Physics 6321

Section 14456

Classical Electrodynamics

Text - *Classical Electrodynamics* - J. D. Jackson

Ed Hungerford	Spring 2013
Office: S&R 1, Room 408	5:30 and 7:00 MW
http://mep.phys.uh.edu	S&R 1 Room 634

Week Beginning	Lecture	Subject	Homework	Due Date
Jan. 13	Ch 4,5,6	Fields, Potentials	Set 1	Jan. 23
Jan. 20	Ch 4,5,6,7	Energy, Potentials, Multipoles	Set 2	Jan. 30
		*****MLK Day Jan 21 *****		
Jan. 27	Ch 6,7,8	Symmetries, Gauge Invariance, etc	Set 3	Feb. 6
Feb. 3	Ch 7,8	Waves, Guided Waves	Set 4	Feb. 13
Feb. 10	Ch 8,	Guided Waves, Tensors	Set 5	Feb. 20
Feb. 17	Ch 11	Tensors, Relativity	Set 6	Feb. 27
Feb. 24	Ch 11, 12	Relativity, Dynamics	Set 7	Mar. 6
Mar. 3	Ch 12	Energy, Lagrangians	Set 8	Mar 20
		*****Mid Term Exam-Week of Mar. 10 *****		
Mar. 10	Ch 9	Multipoles, Vector Spherical Harmonics		
		*****Spring Break March 11-16******		
Mar. 17	Ch 9	Radiation	Set 9	Mar. 27
Mar. 24	Ch 14	Accelerating Charge	Set 10	Apr. 3
Mar. 31	Ch 10	Scattering, Diffraction	Set 11	Apr. 10
Apr. 7	Ch 13	Collisions, Cherenkov Radiation	Set 12	Apr. 17
Apr. 14	Ch 15	Bremsstrahlung, Synchrotron Radiation	Set 13	Apr. 24
Apr. 21	Ch 16	Radiation Damping	Set 14	May 1
Apr. 28				

Last Date to Drop — Mar. 27
Last Day of Class — Apr. 29
Final Exam — May 3 5-8pm

Midterm	40%
Homework	20%
Final	40%

Homework will be collected and graded each week as assigned. It will not be accepted later than the due date. Solutions to the homework and exams will be posted on the website. The midterm exam will be "in-class" sometime during the week of the 9^{th} of March. The final exam is scheduled in the University Calendar. Both exams will be 3 hours long and are **CLOSED** book. **The exams must be completed independently by you. You may NOT discuss the exam with anyone except the instructor**.

Reference Books

1. Classical Eectrodynamics; J. Schwinger - Advanced Book Program
2. Electromagnetic Theory; Bo Thilde - Webbook

3. Electrodynamics; Fulvio Meli - University of Chicago
4. Electrodynamics; C. Btau - Wiley
5. Introduction to Electrodynamics; D. Griffiths - Wiley

Course Content

1. Review
   (a) Fields in terms of potentials
   (b) Multipole expansions
   (c) Cylindrical waveguides and fiber optics
   (d) gauge invarience
   (e) Continuous groups
2. Relativity
   (a) postulates
   (b) Lorentz transformation from postulates
   (c) General time, length, velocity, acceleration transformations
   (d) images of objects in relativistic motion
   (e) Doppler shift - headlight effect
   (f) twin paradox
   (g) 4-vector notation 4-space, 4-velocity, 4-momentum, 4-potential, etc
3. Covarient Notation
   (a) tensor algebra
   (b) differential forms, transfornations
   (c) covarient 4-vectors, field tensor
   (d) covarient Maxwell's equations
   (e) covarient stress tensor
   (f) Lorentz group
   (g) spin, angular momentum, Thomas precession
4. Lagrangian Formulation
   (a) Classical EM Field
   (b) Charged Particle
   (c) gauge transformation
   (d) phase space
5. Radiation
   (a) scalar spherical harmonics, spherical eigenfunctions and operators

(b) expansion of a vector field in vector spherical harmonics
(c) generalized radiation problem, multipole radiation
(d) Multipole connection to the source geometry
(e) covarient Green's function
(f) radiation and coherence

6. Bremsstrahlung
   (a) Lienard-Wiechart Potentials
   (b) radiation from an accelerated charge
   (c) interpretation in terms of causality
   (d) radiation in collisions
   (e) Cherenkov radiation
   (f) synchrotron radiation
   (g) free electron lasers

7. Scattering
   (a) Integral formulation
   (b) peturbation series and Green's function
   (c) Rutherford scattering
   (d) virtural photon and the Weizsacker-Williams formulation
   (e) Thomson scattering
   (f) Diffraction - Scalar and Vector

8. Plasma
   (a) Debye shielding
   (b) pinch and magnetic mirrors
   (c) magnetohydrodynamic waves
   (d) nonlinear behavior in RF accelerators

Homework Sets

Set1.

