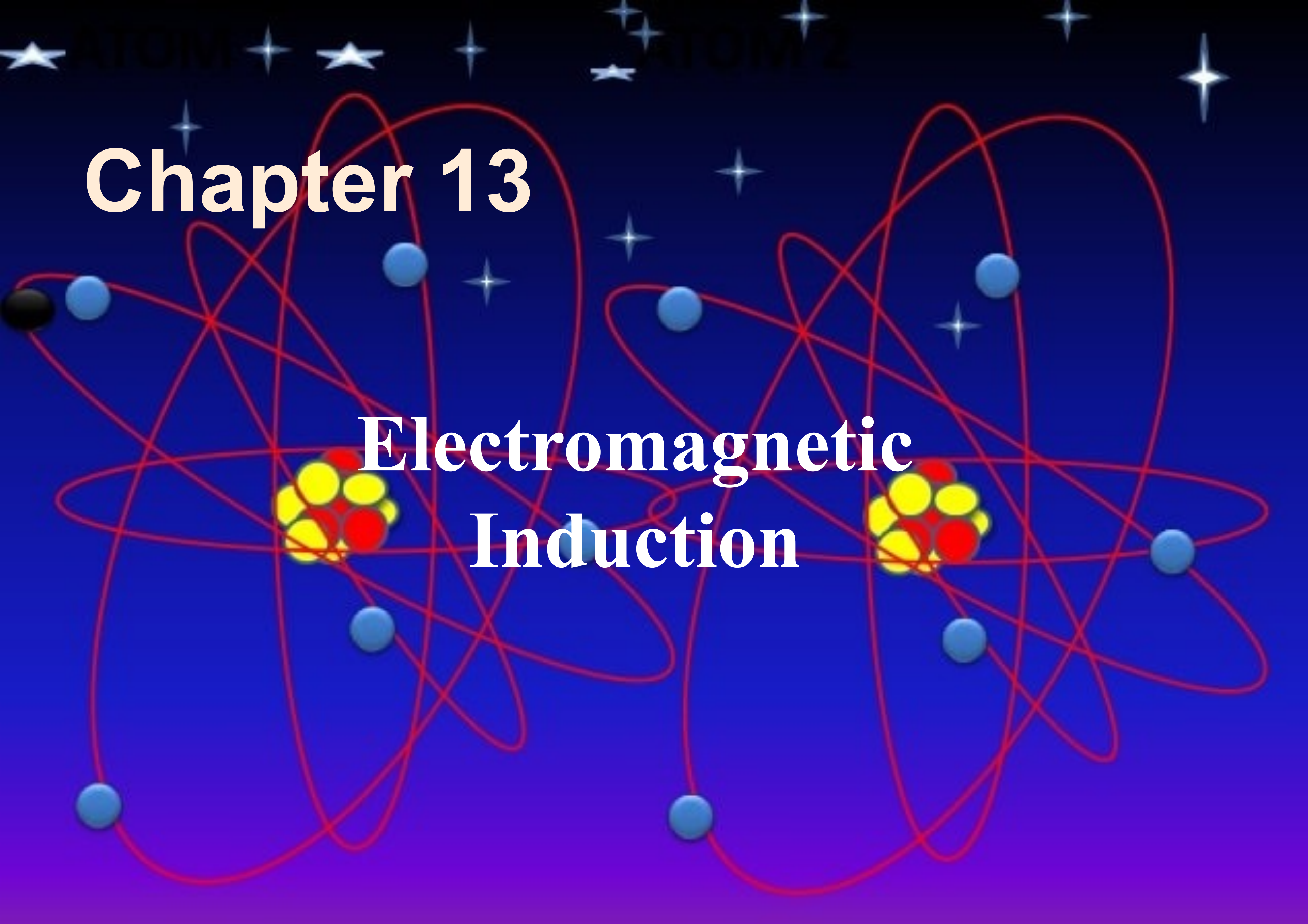


Chapter 13

Electromagnetic Induction

Learning Goals for Chapter 13

Looking forward at …

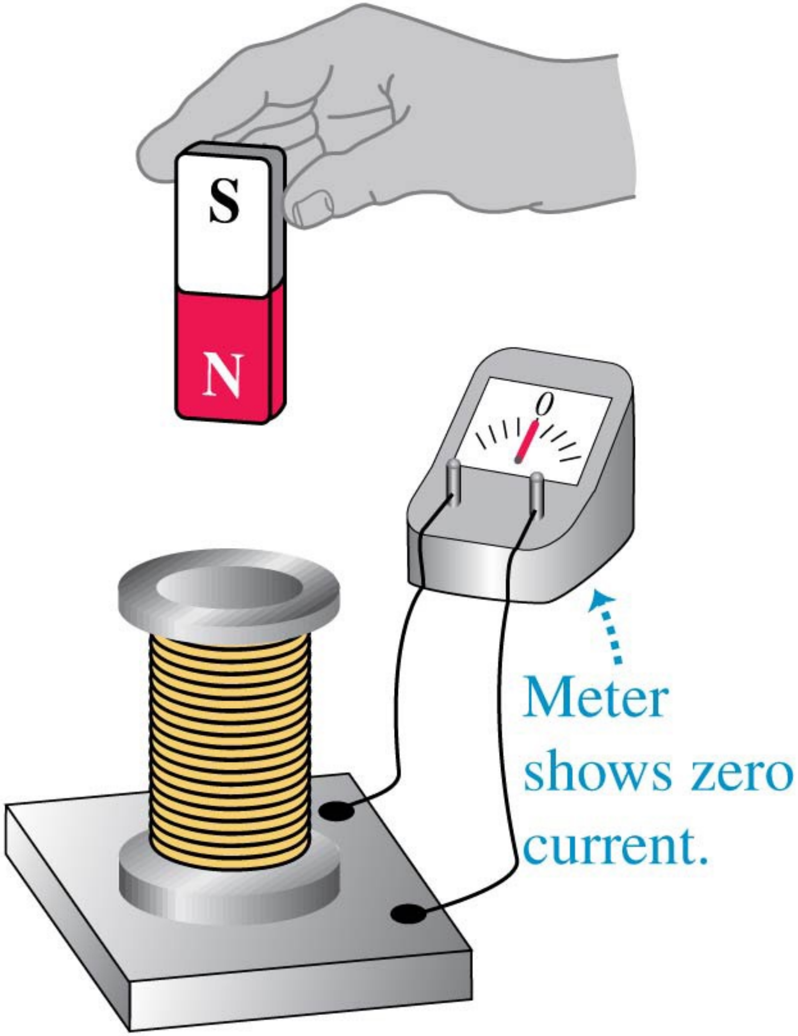
- how Faraday’s law relates the induced emf in a loop to the change in magnetic flux through the loop.
- how to determine the direction of an induced emf.
- how a changing magnetic flux generates a circulating electric field.
- the four fundamental equations that completely describe both electricity and magnetism.
- the remarkable electric and magnetic properties of superconductors.

Introduction

- The card reader at a gas station scans the information that is coded in a magnetic pattern on the back of your card.
- Why must you remove the card quickly rather than hold it motionless in the card reader's slot?
- Energy conversion makes use of electromagnetic induction.
- Faraday's law and Lenz's law tell us about induced currents.
- Maxwell's equations describe the behavior of electric and magnetic fields in *any* situation.

Induction experiment: Slide 1 of 4

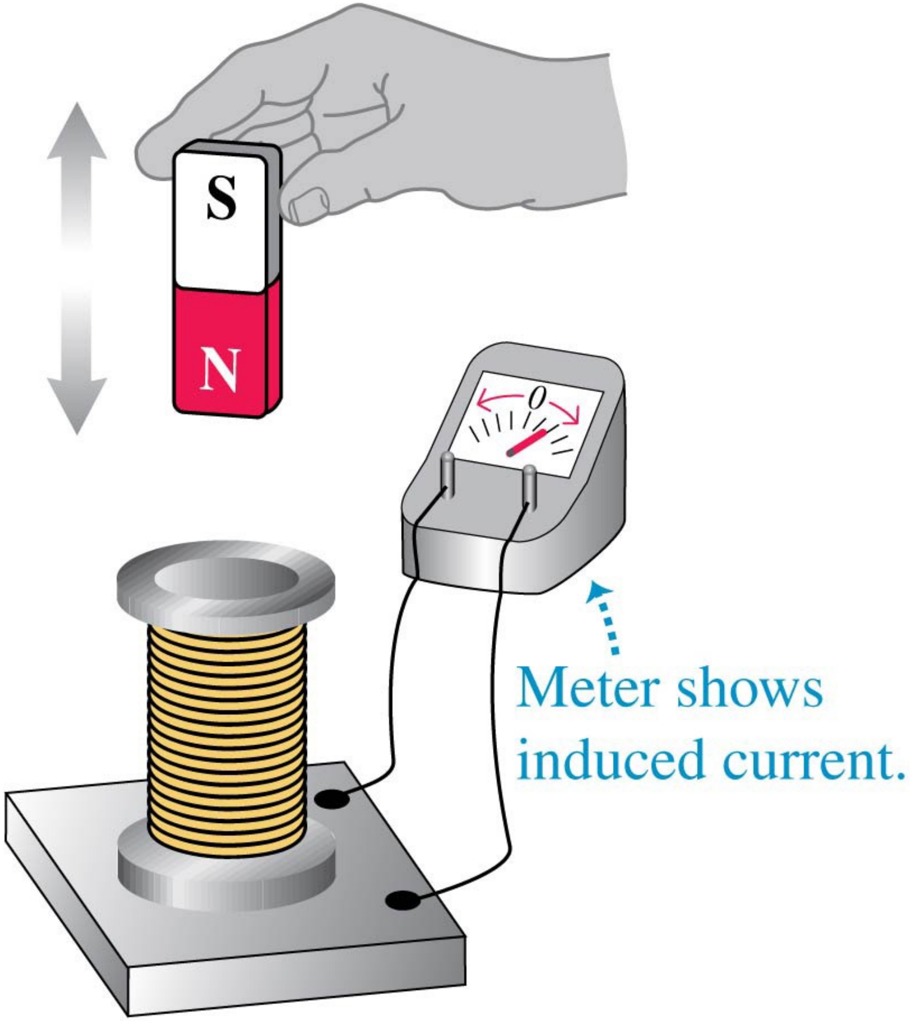
- During the 1830s, several pioneering experiments with magnetically induced emf were carried out.
- In the figure shown, a coil of wire is connected to a galvanometer.
- When the nearby magnet is stationary, the meter shows no current.



