# VECTORS IN PHYSICS

Chapter 3

Units of Chapter 3

- Scalars Versus Vectors

- The components of a vector

- Adding and subtracting vectors

- Unit vectors

- Position, Displacement, Velocity, and Acceleration Vectors

- Relative Motion
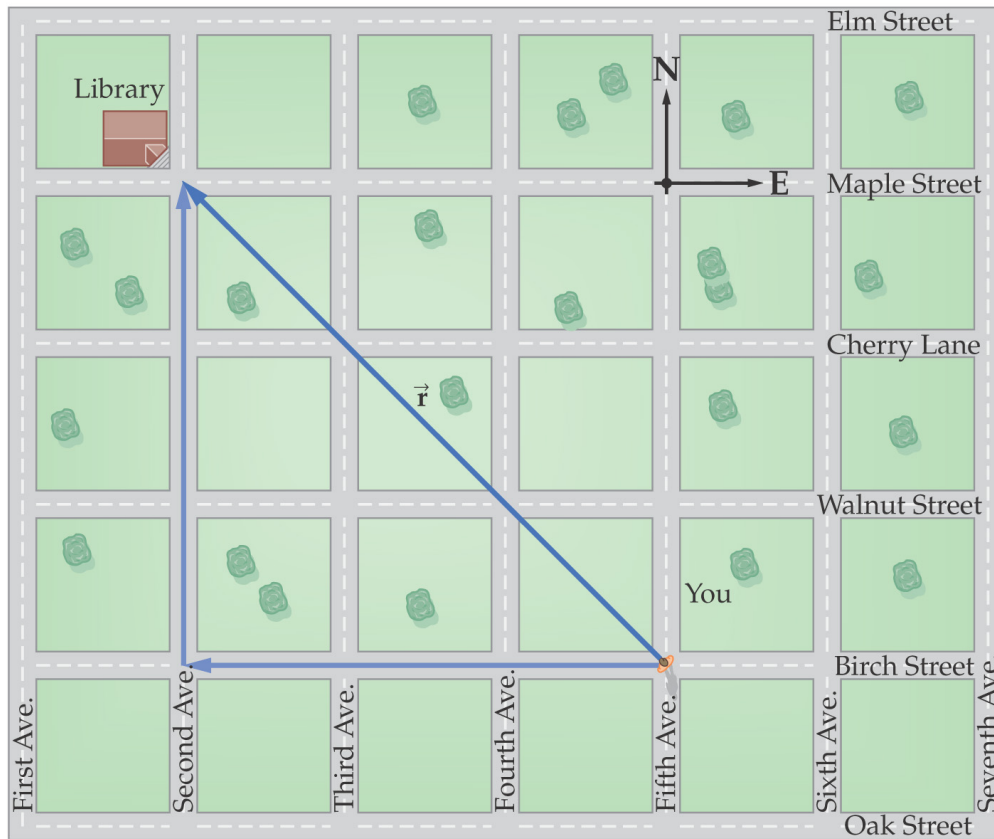
3-1 Scalars versus vectors

- **Scalar**: number with units.
- **Vector**: quantity with magnitude and direction.
- How to get to the library: need to know how far and which way
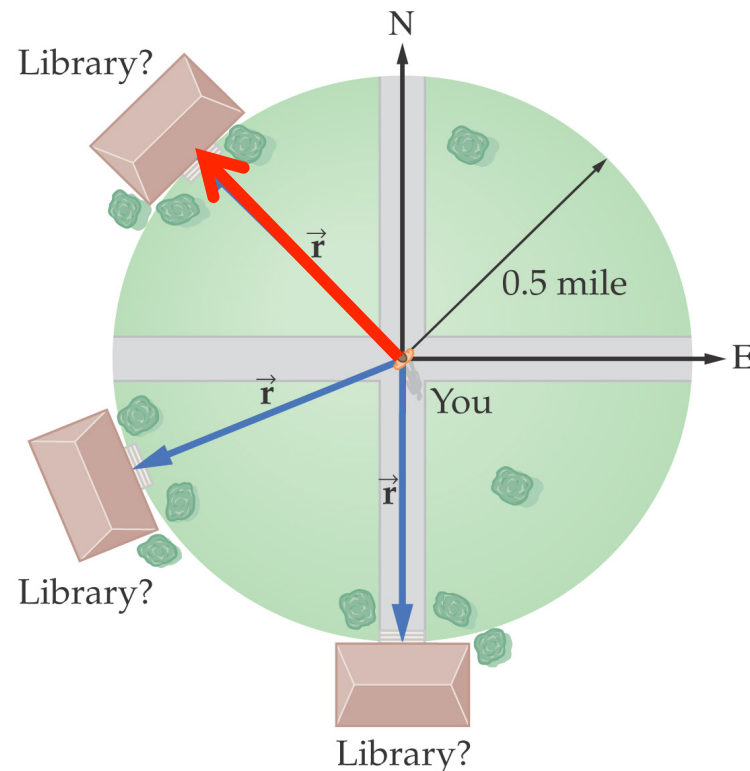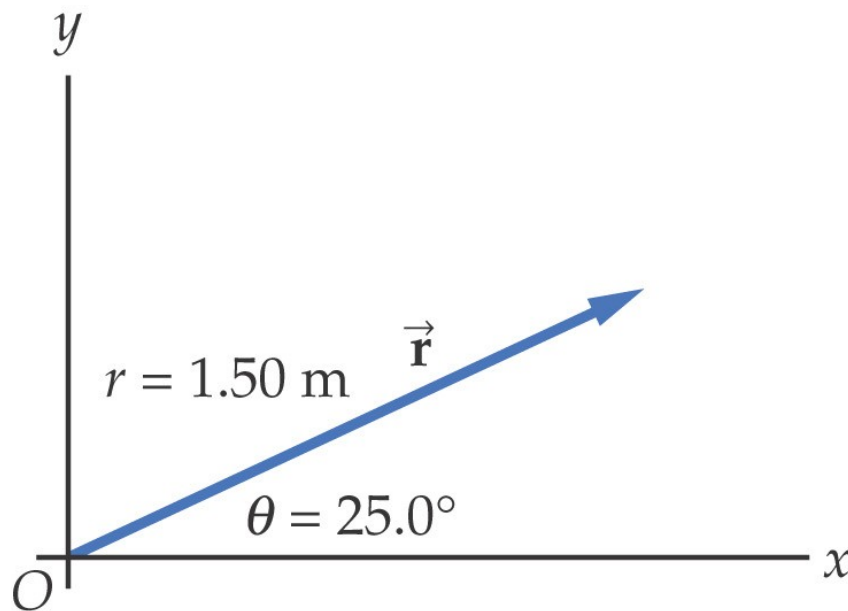

0.5 mi northwest

3-2 The Components of a Vector

- Even though you know how far and in which direction the library is, you may not be able to walk there in a straight line:
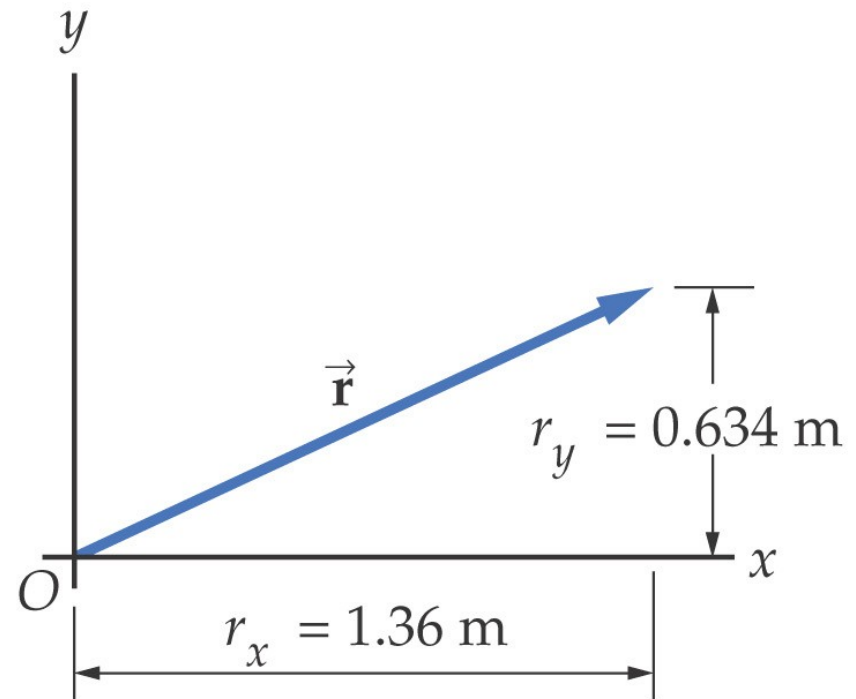
3-2 The Components of a Vector

