SET 8

8.7)

The magnetic field is $\vec{B} = \mu_0 nI\hat{z}$ The torque is $N = \vec{r} \times \vec{F}$ and $\vec{F} = I'\, d\vec{l} \times \vec{B}$ where I' is the leakage current through the spoke. We neglect any induced field because the discharge is slow. The force on spoke is;

$$d\vec{F} = -\mu_0 I' nI d\rho\hat{\phi}$$

The torque on the cylinders is then ;

$$N = \int_a^R d\rho \times d\vec{F}$$

$$N = (1/2)I'\mu_0 nI(R^2 - a^2)\hat{z}$$

Since $\dfrac{\vec{L}}{dt} = N$

$$\vec{L} = \int_0^\infty dt\, \vec{N} = -(1/2)\mu_0 QnI(R^2 - a^2)\hat{z}$$

8.10)

The momentum density is $\vec{\mathcal{P}} = \vec{S}/c^2$. The magnetic field inside the sphere is;

$$\vec{B} = \begin{bmatrix} (2/3)\mu_0\vec{M} & \text{inside} \\ (4/3)\pi R^3\vec{M} & \text{outside} \end{bmatrix}$$

In terms of the magentic moment;

$$\vec{B} = \begin{bmatrix} (2/3)\mu_0\vec{M} & \text{inside} \\ \dfrac{\mu_0}{4\pi r^3}[3(\vec{m}\cot\hat{r})\hat{r} - \vec{m}] & \text{outside} \end{bmatrix}$$

The electric field in terms of its polarization and dipole moment is;

$$\vec{E} = \left[\begin{array}{cc} -(1/3\epsilon_0)\vec{P} & \text{inside} \\ \frac{1}{4\pi\epsilon_0 r^3}[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}] & \end{array} \right]$$

Substitute to get the momentum density and integrate over the volume to find the momentum in the field. There is a substantial amount of algebra. The result is ;

$$\vec{p} = \left[\begin{array}{cc} -(8/27)\mu_0\pi R^3(\vec{P}\times\vec{M}) & \text{inside} \\ -(4/27)\mu_0\pi R^3(\vec{P}\times\vec{M} & \text{outside}) \end{array} \right]$$

Adding the two components one obtains

$$\vec{p} = -(4/9)\mu_0\pi R^3(\vec{P}\times\vec{M})$$

8.11)

The magnetic field due to a rotating charge with density, $\sigma = e/(4\pi r^2)$ is

$$\vec{B} = \left[\begin{array}{cc} (2/3)\mu_0\sigma R\vec{\omega} & \text{inside} \\ \frac{2\mu_0\sigma\omega R^4}{3r^3}cos(\theta)\hat{r} + \frac{\mu_0\sigma\omega R^4}{3r^3}sin(\theta) & \text{outside} \end{array} \right]$$

The elecgtric field is;

$$\vec{E} = \left[\begin{array}{cc} 0 & \text{inside} \\ \frac{e}{4\pi\epsilon_0 r^2} & \text{outside} \end{array} \right]$$

The energy density is $\mathcal{W} = (1/2\epsilon)E^2 + (\mu_0/2)B^2$ Substitute for the fields and integrate over the volume.

$$W = \frac{e^2}{8\pi\epsilon_0 R} + \frac{\mu_0 e^2 R\omega^2}{36\pi}$$

To find the angular momentum, the angular momentun density is; $\vec{\mathcal{L}} = \vec{r}\times\vec{S}/c^2$. Sunstitute for the fields and integrate. By symmetry the angular momentum is in the $\hat{z}$ direction.

$$\vec{L} = \frac{2\mu_0\omega R}{48}\hat{z}$$

8.14)

The magentic field is;

$$\vec{B} \;=\; \begin{bmatrix} \mu_0 N I \hat{z} & \text{inside} \\ 0 & \text{outside} \end{bmatrix}$$

The electroc field is

$$\vec{E} \;=\; .\frac{q}{4\pi\epsilon_0}[(x-a)\hat{x} + y\hat{y} + z\hat{z}]$$

Substitute into the value of the momentum density;

$$\vec{\mathcal{P}} \;=\; \vec{E} \times \vec{H}/c^2 \;=\; \frac{\mu_0 qnI[(a-x)\hat{y} + y\hat{x}}{4\pi\epsilon_0}[(x-a)^2 + y^2 + z^2]^{3/2}$$

Upon integration over the volume the x component cancels by symmetry. Intgration for $-\infty < z < \infty$ yields

$$p_y \;=\; \frac{\mu_0 qnI}{4\pi\epsilon_0} \int_{-R}^{R} dx \int_{-R}^{R} dy \, \frac{a-x}{(a-x)^2 + y^2}$$

Change to Polar coordinates $x = \rho\, cos(\phi)$, $y = \rho\, sin(\phi)$, $dx\, dy = \rho\, d\rho\, d\phi$ and integrate.

$$p_y \;=\; \frac{\mu_0 qnIR^2}{2a}$$

To get the angular momentum;

$$\vec{\mathcal{L}} \;=\; \vec{r} \times \vec{\mathcal{P}} \;=\; \frac{\mu_0 qnI}{4\pi\epsilon_0[(x-a)^2 + y^2 + z^2]^{3/2}}[-x(x-a)\hat{z} - y^2\hat{z} + (x-a)z\hat{x} + yz\hat{y}]$$

Symmetry removes components in $\hat{x}$ and $\hat{y}$ Change to polar coordinates an integrate. The result is;

$$L_z \;=\; -\frac{\mu_0 qnI}{2\pi} \int \rho\, d\rho \int d\phi \, \frac{s^2 - as\, cos(\phi)}{s^2 + a^2 - 2as\, cos(\phi)}$$

$$L_z \;=\; 0$$