Meter shows zero current.

Induction experiment: Slide 2 of 4

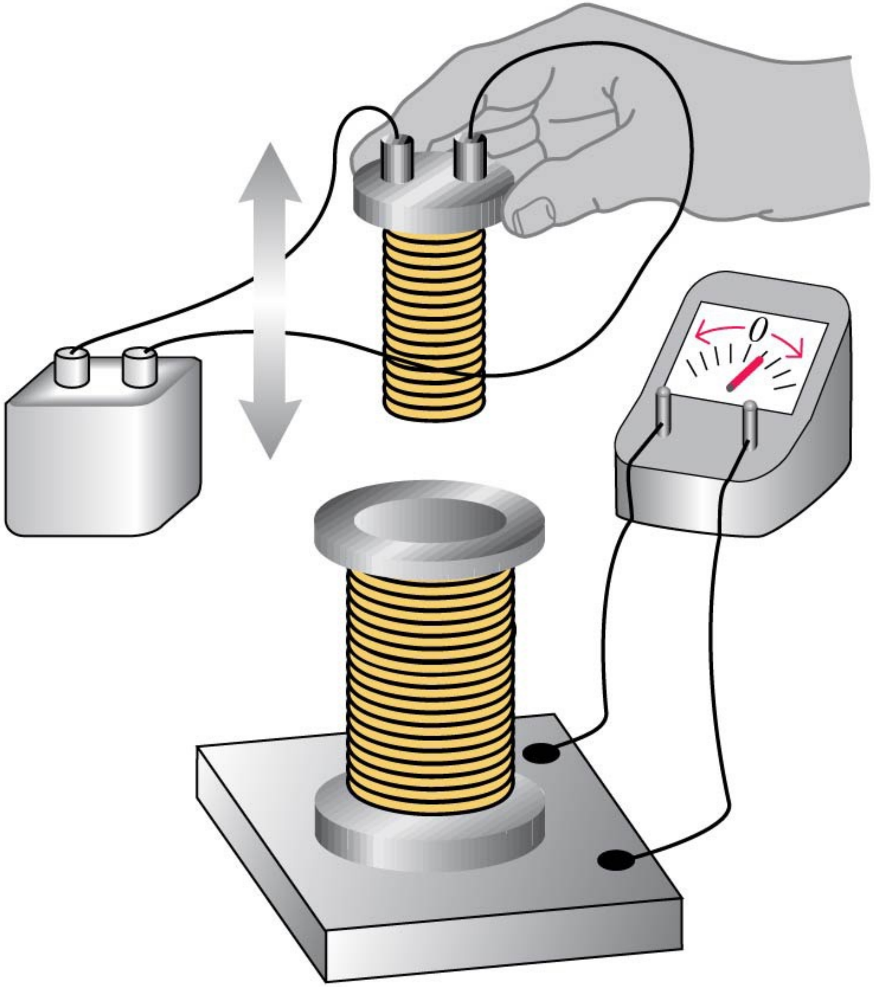
- When we move the magnet either toward or away from the coil, the meter shows current in the circuit, but only while the magnet is moving.
- We call this an **induced current**, and the corresponding emf required to cause this current is called an **induced emf**.



Meter shows induced current.

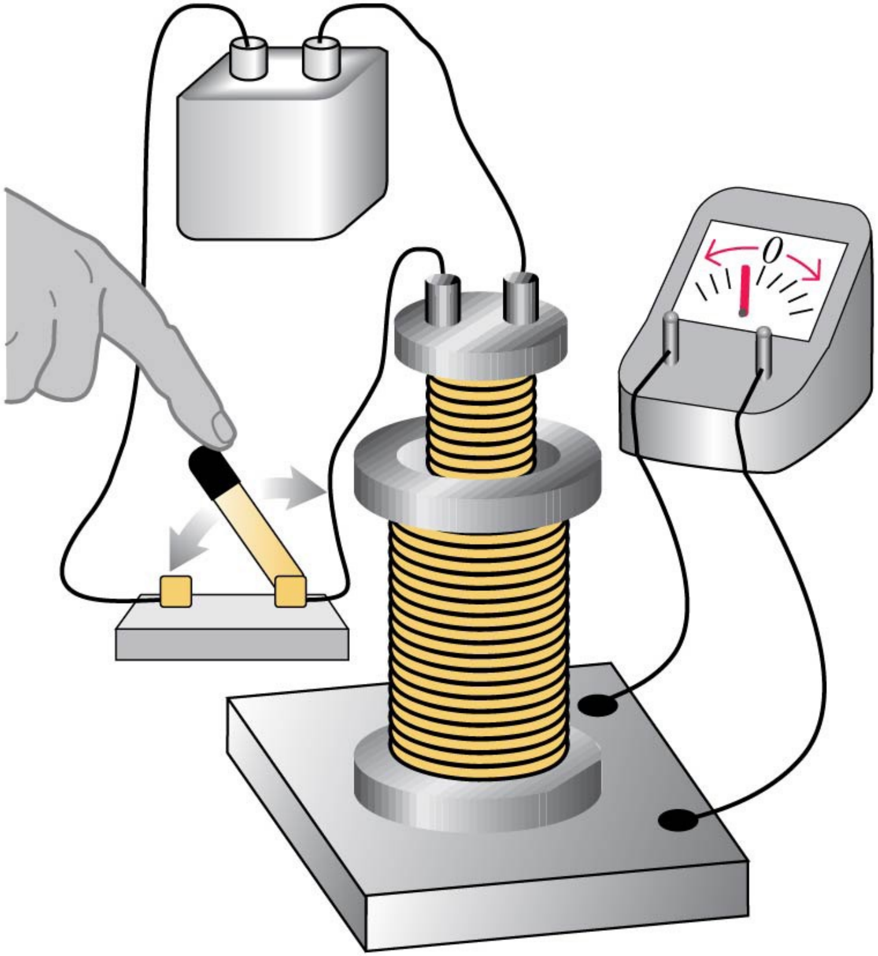
Induction experiment: Slide 3 of 4

- In this figure we replace the magnet with a second coil connected to a battery.
- When we move the second coil toward or away from the first, there is current in the first coil, but only while one coil is moving relative to the other.