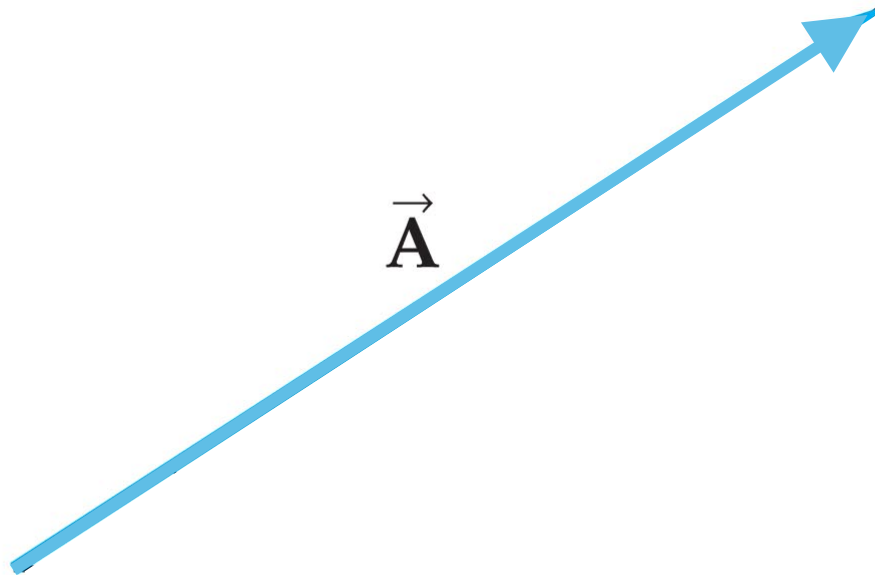• Can resolve vector into **perpendicular components** using a two-dimensional coordinate system:

(a)

(b)

3-2 The Components of a Vector

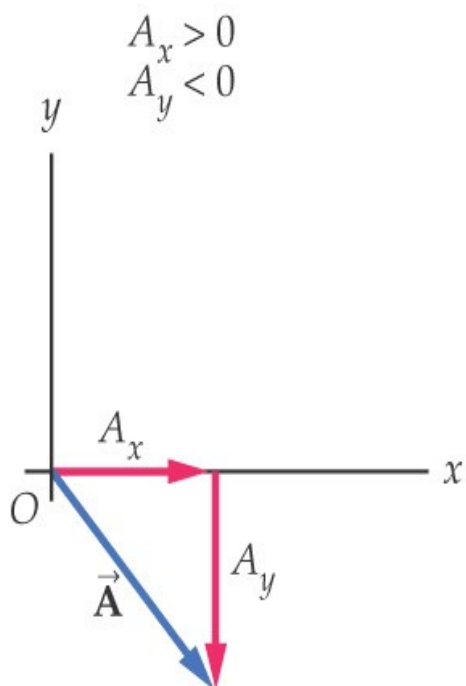- Length, angle, and components can be calculated from each other using trigonometry:

$$\vec{A}$$

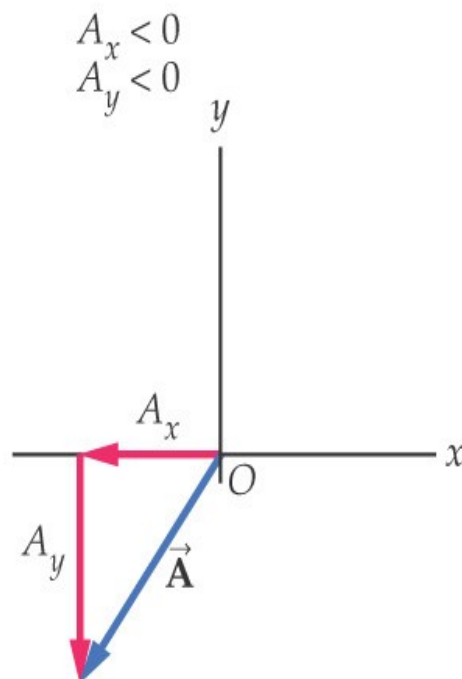Given the magnitude and direction of a vector, find its components:

$A_x = A \cos\theta$
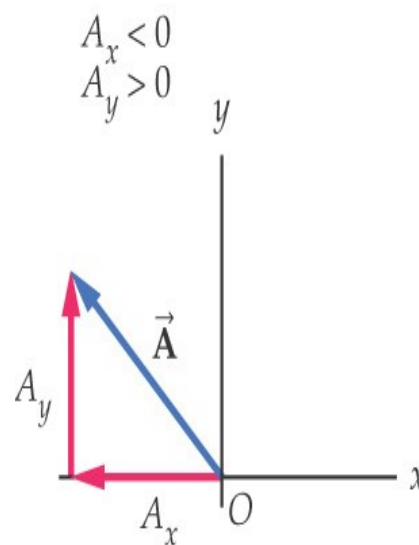
$A_y = A \sin\theta$

3-2 The Components of a Vector

- Signs of vector components:

$A_x > 0$
$A_y < 0$

(a)

$A_x < 0$
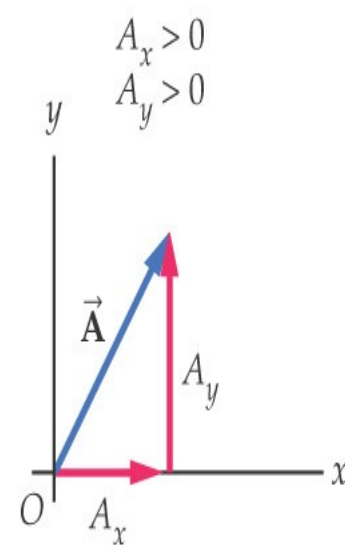$A_y < 0$

(b)

$A_x < 0$
$A_y > 0$

(c)