1.) Two long, straight wires are separated by a distance, d, and each carries a current, I, which flows in opposite directions. Find the vector potential, $\vec{A}$, and show that, $\vec{\nabla} \cdot \vec{A} = 0$. Choose an origin half-way between, and in the plane of, the wires.

2.) Derive equation 5.39 in the text

3.) Dirac proposed a vector potential for a magnetic monopole (eqn 6.161);

$$\vec{A} = \frac{g}{4\pi} \int_L d\vec{l'} \times \frac{\vec{x} - \vec{x}'}{|\vec{x} - \vec{x}'|^3}$$

In this equation g is the strength of the magnetic charge, and the integral is over the string path, L, from ∞ to the position of the pole. The monopole is independent of the string path, so put the pole at the origin and the string along the negative z axis. Find $\vec{A}$ and verify that $\vec{\nabla} \times \tilde{\mathbf{A}}$ produces a magnetic field proportional to $\frac{g}{r^2}$

Set 2.

1.) A volume charge, Q, has a surface described by the function;

$$\mathrm{R}(\theta) = \mathrm{R}_0[1. + \beta\, \mathrm{P}_2(\cos(\theta))];$$

Find the multipole expansion.

2.) A localized charge distribution has charge density;

$$\rho(r) \;=\; (1/64\pi)\, r^2\, e^{-r}\, sin^2(\theta)$$

Find the multipole expansion.

3.) The l^{th} term in the multipole expansion of the potential is specified by the q_{lm} multipole moments. Find the multipole moments through l=2 in Cartesian coordinates.

Set 3.

1.) A uniform, conducting spherical shell of radius, R, mass, M, and resistivity, σ, is placed in a uniform magnetic field parallel to the z axis, $B_0\hat{z}$. The sphere is given an initial angular rotational velocity, ω_0 about the z axis. How much time is required for the angular velocity of the sphere to decrease by a factor of 2?

2.) Show the retarded potentials satisfy the Lorentz gauge condition.

3.) The 4-vector potential of a plane electromagnetic wave traveling in the $\hat{k}$ direction is; $A_\alpha = a^\alpha\, exp(ik^\alpha r_\alpha)$ where $a_\alpha = (\phi/c, \vec{a})$ and $k^\alpha = (\omega/c, \vec{k})$. What is the condition on a_α in the Lorentz gauge ?

Set 4.

1.) Two thin wires lie in a plane, spaced a distance, d, apart. Find the lowest TEM mode of propagation for an electromagnetic wave (current wave) carried by these coductors.

2.) A resonant cavity consists of the hollow within a perfectly-conducting, spherical shell of radius, a. Write the equation for the characteristic frequencies of the cavity assuming H is always in the $\hat{\phi}$ direction (spherical coordinates).

3.) Find the TM cutoff frequencies for a hollow, perfectly-conducting cylinder of radius a.

Set 5.

1.)

For a general orthorgonal, curvilinear coorinate system find the differential area element and the Jacobian for the transformation to this system from a cartesian set

2.)

An antisymmetric array is given by the matrix;

$$\begin{pmatrix} 0 & C_{12} & C_{13} \\ -C_{12} & 0 & C_{23} \\ -C_{13} & -C_{23} & 0 \end{pmatrix}$$

Note that $(C_{23}, -C_{13}, C_{12})$ form an axial vector. Now show that;

$$C_i = (1/2!)\,\epsilon_{ijk}\,C^{jk}$$

If C^{jk} are the elements of a 2-dimensional tensor and ϵ_{ijk} form the elements of the 3-dimensional Levi-Civita tensor.

3.) The operator, $\vec{\nabla} \cdot \vec{\nabla} - \frac{1}{c} \frac{\partial^2}{\partial t^2}$ may be written in 4-dimensional space using $x_4 = ict$. Write this in Cartesian tensor notation and show that it forms a scalar under a Lorentz transformation.

Set 6.

1.) Show that the 4-velocity of a particle is always a unit vector.