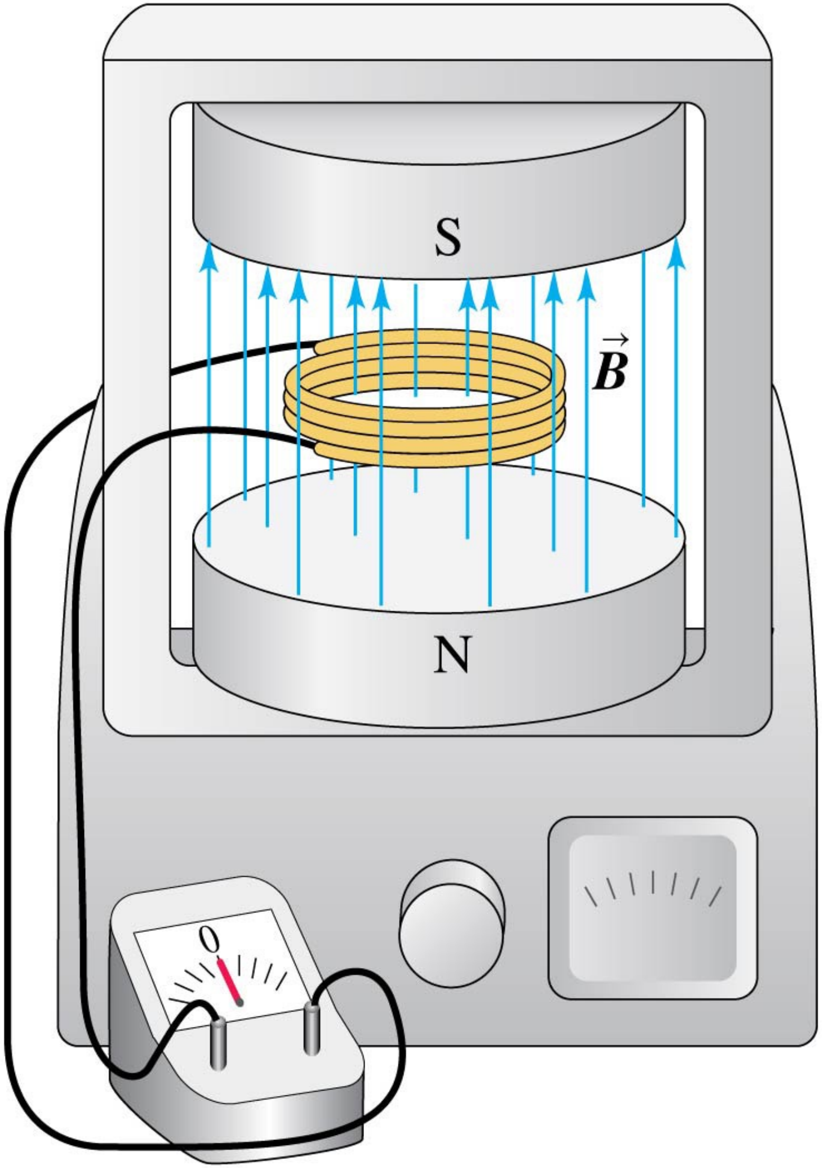
Induction experiment: Slide 4 of 4

- Using the two-coil setup of the previous slide, we keep both coils stationary and vary the current in the second coil by opening and closing the switch.
- The induced current in the first coil is present only while the current in the second coil is *changing*.



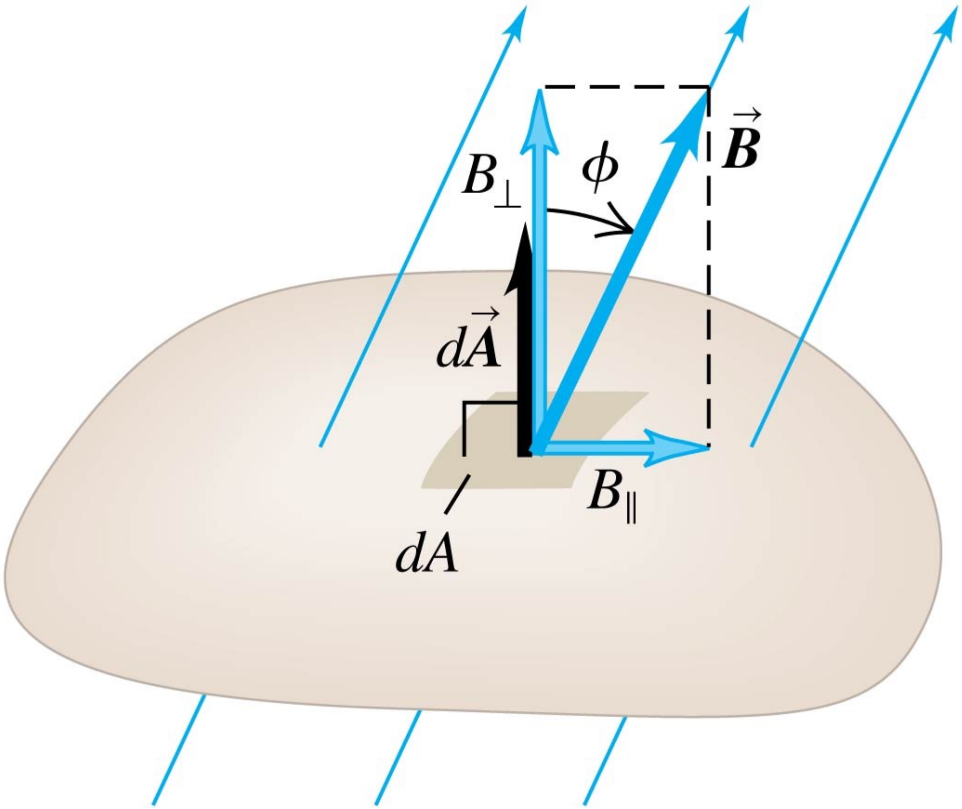
A coil in a magnetic field

- Shown is a coil in a magnetic field.
- When the magnetic field is constant and the shape, location, and orientation of the coil do not change, no current is induced in the coil.
- A current is induced when any of these factors *change*.



Magnetic flux (Review of Section 27.3)

- To define the *magnetic flux*, we can divide any surface into elements of area dA.
- The magnetic flux through the area element is defined to be $d\Phi_B = B_\perp dA$.

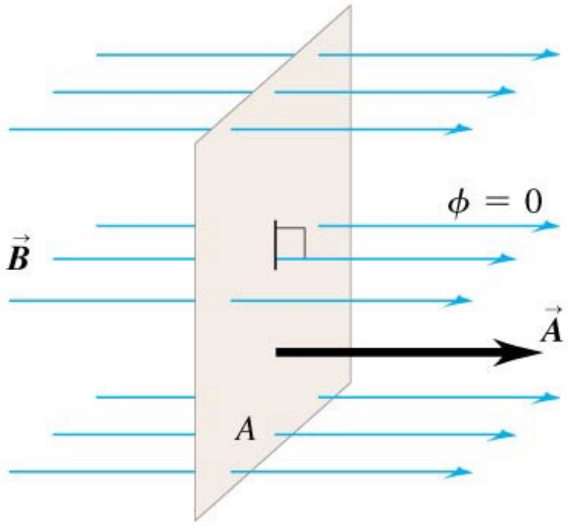


Magnetic flux through a flat area: Orientation 1 of 3

- The maximum magnetic flux through a surface occurs when the surface is face-on to the magnetic field.
- In this case the magnetic flux is simply *BA*.

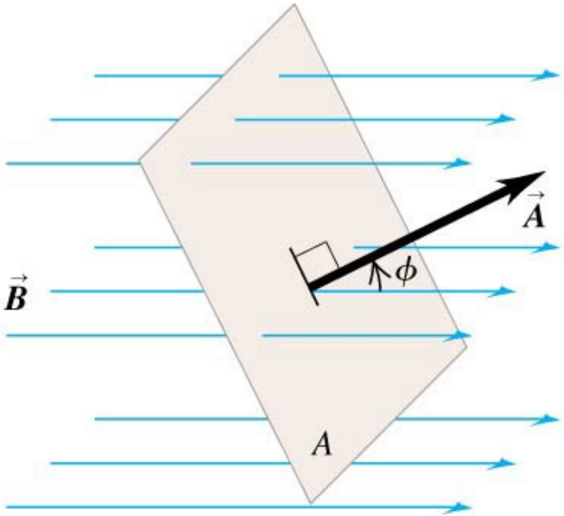
Surface is face-on to magnetic field:

- $\vec{B}$ and $\vec{A}$ are parallel (the angle between $\vec{B}$ and $\vec{A}$ is $\phi = 0$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA$.



Magnetic flux through a flat area: Orientation 2 of 3

- When the surface is at some angle relative to the magnetic field, the magnetic flux is between 0 and BA.

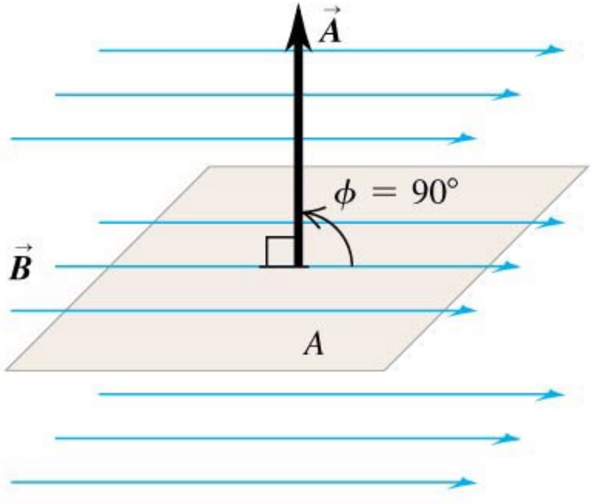


Surface is tilted from a face-on orientation by an angle ϕ:

- The angle between $\vec{B}$ and $\vec{A}$ is ϕ.
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\phi$.

Magnetic flux through a flat area: Orientation 3 of 3

- When the surface is edge-on to the magnetic field, the magnetic flux through the surface is zero.



Surface is edge-on to magnetic field:

- $\vec{B}$ and $\vec{A}$ are perpendicular (the angle between $\vec{B}$ and $\vec{A}$ is $\phi = 90°$).
- The magnetic flux $\Phi_B = \vec{B} \cdot \vec{A} = BA\cos 90° = 0$.