$A_x > 0$
$A_y > 0$

(d)
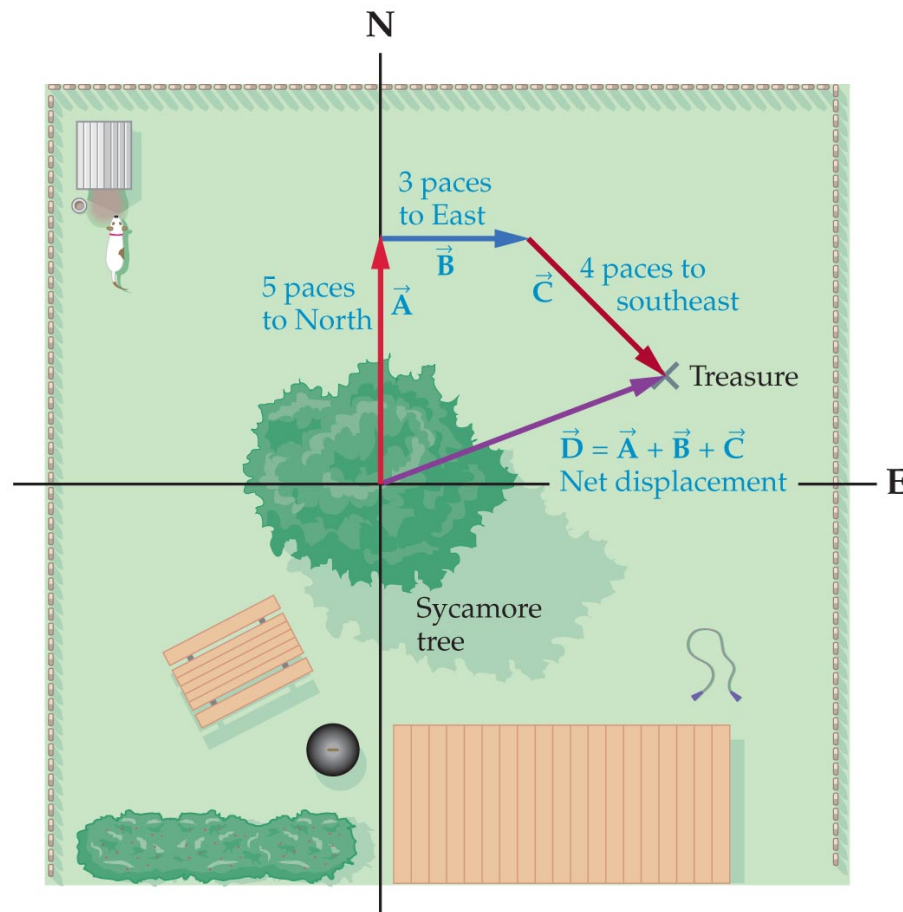
3-3 Adding and Subtracting Vectors

- Adding vectors graphically: Place the tail of the second at the head of the first. The sum points from the tail of the first to the head of the last.
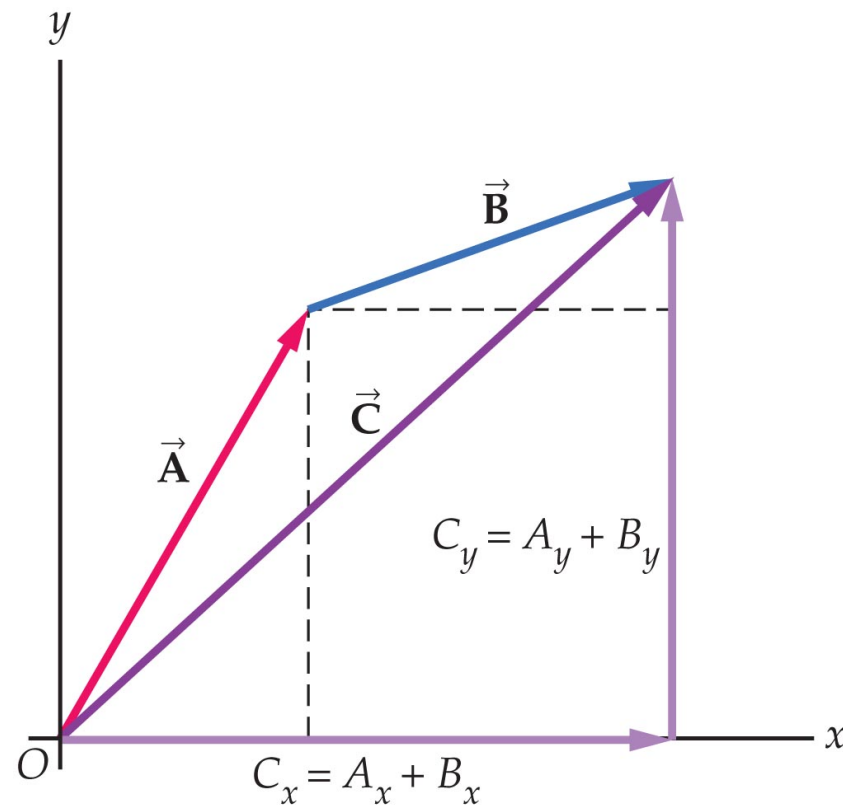
3-3 Adding and Subtracting Vectors

- Adding vectors using components:

1. Find the components of each vector to be added.

2. Add the *x*- and *y*-components separately.

3. Find the resultant vector.

3-3 Adding and Subtracting Vectors

• Adding vectors using components:

$$C_y = A_y + B_y$$

$$C_x = A_x + B_x$$

(b)
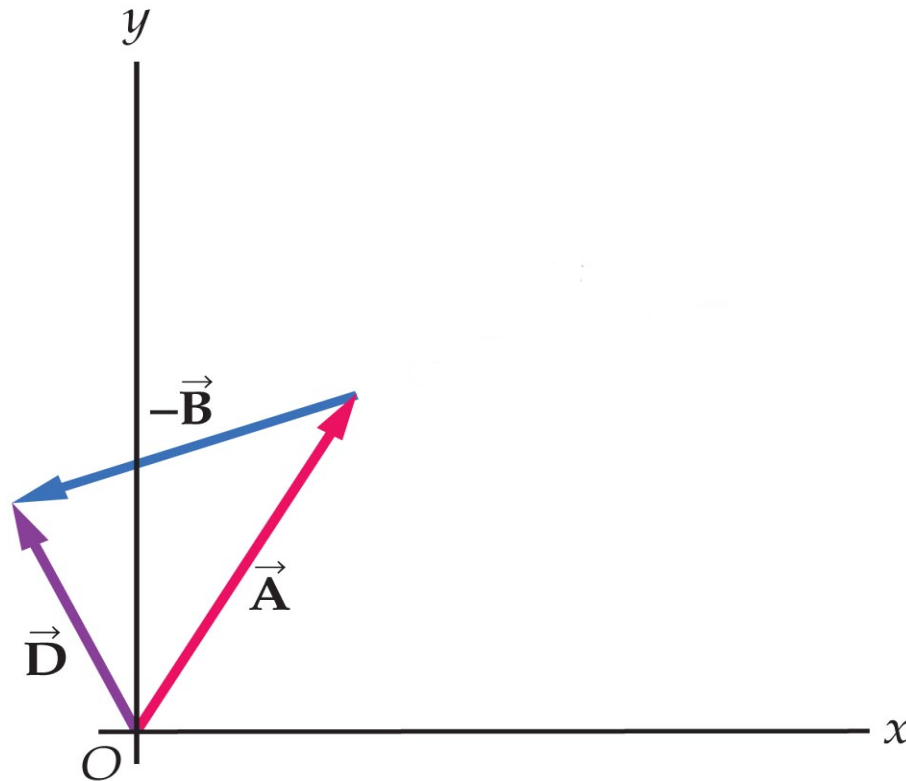
3-3 Adding and Subtracting Vectors

- Subtracting Vectors: The negative of a vector is a vector of the same magnitude pointing in the opposite direction.

Here,

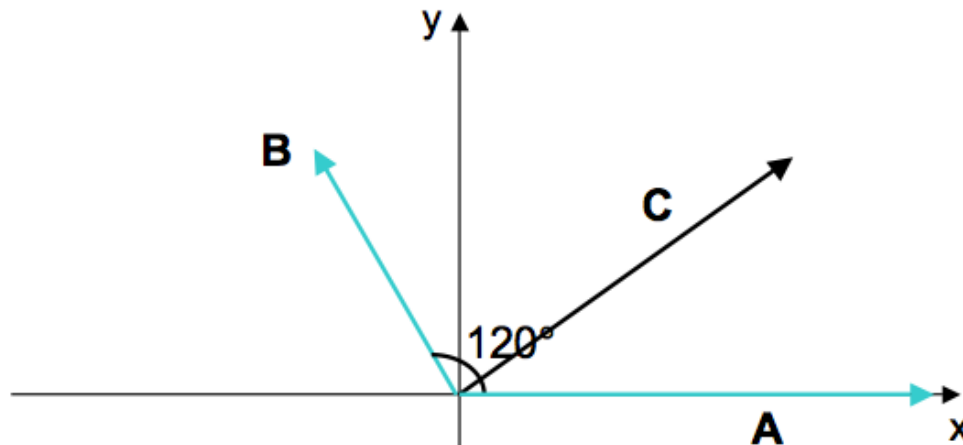$$\vec{D} = \vec{A} - \vec{B}$$

(b)

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Example

Vector **A** has a length of 5.00 meters and points along the x-axis. Vector **B** has a length of 3.00 meters and points 120° from the +x-axis. Compute **A+B** (=**C**)

