SET 6

8.1)

The power is obained from the Poynting vector. The E field is obtained from Gauss' law by placing a uniform charge per unit lenght λ on the inner conductor. Direction is determined by symmetry to be radial.

$$\int \vec{E} \cdot d\vec{a} \; - \; E(2\pi r L) \; = \; Q/\epsilon$$

$$\vec{E} \; = \; \frac{\lambda}{2\pi\epsilon_0 r}\,\hat{r}$$

The magnetic field is obtained from Ampere's law. Assune a current, I, on the inner conductor. Direction is determined by the right hand rule.

$$\oint \vec{B} \cdot d\vec{l} \; = \; \mu I$$

$$B \; = \; \frac{\mu I}{2\pi r}\hat{\phi}$$

We use cylindrical coordinates so that $\hat{r} \times \hat{\phi} \; = \; \hat{z}$ Then;

$$\vec{S} \; = \; (EB/\mu)\hat{z} \; = \; \frac{\lambda I}{4\pi\epsilon_0 r^2}\hat{z}$$

The Poynting vector is the power through an area. A small cylindrical area is ;

$$da \; = \; r dr \, d\phi$$

Thus the total power is;

$$P \; = \; \int \; \frac{\lambda I}{4\pi^2\epsilon_0 r^2} r dr \, d\phi =, = \; \frac{\lambda I}{2\pi\epsilon} ln(b/a)$$

Integration $\int \vec{E} \cdot d\vec{r} \; = \; V$ so by completing the integral and substitution;

$$P \; = \; V I$$

8.3)

Use the value for the magnetic field found in example 5.11 which is uniform and in the $\hat{z}$ direction.

$$\vec{B} = (2/3)\mu_0 \sigma R \omega \hat{z} \; r < R$$

We compute the force by choosing a surface that encloses the top half of the sphere. Choose a hemispherical surface of large radius closed below by the (x,y) plane. We need only to find the force due to the stress tensor at the (x,y) plane. Then insert the B field into the stress tensor and evaluate $\int T_{zz} \cdot d\vec{a}$ on the (x,y) plane.

$$T_{zz} = (2/9)\mu_0 \sigma^2 R^2 \omega^2 \; r < R$$

$$T_{zz} = (1/18)\mu_0 \sigma^2 (R^8/r^2) \omega^2 \; r < R$$

Then the force in the $\hat{z}$ direction is;

$$F = -2\pi \int_0^R r\,dr\,(1/18)\mu_0 \sigma^2 (R^8/r^2)\omega^2 \; - \; 2\pi \int r\,dr\,(2/9)\mu_0 \sigma^2 R^2 \omega^2$$

$$F = -(\pi/4)\mu_0 \omega^2 \sigma^2 R^4$$

8.5)

Draw a surface surrounding a capacitor plate. The electric field is $E = \sigma/\epsilon$ and has a direction perpendicular to the plate. Substitute into the stress tensor. The diagonal elements are the only non-zero terms. Evaluate the force through a surface perpendicular to a plate.

$$F_z \hat{z} = \int T_{zz} \cdot d\vec{a} = -\frac{\sigma^2}{2\epsilon_0}$$

8.6)

The momentum per unit volume is $\vec{P} = \epsilon \vec{E} \times \vec{B}$. This is to be integated over the volume between the plates.

$$P_{Total} = \epsilon_0 EB(area\, d)$$

The force is the time change of the momentum so that the force on a wire is the integration on the above value over the time of discharge. This same result can also be obtained from the Lorentz force caused by the time change in magnetic flux through a loop in the x,z plane.