Faraday's law of induction

- When the magnetic flux through a single closed loop changes with time, there is an induced emf that can drive a current around the loop:

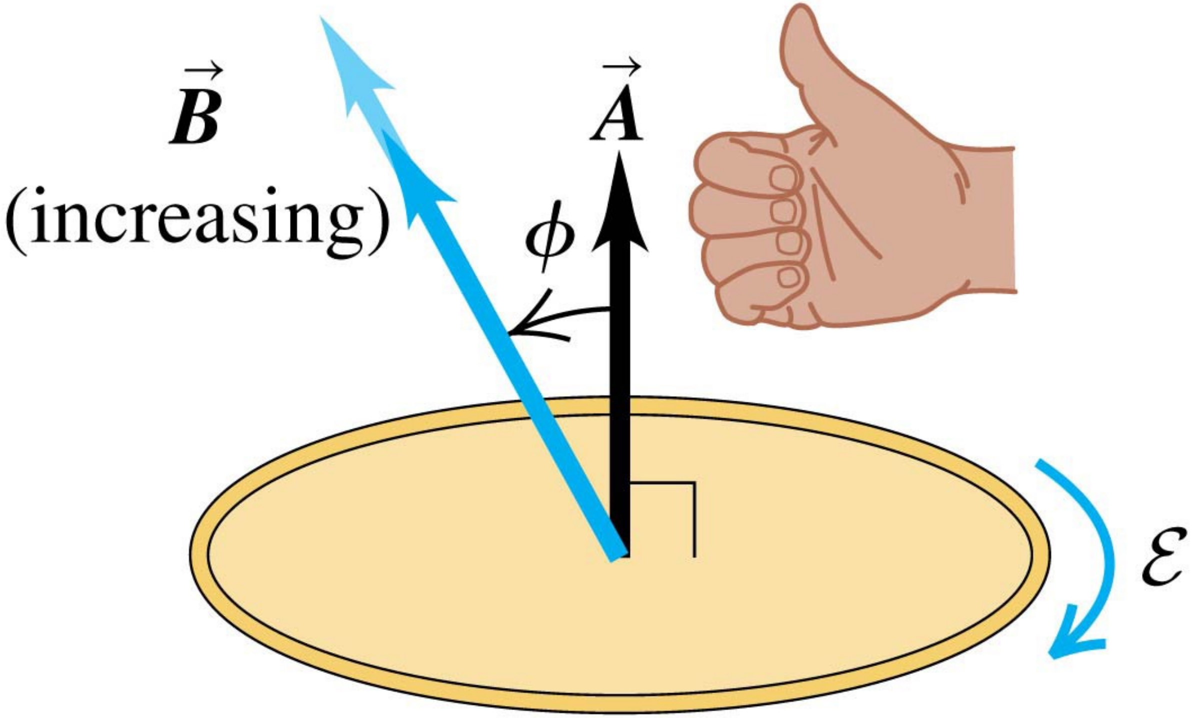
Faraday's law:
The induced emf in a closed loop ...

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

... equals the negative of the time rate of change of magnetic flux through the loop.

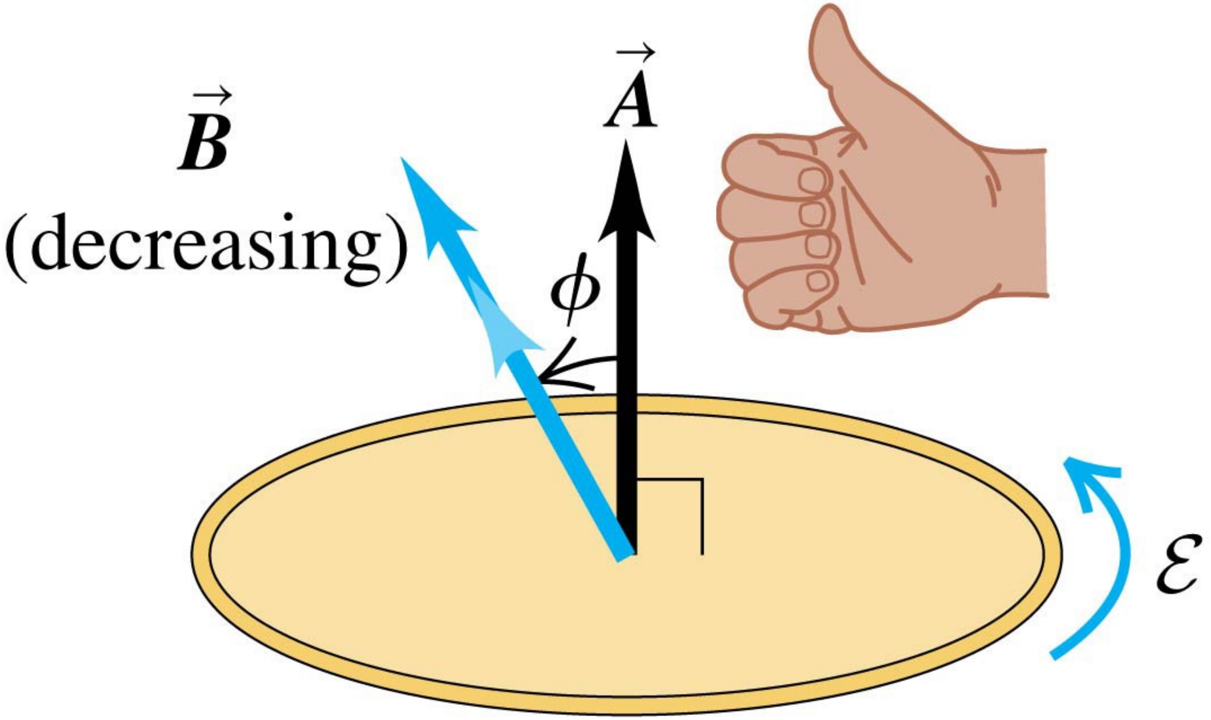
- Recall that the unit of magnetic flux is the weber (Wb).
- 1 T · m^2 = 1 Wb, so 1 V = 1 Wb/s.

Determining the direction of the induced emf: Slide 1 of 4



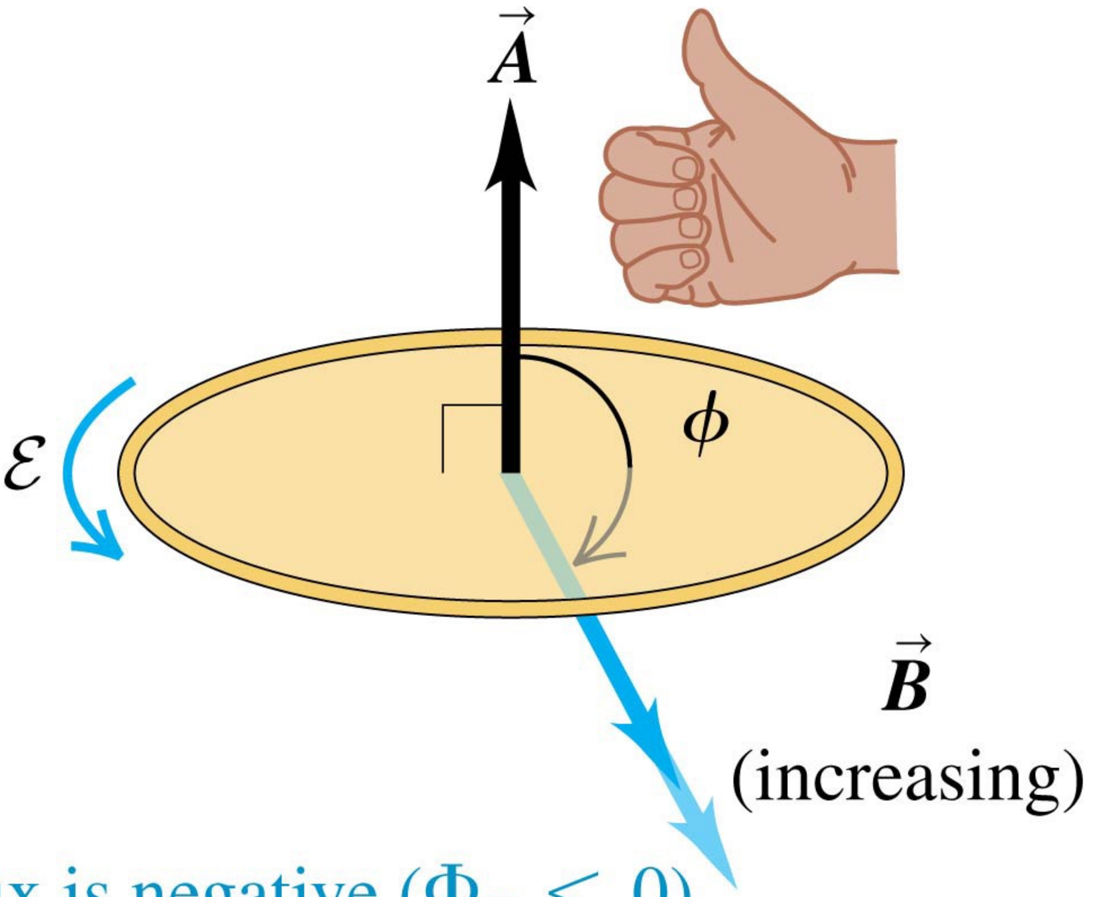
- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming more positive ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