Example continued
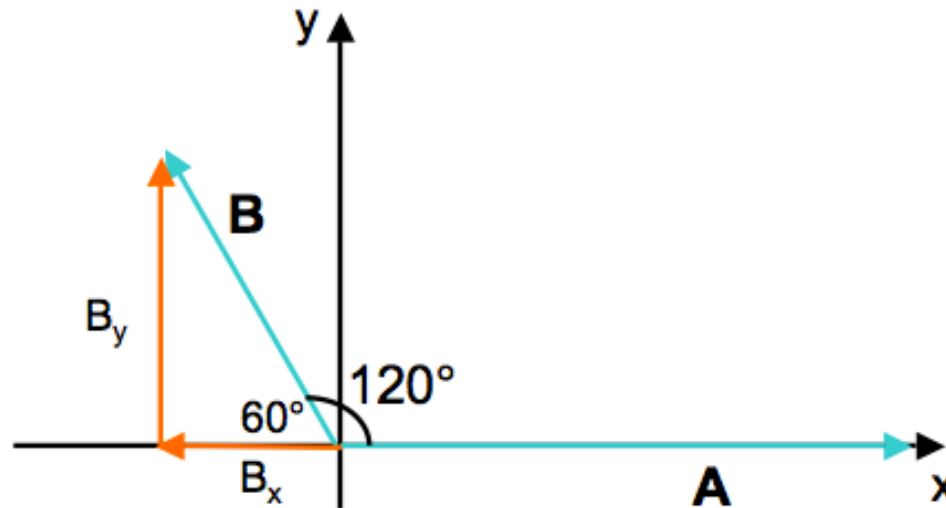
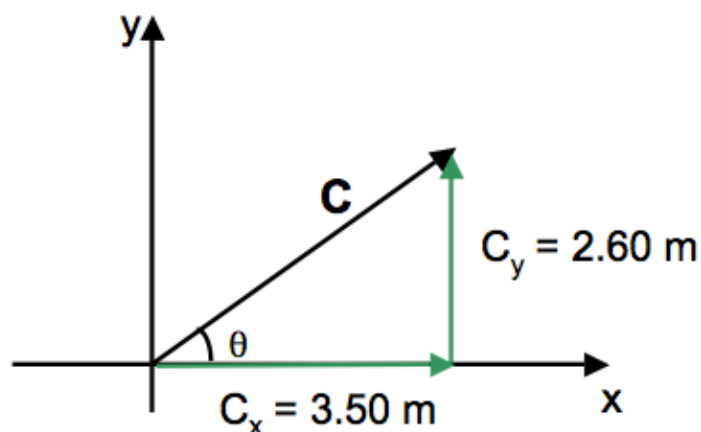$$\sin 60° = \frac{B_y}{B} \Rightarrow B_y = B\sin 60° = (3.00\,\text{m})\sin 60° = 2.60\,\text{m}$$

$$\cos 60° = \frac{-B_x}{B} \Rightarrow B_x = -B\cos 60° = -(3.00\,\text{m})\cos 60° = -1.50\,\text{m}$$

and A_x = 5.00 m and A_y = 0.00 m

Example continued

The components of **C**:

$$C_x = A_x + B_x = 5.00\,\text{m} + (-1.50\,\text{m}) = 3.50\,\text{m}$$

$$C_y = A_y + B_y = 0.00\,\text{m} + 2.60\,\text{m} = 2.60\,\text{m}$$

$C_y = 2.60\,\text{m}$

$C_x = 3.50\,\text{m}$

The length of C is:

$$C = |\mathbf{C}| = \sqrt{C_x^2 + C_y^2}$$
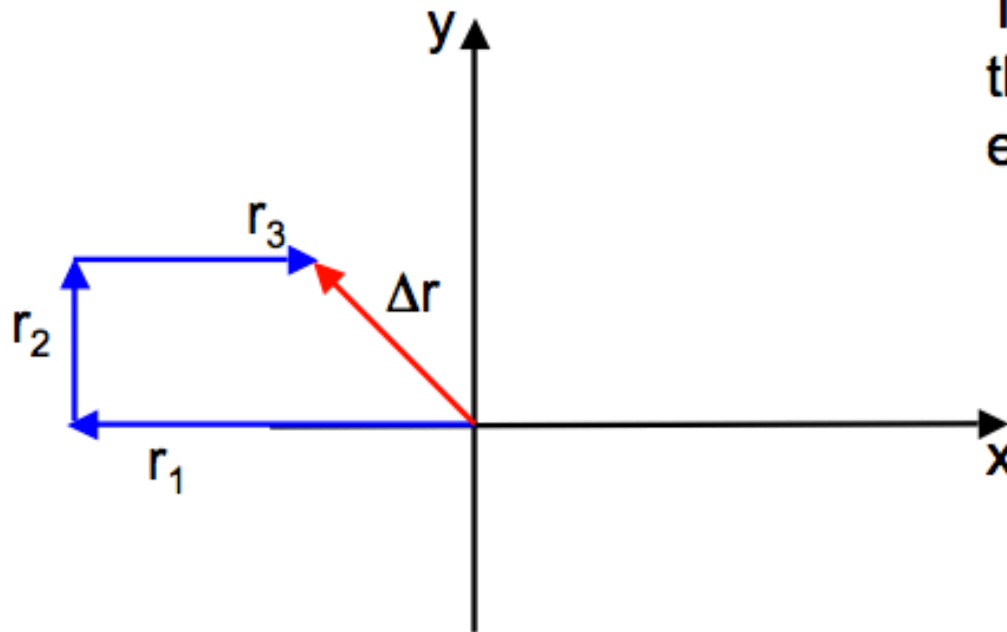
$$= \sqrt{(3.50\,\text{m})^2 + (2.60\,\text{m})^2}$$

$$= 4.36\,\text{m}$$

The direction of C is: $\tan\theta = \dfrac{C_y}{C_x} = \dfrac{2.60\,\text{m}}{3.50\,\text{m}} = 0.7429$

$$\theta = \tan^{-1}(0.7429) = 36.6° \quad \text{From the +x-axis}$$

Example

• Margaret walks to the store using the following path: 0.500 miles west, 0.200 miles north, 0.300 miles east. What is her total displacement? Give the magnitude and direction.
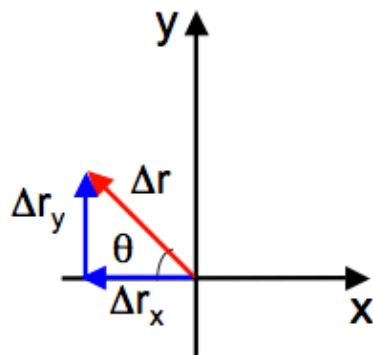

Take north to be in the +y direction and east to be along +x.

Example continued

The displacement is $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$. The initial position is the origin; what is $\mathbf{r}_f$?

The final position will be $\mathbf{r}_f = \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3$. The components are $r_{fx} = -r_1 + r_3 = -0.2$ miles and $r_{fy} = +r_2 = +0.2$ miles.
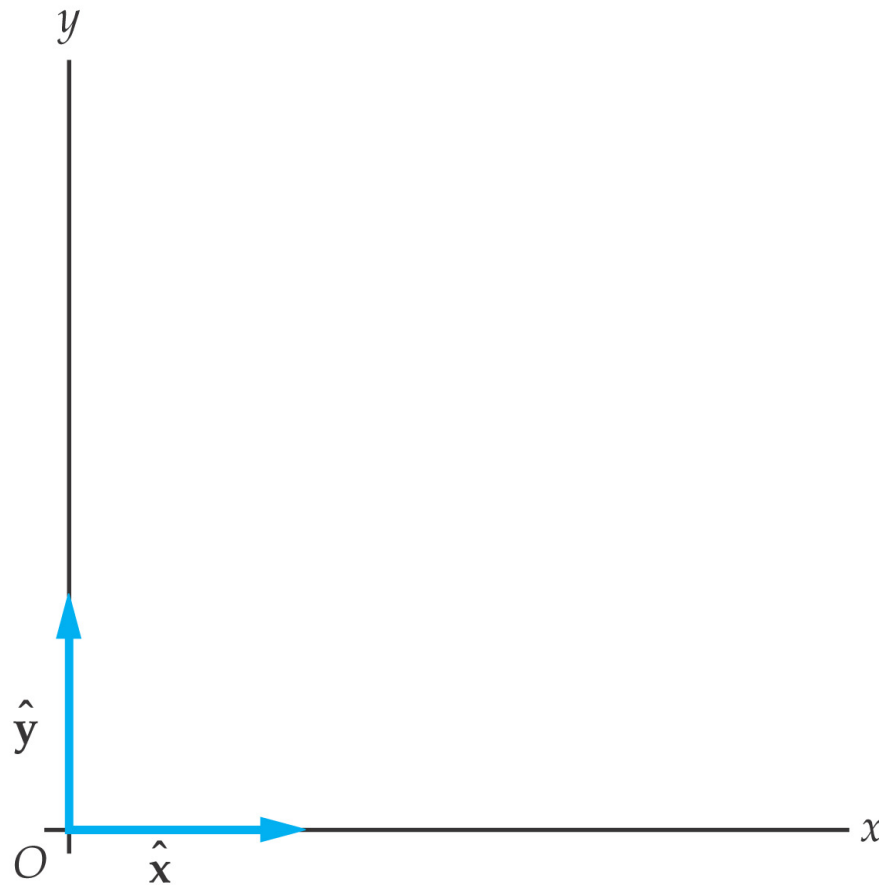

