SET 9

9.2)

The function to test is $f(z,t) \; = \; A\, sin(kz)\, cos(kct)$

It is easily shown that this is a solution to the wave equation;

$$\frac{\partial^2 f}{\partial z^2} \; - \; (1/c)^2 \, \frac{\partial^2 f}{\partial t^2}$$

by substitution. The equation can be experssed as a standing wave by superimposing waves moving in opposite directions. Thus

$$f \; = \; (A/4i)[e^{ikz} e^{-ikz}][e^{ict} + e^{-ict}]$$

$$f \; = \; (A/4i)[e^{i[kz+ct]} \; + \; e^{-i[kz-ct]} \; + \; e^{i[kz-ct]} \; + \; e^{-i[kz+ct]}]$$

9.4)

Apply a Fourier transform to the solution $f(x,t)$

$$/ \; f(k,t) \; = \; \frac{1}{\sqrt{1/2\pi}} \int \, dk \, f(z,t) e^{-kx}$$

$$f(z,t) \; = \; \frac{1}{\sqrt{1/2\pi}} \int \, dt \, / \; f(k,t) e^{ikx}$$

Separation of variables allows a product solution a function of x and a function of k. If the above is a solution to the wave equation then the time component must have the form $e^{i\omega t}$. Thefore

$$f(z,t) \; = \; \frac{1}{\sqrt{1/2\pi}} \int \, dt \, / \; f(k,t) e^{i[kx-\omega t]}$$

9.11)

This was worked in class. See notes on waves

9.13)

This was worked in class. See notes on waves incident on a boundary.