Determining the direction of the induced emf: Slide 2 of 4



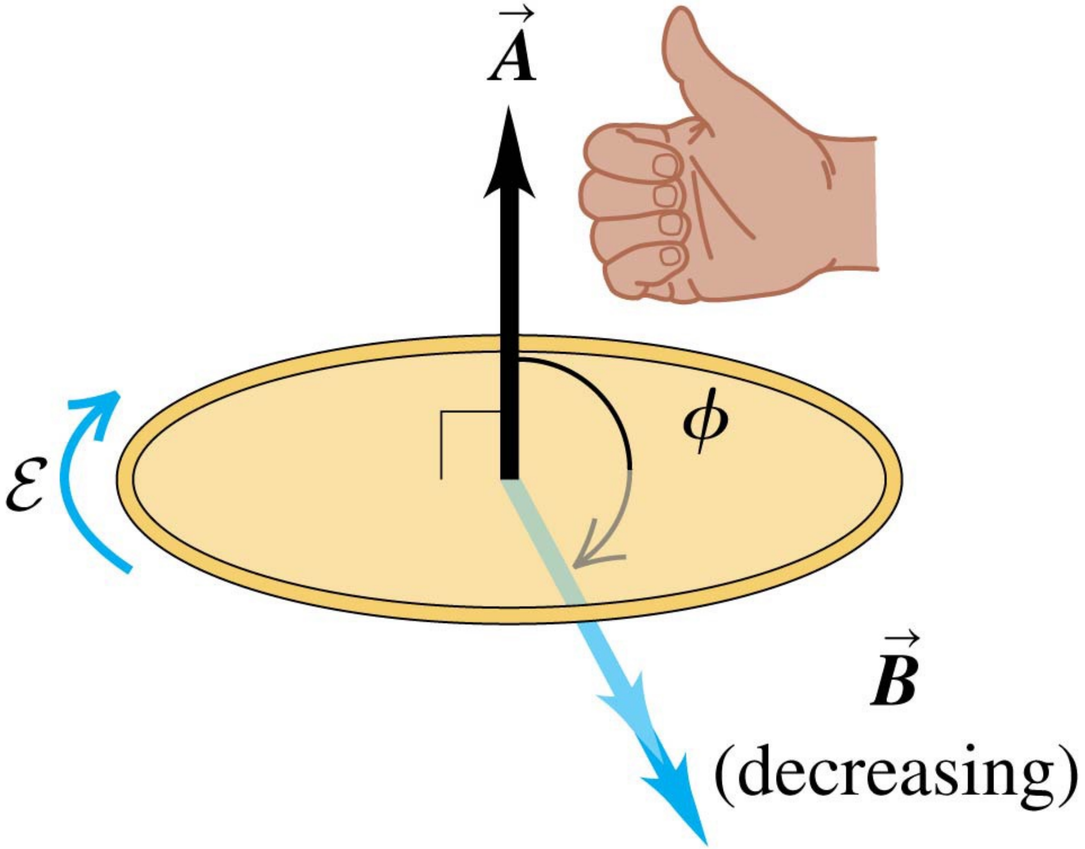
- Flux is positive ($\Phi_B > 0$) ...
- ... and becoming less positive ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).

Determining the direction of the induced emf: Slide 3 of 4



- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming more negative ($d\Phi_B/dt < 0$).
- Induced emf is positive ($\mathcal{E} > 0$).

Determining the direction of the induced emf: Slide 4 of 4



- Flux is negative ($\Phi_B < 0$) ...
- ... and becoming less negative ($d\Phi_B/dt > 0$).
- Induced emf is negative ($\mathcal{E} < 0$).

Faraday’s law for a coil

- A commercial alternator uses many loops of wire wound around a barrel-like structure called an armature.
- The resulting induced emf is far larger than would be possible with a single loop of wire.

- If a coil has N identical turns and if the flux varies at the same rate through each turn, total emf is:

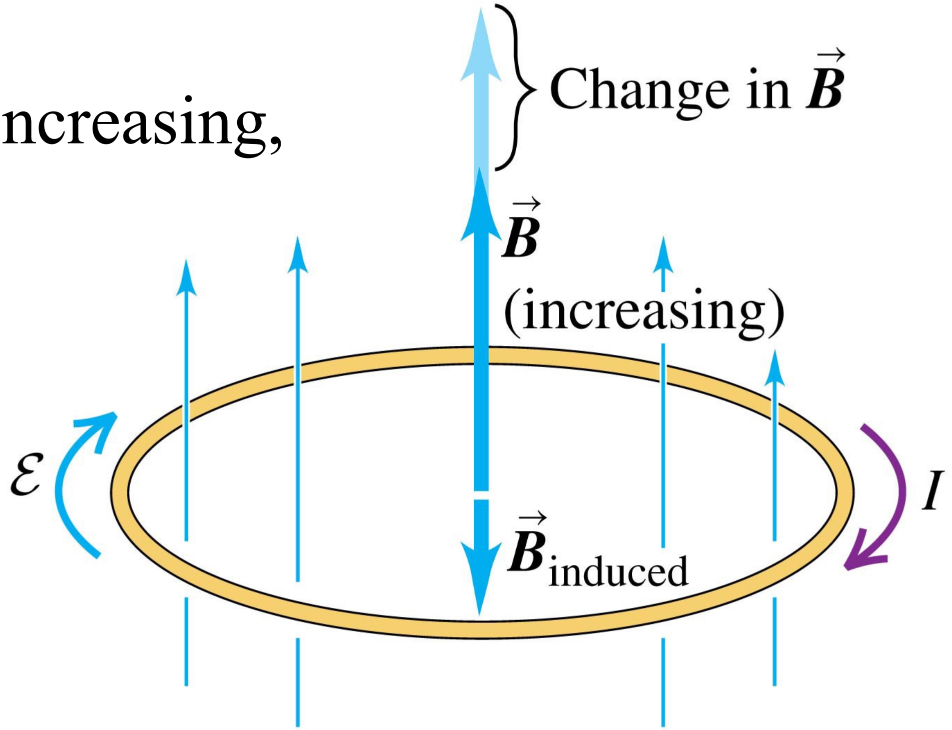
$$\mathcal{E} = -N\frac{d\Phi_B}{dt}$$

Lenz’s law

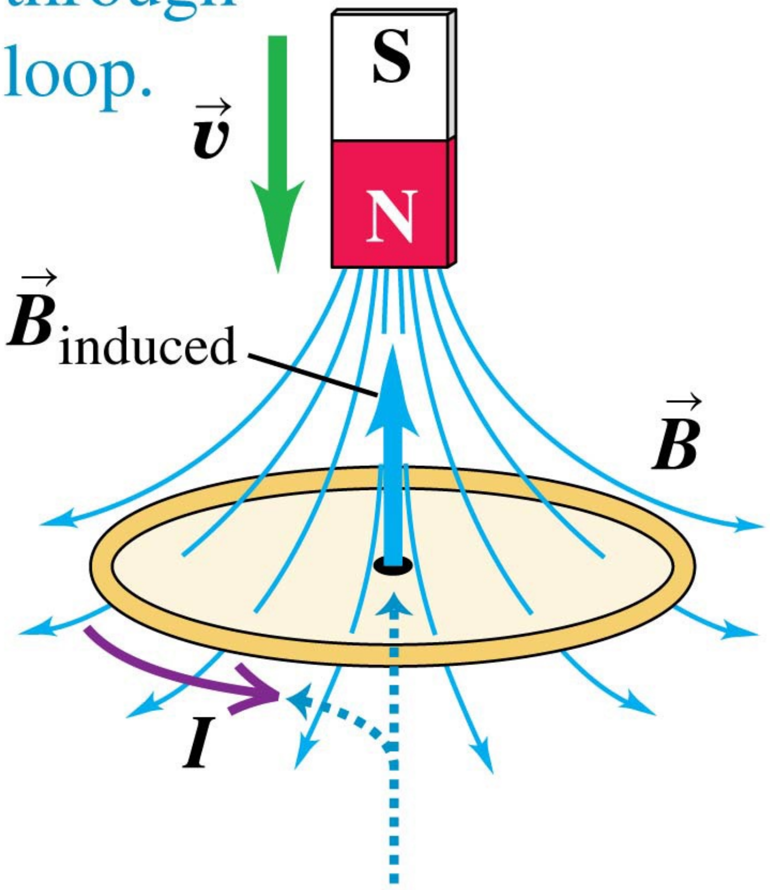
- Lenz’s law is a convenient method for determining the direction of an induced current or emf:

> **The direction of any magnetic induction effect is such as to oppose the cause of the effect.**