2.) An aircraft can fly faster than the speed of light in vacuum. After flying 6000 km in 10^{-2}s the plane crashes. Show that there is a Lorentz frame in which the plane crashes before it begins its flight.

3.) Show that it is possible to out-run a light ray if you are given a sufficient head start, and your feet apply a constant acceleration.

Set 7.

1.) Show explicitedly that Maxwell's equations are consistent with special relativity. What asumptions are necessary for charge, volume, and current transformations?

2.) Assume a rocket leaves the earth in 2000. It uses accelerations of magnitude, g, always in a straight line. It accelerates for 5 years away from earth, decelerates for 5 years, turns around and accelerates for 5 years, and finally decelerates for 5 years, returning to its starting point (all times measured in the rest frame of the rocket). When the rocket, leaves one identical twin is placed onboard and the other is left behind. The one on board ages 40 years. How old is the twin on earth?

3.) In problem 2 above, what was the maximum distance the rocket traveled from earth?

Set 8.

1.) A cylinder of radius, a, and length, L, has a uniform surface charge, σ, on the circular end caps and no charge on the cylindrical surface. Find the force between the top half and bottom half of the cylinder.

2.) For an infinite parallel plate capacitor with separation between the plates of d and charge per unit area, σ, on the surfaces, find the elements of the Maxwell stress tensor and the force per unit area between the plates using this tensor.

3.) For an electric charge, q, and a magnetic charge, g, separated by a distance, d, find the total angular momentum stored in the resulting electromagnetic fields.

Set 9.

1.) Derive the expression for the longitudinal precession of the spin of a particle in an external electric and magnetic field. (eqn 11.171)

2.) For a particle of mass, m, moving in an external scalar potential, ψ, consider the action:

$$\mathrm{S} = \int \mathrm{L}dt,$$

with $\mathrm{L} = -(\mathrm{m}+\mathrm{g}\psi)\sqrt{1-\mathbf{v}^2}$ and g a dimensionless coupling constant. Find the equations of motion for $\mathbf{x}(\mathrm{t})$ and show that they form the space components of a consistent 4-vector equation.

3.) Show that a gauge transformation $A_\mu \rightarrow A_\mu + \partial\Lambda/\partial x_\mu$ of the potentials in the charged particle Lagrangian generates an equivalent Lagrangian.

Set 10.

1.) A volume charge, Q, has a surface described by the function;

$\mathrm{R}(\theta) = \mathrm{R}_0[1. + \beta \mathrm{P}_2(\cos(\theta))]$;

where β is small and varies with time as $e^{-i\omega t}$. This provides a model for surface waves on a sphere. Keeping terms to first order in β and using the long wavelength approximation, find the multipole moments of the radiation.

2.) The north pole of the earth (really magnetically a south pole) is not aligned with the rotational axis of the earth (it is off by 11°). Find the radiated power from the earth due to magnetic dipole radiation.

3.) Find the rate of angular momentum radiated from a classical electric dipole.(see problem 9:8)

Set 11.

1.) An electron is constantly decelerated from a velocity, v_0 down to zero. What fraction of its kinetic energy is lost to radiation?

2.) If quantum mechanics were not valid, how long would it take an electron to spiral into the nucleus? (neglect radiation damping)

3.) A heavy particle of charge Ze, Mass, M, collides with a free electron. Assume non-relativistic velocities and that the electron is initially at rest. Find the energy transferred to the electron as a function of the impact parameter.

Set 12.

1.) Find the total radiation from a relativistic particle which passes with impact parameter, d, through a Couloumb field with fixed center. (potential $= \frac{1}{4\pi\epsilon_{zero} r}$)

2.) Derive equation 15.61 of the text.

3.) For water, n=1.33, find the Cerenkov angle and number of photons radiated for a cosmic ray muon of momentum 2 GeV.

Set 13.

1.) Find the diffraction due to a circular hole in a conducting sheet using the Kirchoff equations for plane waves incident at an oblique angle with polarizations perpendicular to the plane of incidence.

2.) Find the scattering from a small, lossy dielectric sphere that can be represented by a dielectric constant, ϵ, and a conductivity, ρ. The skin depth is small compared to the radius.

3.) Find the total power scattered from a conducting sphere of small radius when illuminated by a plane electromagnetic wave of frequency, ω.

Set 14.

1.) In problem 2 of set 12 above, repeat the calculation including radiation damping.

2.) What additional force would be needed to maintain an electron in problem 1 set 14 in the same orbit. (do this calculation non-relativistically)

3.) Work out problem 16.7 of the text.