Using the figure, the magnitude and direction of the displacement are

$$\left|\Delta\mathbf{r}\right| = \sqrt{\Delta r_x^2 + \Delta r_y^2} = 0.283 \text{ miles}$$

$$\tan\theta = \frac{\left|\Delta r_y\right|}{\left|\Delta r_x\right|} = 1 \text{ and } \theta = 45° \qquad \text{N of W.}$$
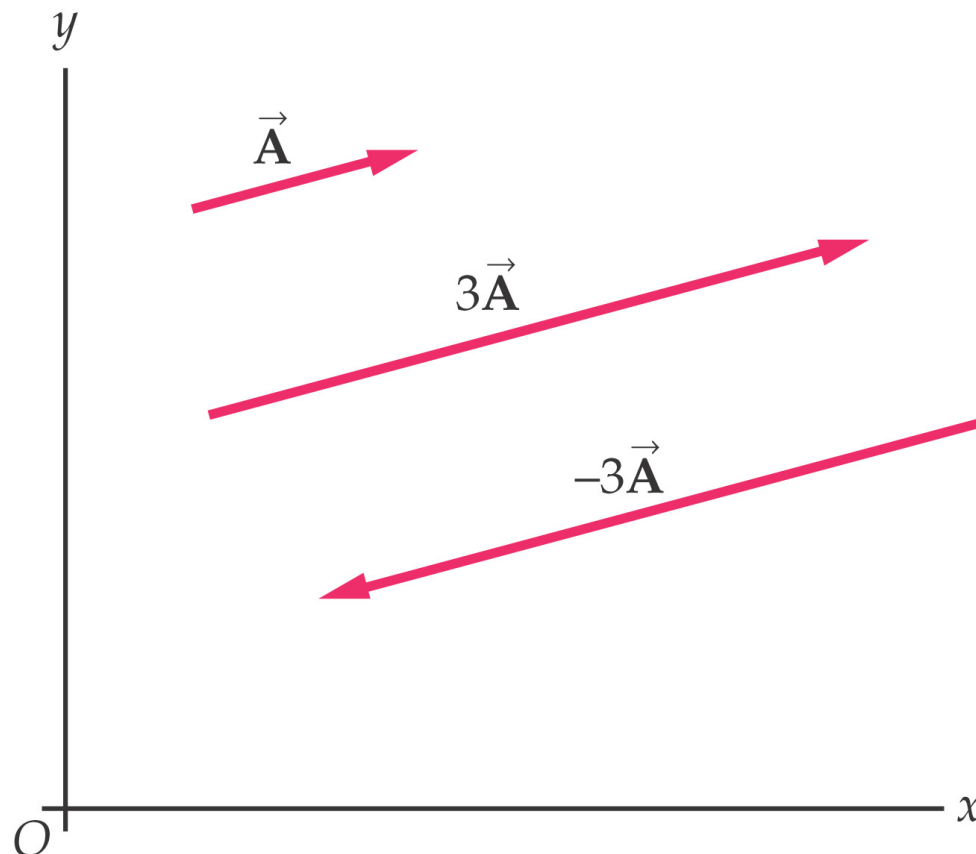
3-4 Unit Vectors

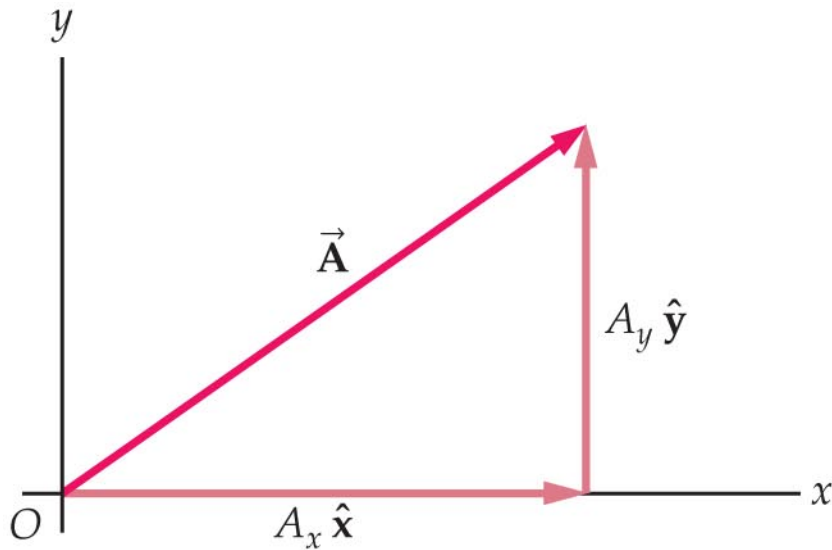- Unit vectors are dimensionless vectors of unit length.

3-4 Unit Vectors

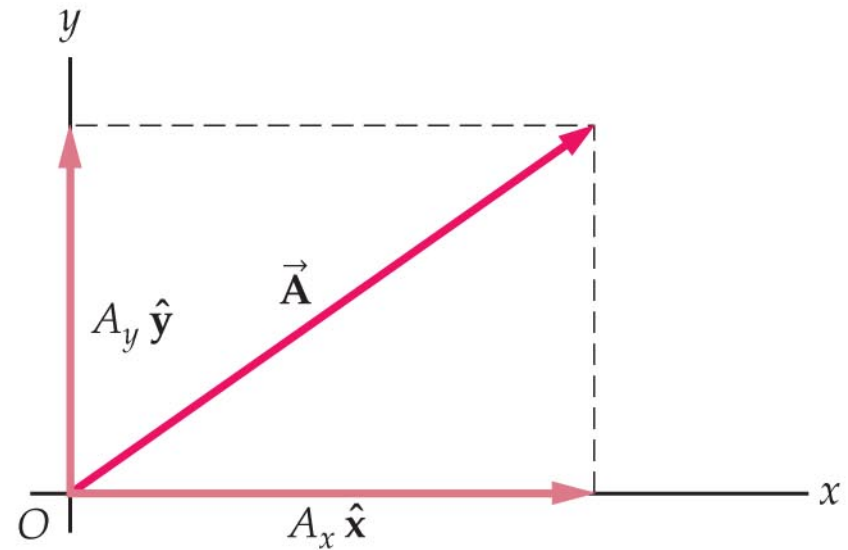- Multiplying unit vectors by scalars: the multiplier changes the length, and the sign indicates the direction.

3-4 Unit Vectors

- Unit vectors provide a useful way to keep track of the x and y components of a vector:

(a)

(b)
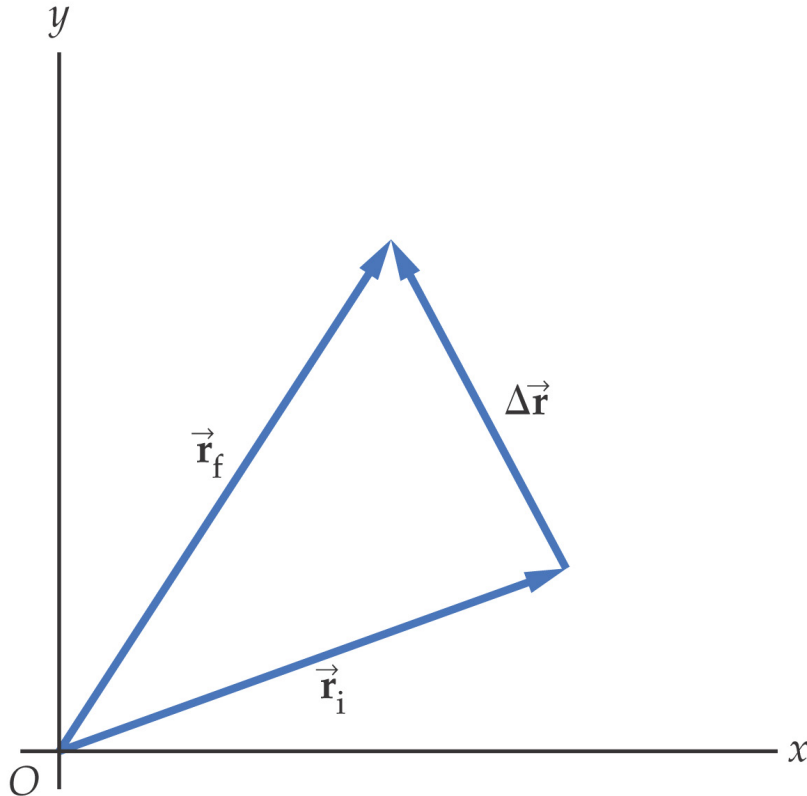
3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Position vector $\vec{r}_f$ points from the origin to the location in question.