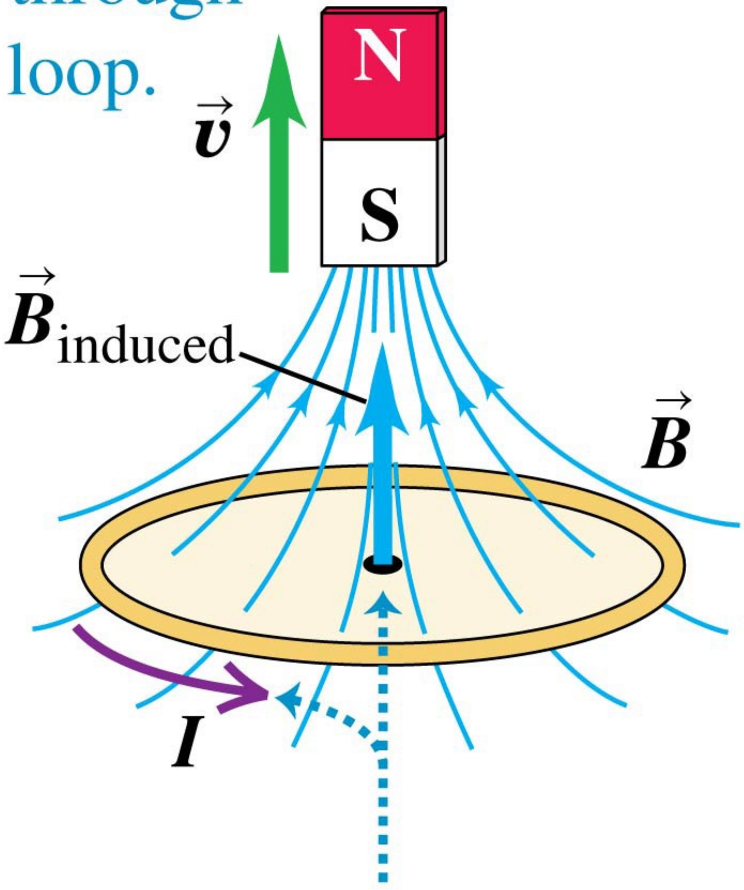
- For example, in the figure there is a uniform magnetic field through the coil.
- The magnitude of the field is increasing, so there is an induced emf driving a current, as shown.



Lenz’s law and the direction of induced current



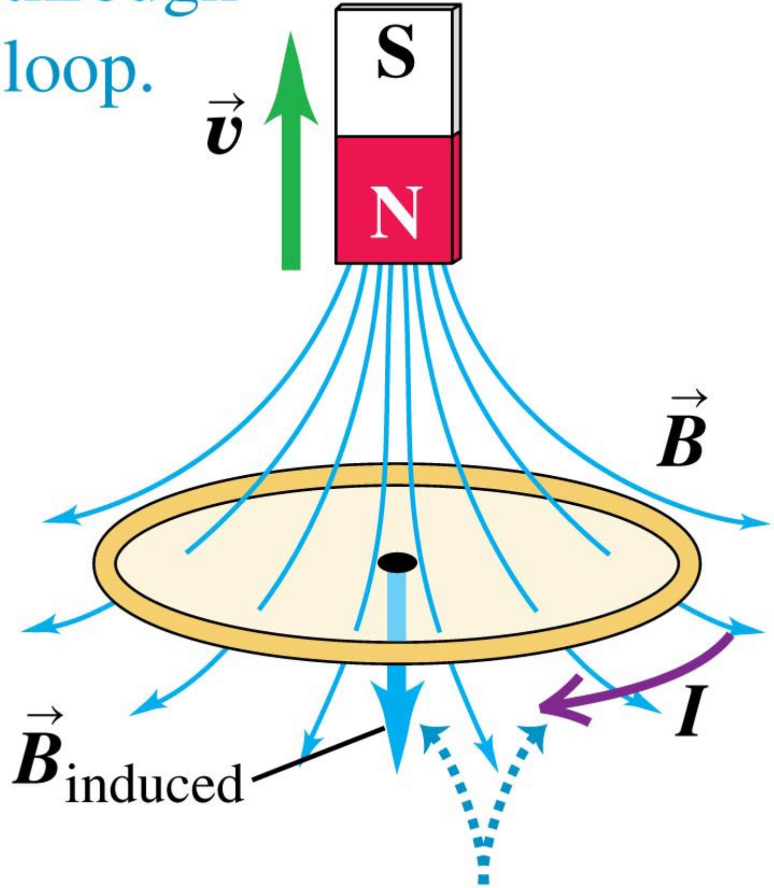
Motion of magnet causes *increasing downward* flux through loop.



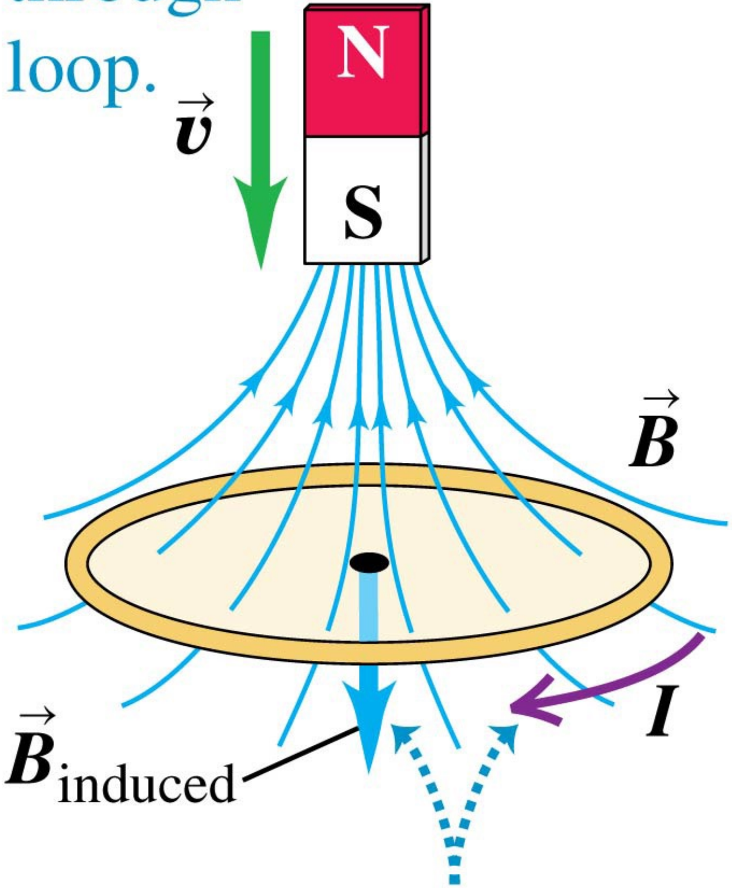
Motion of magnet causes *decreasing upward* flux through loop.

Lenz’s law and the direction of induced current

Motion of magnet causes *decreasing downward* flux through loop.



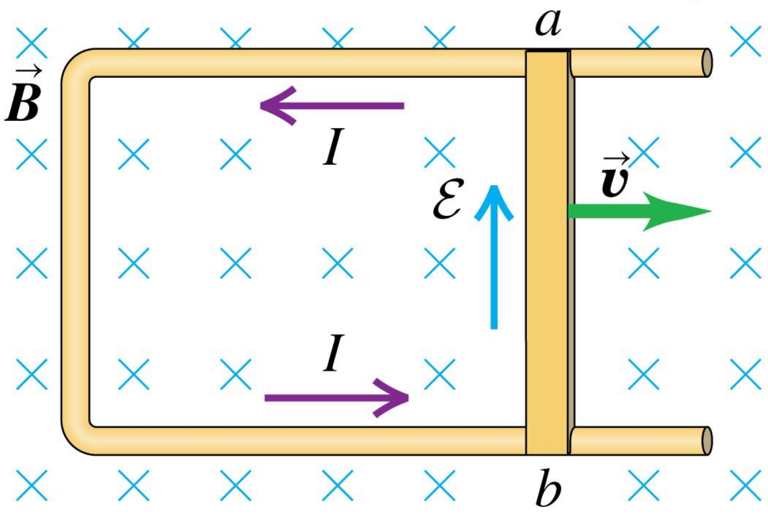
Motion of magnet causes *increasing upward* flux through loop.



Motional electromotive force

- When a conducting rod moves perpendicular to a uniform magnetic field, there is a **motional emf** induced.

Rod connected to stationary conductor



The motional emf $\mathcal{E}$ in the moving rod creates an electric field in the stationary conductor.

