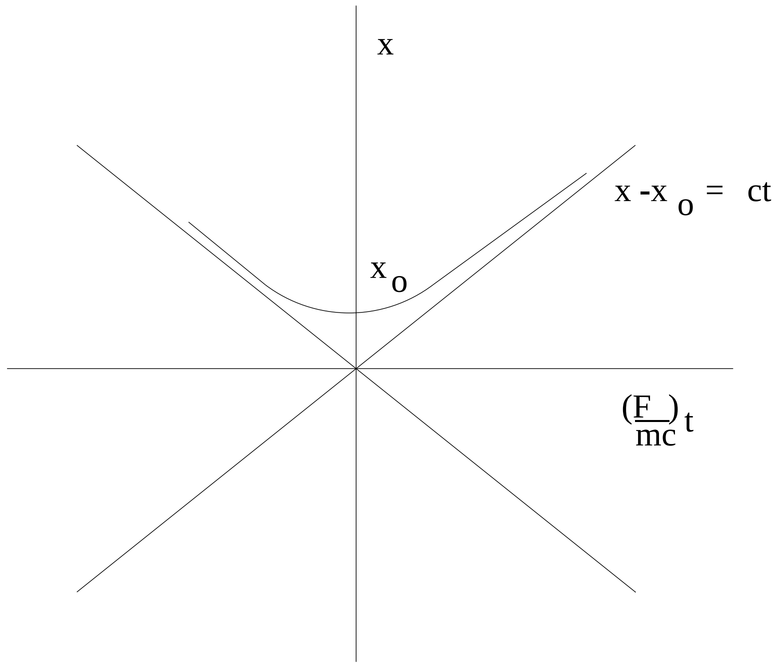

Figure 1: The figure shows the light cone in 2 dimensional space-time

12.43)

The electric field of a moving charge is

$$\vec{E} = \frac{1}{4\pi\epsilon} \frac{q[1-\beta^2]}{[1-\beta^2\, sin^2(\theta)]^{3/2}} \frac{\hat{r}}{r^2}$$

Integrate this to get the flux out of a spherical surface.

$$\oint \vec{E} \cdot d\vec{a} = \oint Er^2\, sin(\theta)\, d\theta d\phi$$

This integrates to q/ϵ_0

The charge is moving so a magnetic field is produced. This obtained by a field transformation fromteh static electric field at rest.

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q[1-\beta^2]\, sin(\theta)}{[1-\beta^2\, sin^2(\theta)]^{3/2}} \frac{\hat{\phi}}{r^2}$$

The Poynting vector is $\vec{S} = \vec{E} \times \vec{H}$ with $B = \mu H$. Substitute to obtain;

$$\vec{S} = -\frac{cq^2}{16\pi^2\epsilon_0} \frac{[1-\beta^2]^2\, \beta\, sin(\theta)}{r^4[1-\beta^2\, sin^2(\theta)]^3}\, \hat{\theta}$$

12.50)

This problem requires matrix multiplication of the Field tensors $\mathcal{F}$ and $\mathcal{G}$

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$$\sum F^{ij}F_{ij} = 2[B^2 - (E/c)^2]$$

$$\sum G_{ij}G_{ij} = -2[B^2 - (E/c)^2]$$

$$\sum F^{ij}G_{ij} = -(4/c)\vec{E} \cdot \vec{B}$$