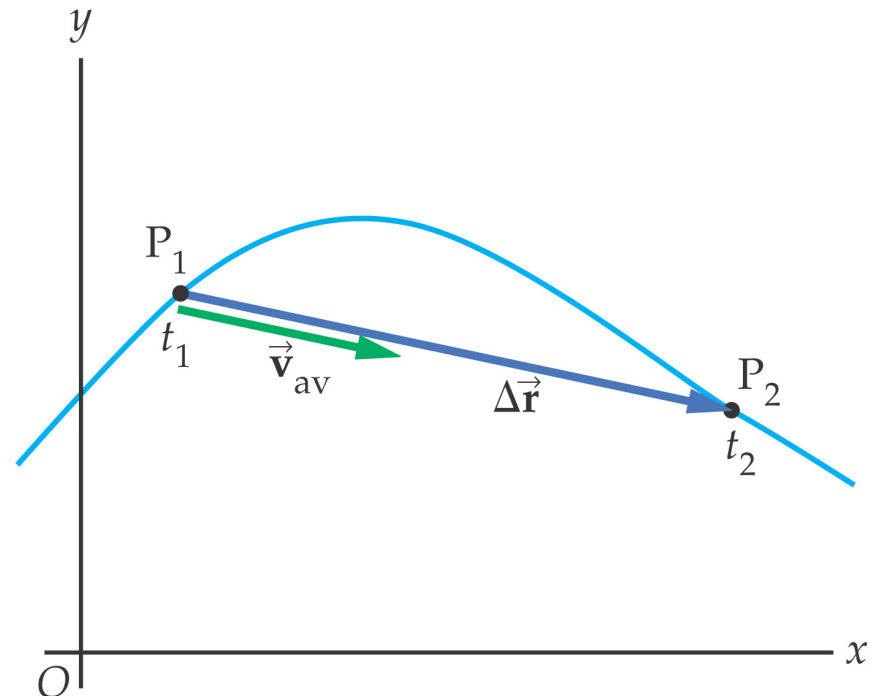The displacement vector $\Delta\vec{r}$ points from the original position to the final position

3-5 Position, Displacement, Velocity, and Acceleration Vectors

• Average velocity vector:

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$$

So $\vec{v}_{av}$ is in the same direction as $\Delta \vec{r}$.

3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Instantaneous velocity vector is tangent to the path:
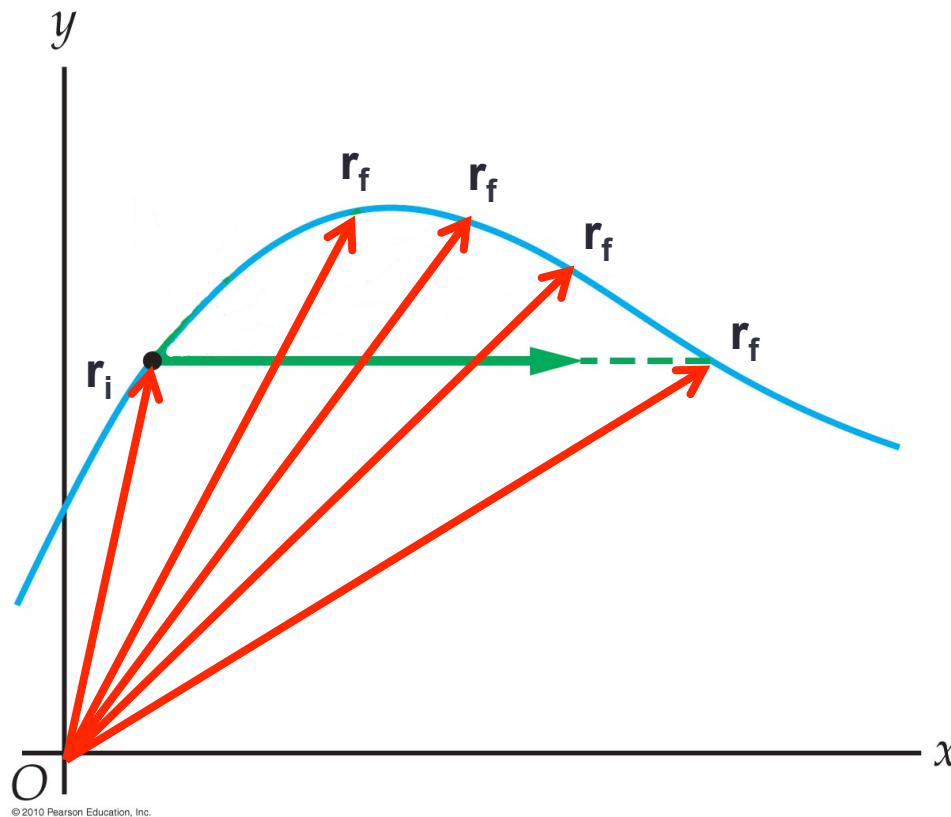
3-5 Position, Displacement, Velocity, and Acceleration Vectors

• Average acceleration vector is in the direction of the change in velocity:

$$\vec{a}_{\text{av}} = \frac{\Delta \vec{v}}{\Delta t}$$

3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Average acceleration is non-zero for constant speed but change of direction:

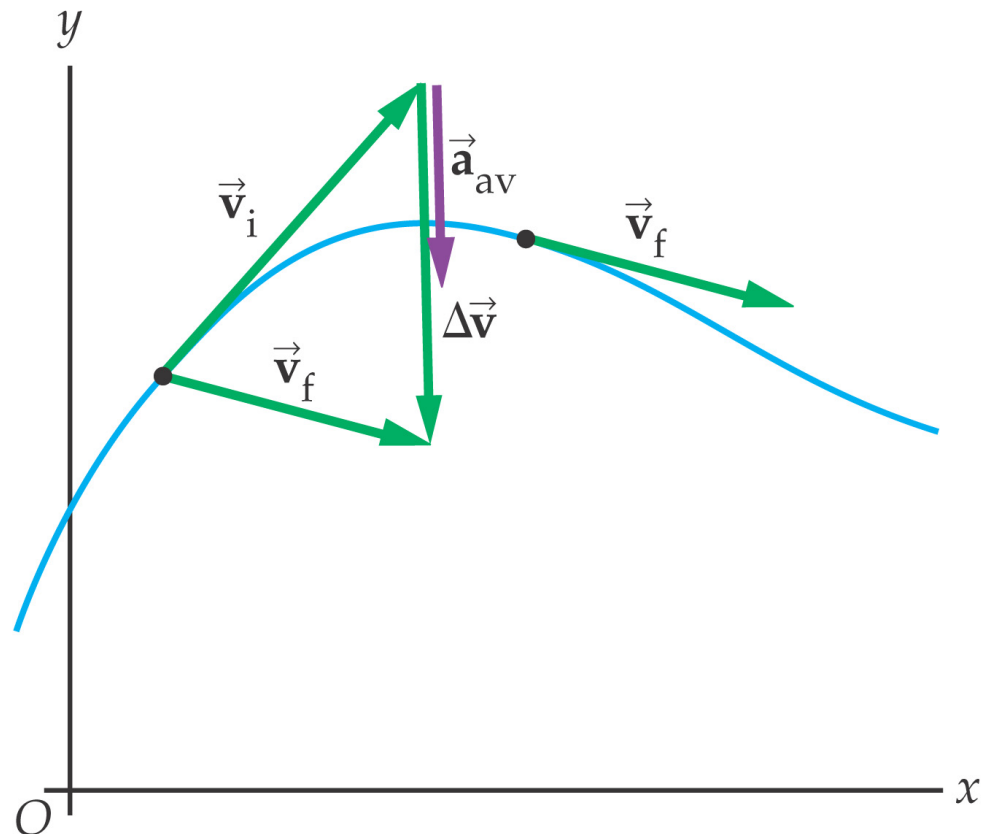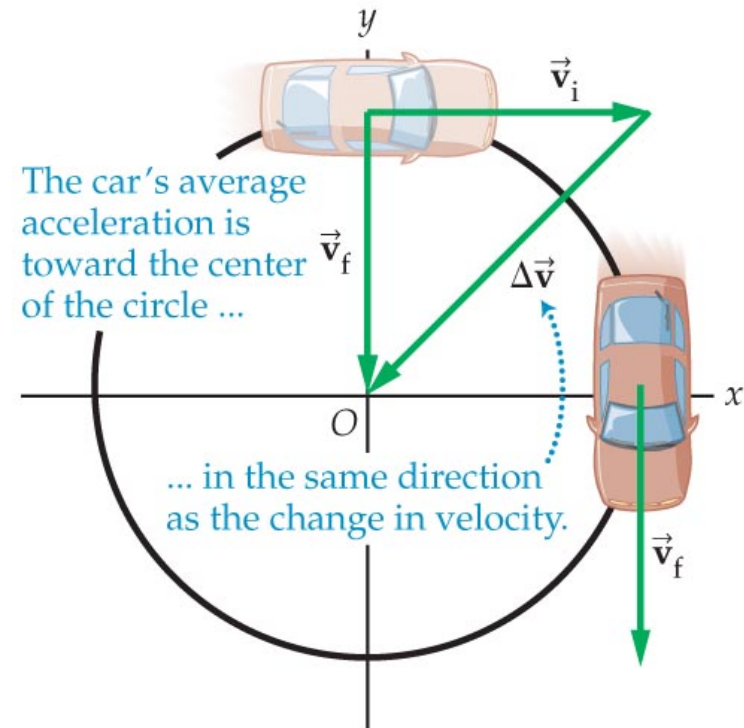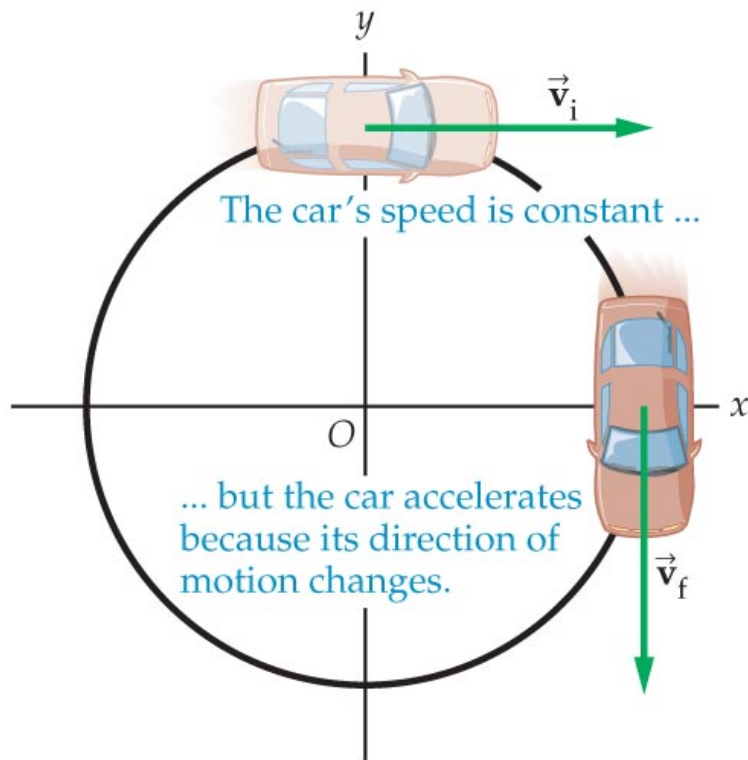# 3-5 Position, Displacement, Velocity, and Acceleration Vectors

- Velocity vector is always in the direction of motion; acceleration vector can point anywhere:
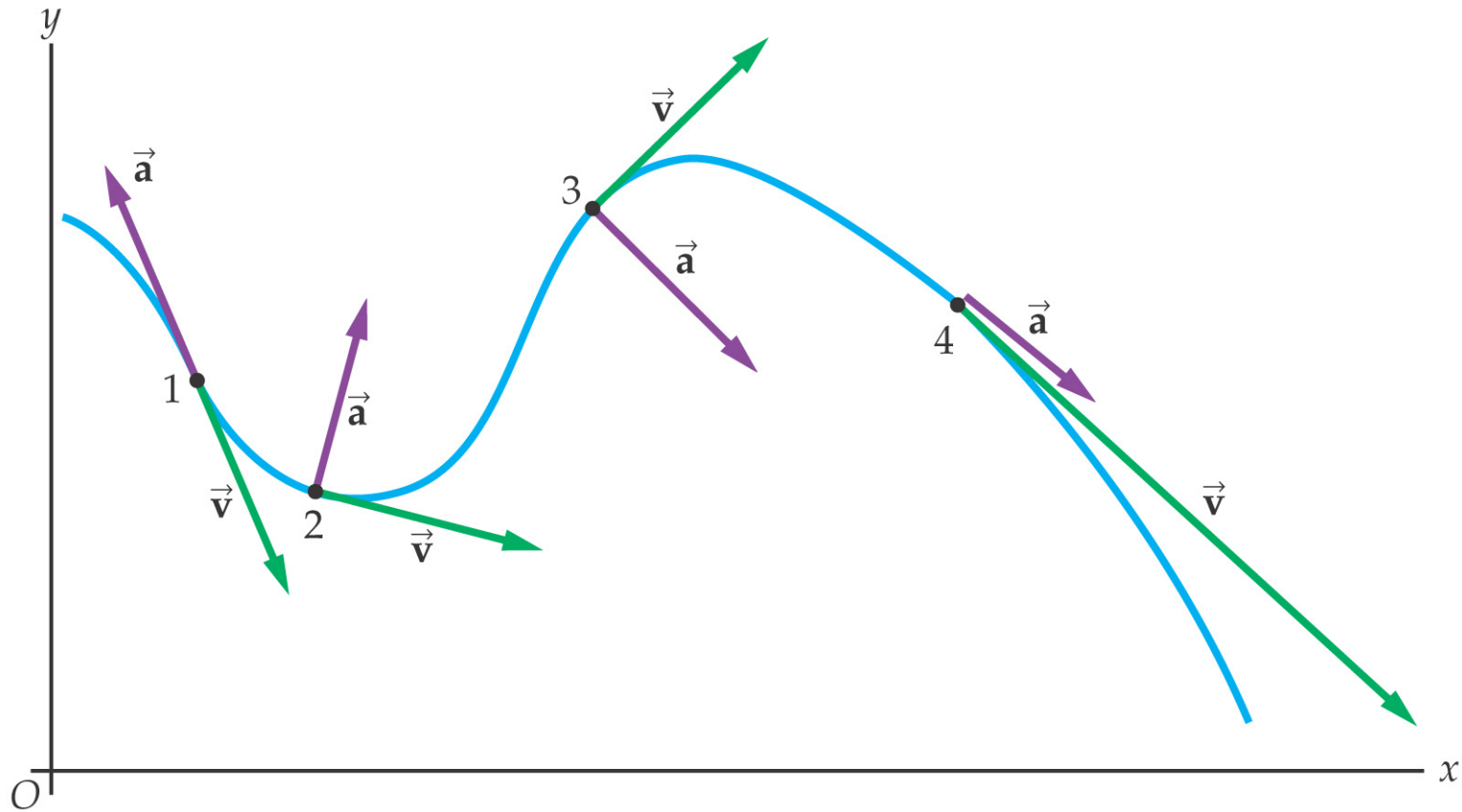
Example

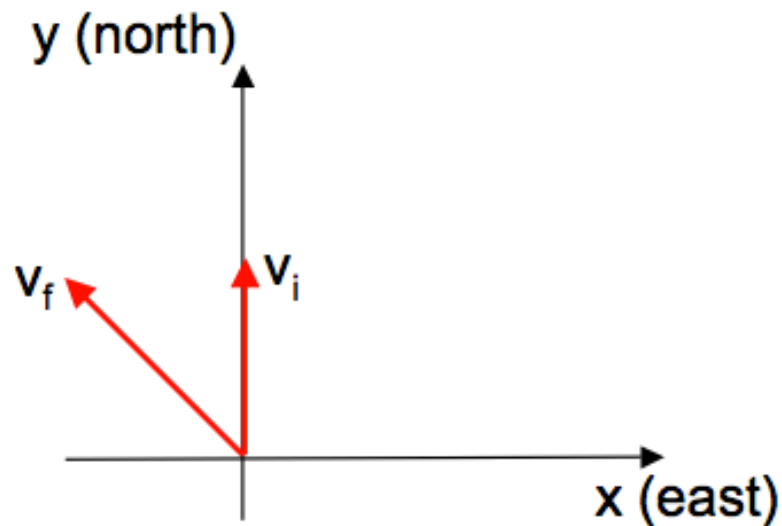• At the beginning of a 3 hour plane trip you are traveling due north at 192 km/hour. At the end, you are traveling 240 km/hour at 45° west of north.