Motional emf, conductor length and velocity perpendicular to uniform $\vec{B}$

$$\mathcal{E} = vBL$$

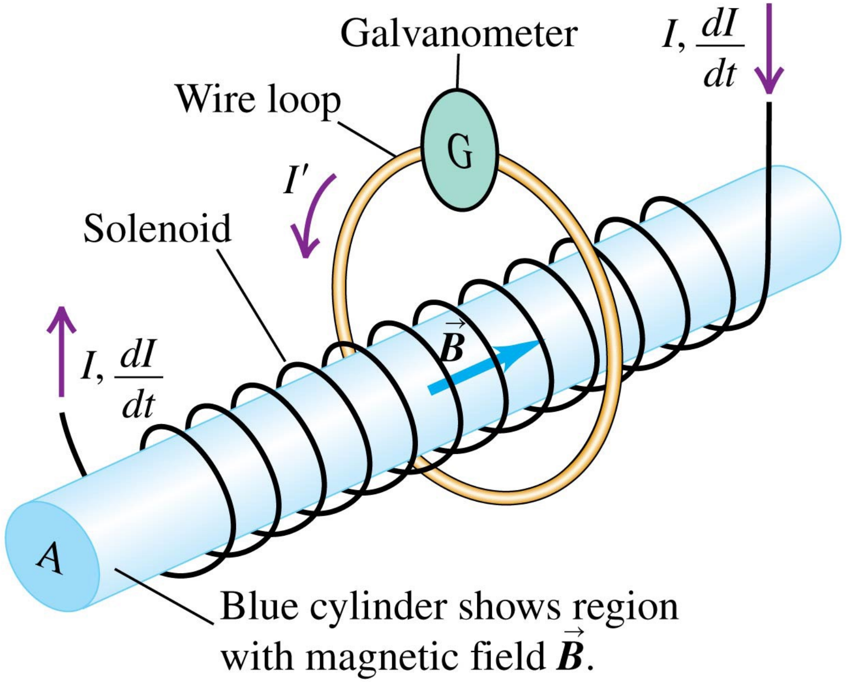
v: Conductor speed

L: Conductor length

B: Magnitude of uniform magnetic field

Induced electric fields

- A long, thin solenoid is encircled by a circular conducting loop.
- Electric field in the loop is what must drive the current.
- When the solenoid current *I* changes with time, the magnetic flux also changes, and the induced emf can be written in terms of **induced electric field**:



Blue cylinder shows region with magnetic field $\vec{B}$.

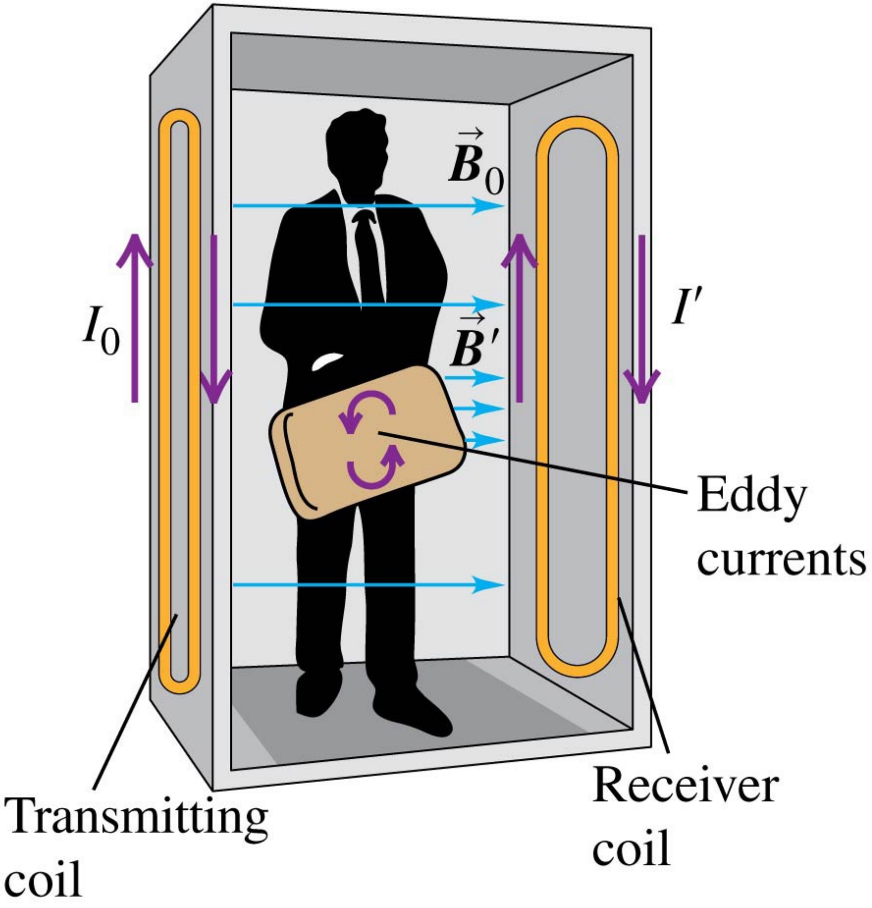
Faraday's law for a stationary integration path:

$$\overbrace{\oint \vec{E}\cdot d\vec{l}}^{\text{Line integral of electric field around path}} = -\underbrace{\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

Eddy currents

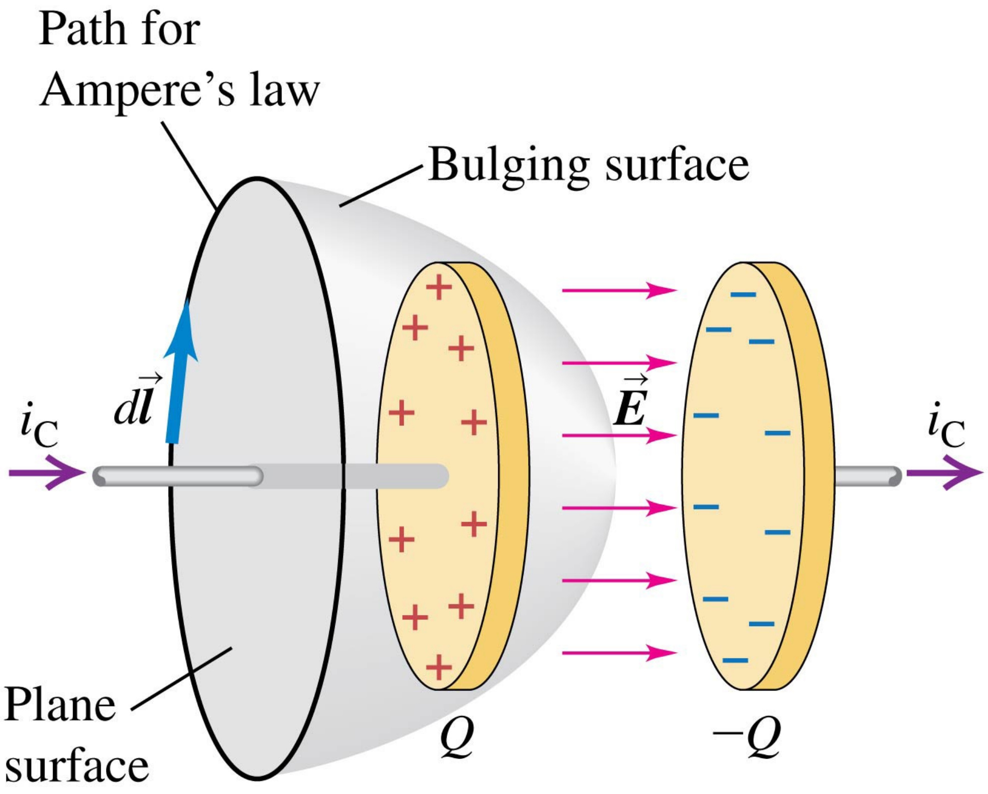
- When a piece of metal moves through a magnetic field or is located in a changing magnetic field, **eddy currents** of electric current are induced.

- The metal detectors used at airport security checkpoints operate by detecting eddy currents induced in metallic objects.



Displacement current

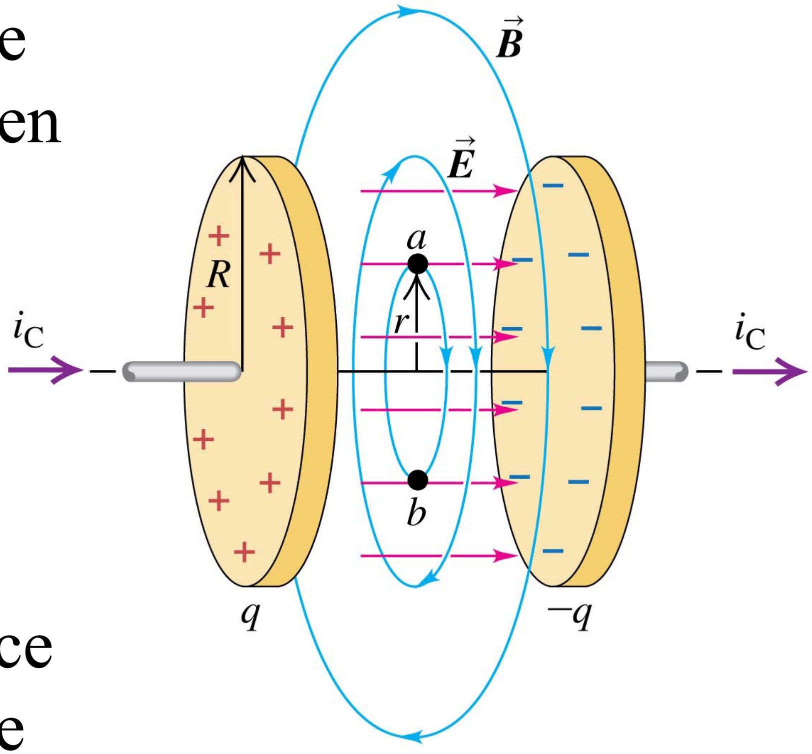
- Ampere’s law is *incomplete*, as can be shown by considering the process of charging a capacitor, as shown.



- For the plane circular area bounded by the circle, I_{encl} is the current i_C in the left conductor.
- But the surface that bulges out to the right is bounded by the same circle, and the current through that surface is zero.
- This leads to a contradiction.

Displacement current

- When a capacitor is charging, the electric field is increasing between the plates.
- We can define a fictitious **displacement current** i_D in the region between the plates.
- This can be regarded as the source of the magnetic field between the plates.



Displacement current through an area

$$i_D = \epsilon \frac{d\Phi_E}{dt}$$

Time rate of change of electric flux through area

Permittivity of material in area

Maxwell’s equations of electromagnetism

- All the relationships between electric and magnetic fields and their sources are summarized by four equations, called **Maxwell’s equations**.
- The first Maxwell equation is Gauss’s law for electric fields from Chapter 22:

Gauss’s law for $\vec{E}$:

$$\overbrace{\oint \vec{E}\cdot d\vec{A}}^{\text{Flux of electric field through a closed surface}} = \frac{Q_{\text{encl}} \longleftarrow \text{Charge enclosed by surface}}{\epsilon_0 \longleftarrow \text{Electric constant}}$$

- The second Maxwell equation is Gauss’s law for magnetic fields from Chapter 27:

Gauss’s law for $\vec{B}$:

$$\overbrace{\oint \vec{B}\cdot d\vec{A}}^{\text{Flux of magnetic field through any closed surface ...}} = 0 \longleftarrow \text{... equals zero.}$$

Maxwell’s equations of electromagnetism

- The third Maxwell equation is this chapter’s formulation of Faraday’s law:

Faraday’s law for a stationary integration path:

$$\overbrace{\oint \vec{E}\cdot d\vec{l}}^{\text{Line integral of electric field around path}} = -\underbrace{\frac{d\Phi_B}{dt}}_{\text{Negative of the time rate of change of magnetic flux through path}}$$

- The fourth Maxwell equation is Ampere’s law, including displacement current:

Ampere’s law for a stationary integration path:

$$\overbrace{\oint \vec{B}\cdot d\vec{l}}^{\text{Line integral of magnetic field around path}} = \underset{\text{Magnetic constant}}{\mu_0}\left(\underset{\text{Conduction current through path}}{i_C} + \underbrace{\overset{\text{Electric constant}}{\epsilon_0}\overbrace{\frac{d\Phi_E}{dt}}^{\text{Time rate of change of electric flux through path}}}_{\text{Displacement current through path}}\right)_{\text{encl}}$$

Maxwell's equations in empty space

- There is a remarkable symmetry in Maxwell's equations.
- In empty space where there is no charge, the first two equations are identical in form.
- The third equation says that a changing magnetic flux creates an electric field, and the fourth says that a changing electric flux creates a magnetic field.

In empty space there are no charges, so the fluxes of $\vec{E}$ and $\vec{B}$ through any closed surface are equal to zero.

$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

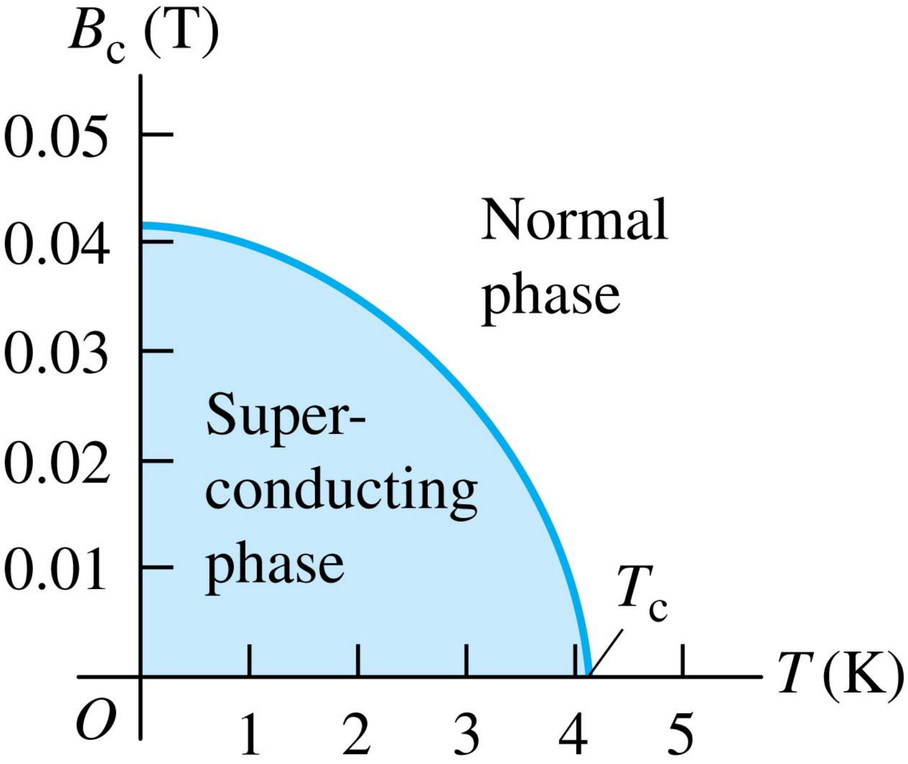
$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

In empty space there are no conduction currents, so the line integrals of $\vec{E}$ and $\vec{B}$ around any closed path are related to the rate of change of flux of the other field.

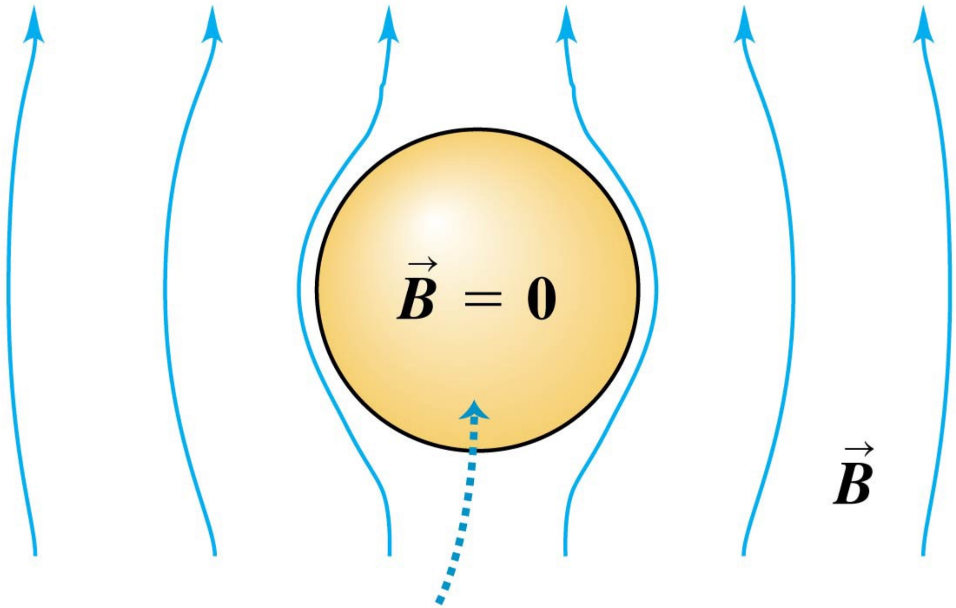
Superconductivity in a magnetic field

- When a superconductor is cooled below its critical temperature T_c, it loses all electrical resistance.
- For any superconducting material the critical temperature T_c changes when the material is placed in an externally produced magnetic field.
- Shown is this dependence for mercury.
- As the external field magnitude increases, the superconducting transition occurs at a lower and lower temperature.



The Meissner effect

- If we place a superconducting material in a uniform applied magnetic field, and then lower the temperature until the superconducting transition occurs, then all of the magnetic flux is expelled from the superconductor.
- The expulsion of magnetic flux is called the **Meissner effect**.



Magnetic flux is expelled from the material, and the field inside it is zero (Meissner effect).

Superconductor levitation

- The **Meissner effect** makes a superconductor a perfect diamagnet.
- A paramagnetic material is *attracted* by a permanent magnet because the magnetic dipoles in the material align with the nonuniform magnetic field of the permanent magnet.
- For a diamagnetic material the magnetization is in the opposite sense, and a diamagnetic material is *repelled* by a permanent magnet.
- The photograph shows the repulsion between a specimen of a high-temperature superconductor and a magnet.