(a) Draw the initial and final velocity vectors.

Example continued

(b) Find $\Delta\mathbf{v}$

The components are

$$\Delta v_x = v_{fx} - v_{ix} = -v_f \sin 45° - 0 = -170 \text{ km/hr}$$

$$\Delta v_y = v_{fy} - v_{iy} = +v_f \cos 45° - v_i = -22.3 \text{ km/hr}$$

The magnitude and direction are:

$$|\Delta\mathbf{v}| = \sqrt{\Delta v_x^2 + \Delta v_y^2} = 171 \text{ km/hr}$$

$$\tan\varphi = \frac{|\Delta v_y|}{|\Delta v_x|} = 0.1312 \Rightarrow \phi = \tan^{-1}(0.1312) = 7.5°$$

South of west

Example continued

(c) What is $\mathbf{a}_{av}$ during the trip?

$$\mathbf{a}_{av} = \frac{\Delta\mathbf{v}}{\Delta t}$$

$$a_{x,av} = \frac{\Delta v_x}{\Delta t} = \frac{-170 \text{ km/hr}}{3 \text{ hr}} = -56.7 \text{ km/hr}^2$$

$$a_{y,av} = \frac{\Delta v_y}{\Delta t} = \frac{-22.3 \text{ km/hr}}{3 \text{ hr}} = -7.43 \text{ km/hr}^2$$

The magnitude and direction are:
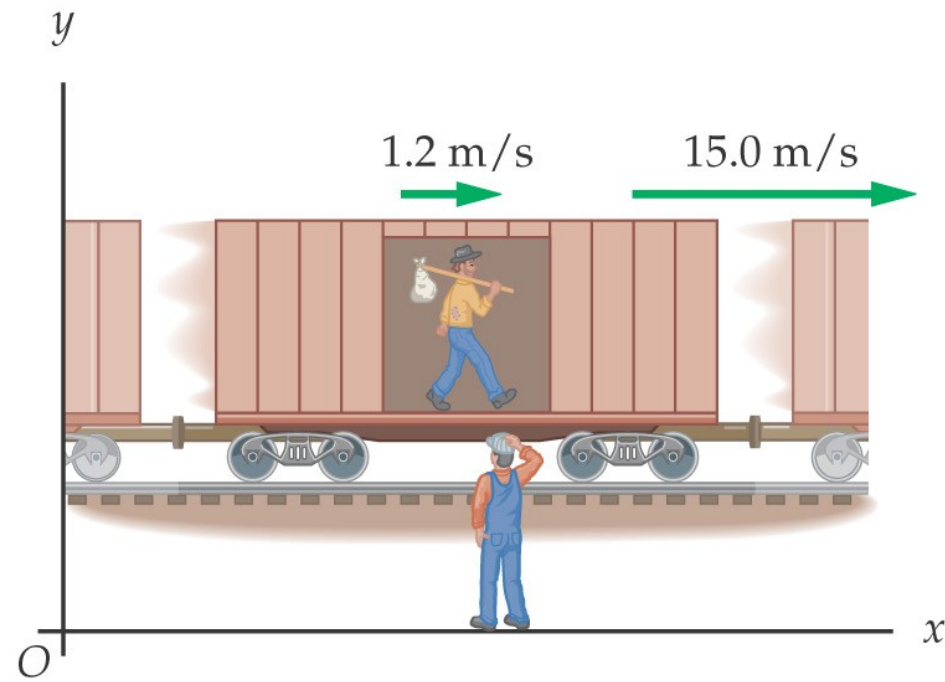
$$\left|\mathbf{a}_{av}\right| = \sqrt{a_{x,av}^2 + a_{y,av}^2} = 57.2 \text{ km/hr}^2$$

$$\tan\phi = \frac{\left|a_{y,av}\right|}{\left|a_{x,av}\right|} = 0.1310 \Rightarrow \phi = \tan^{-1}(0.1310) = 7.5° \qquad \text{South of west}$$

3-6 Relative motion

- The speed of the passenger with respect to the ground depends on the relative directions of the passenger's and train's speeds:
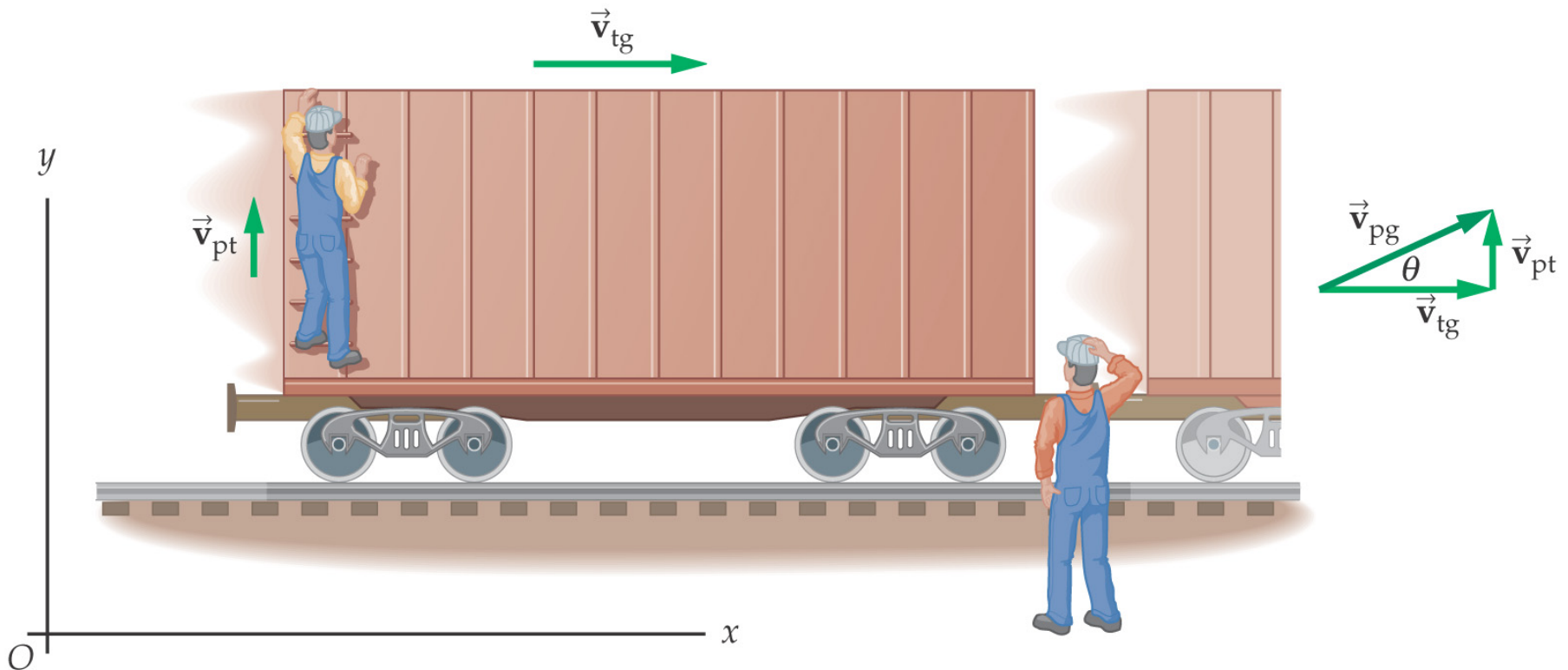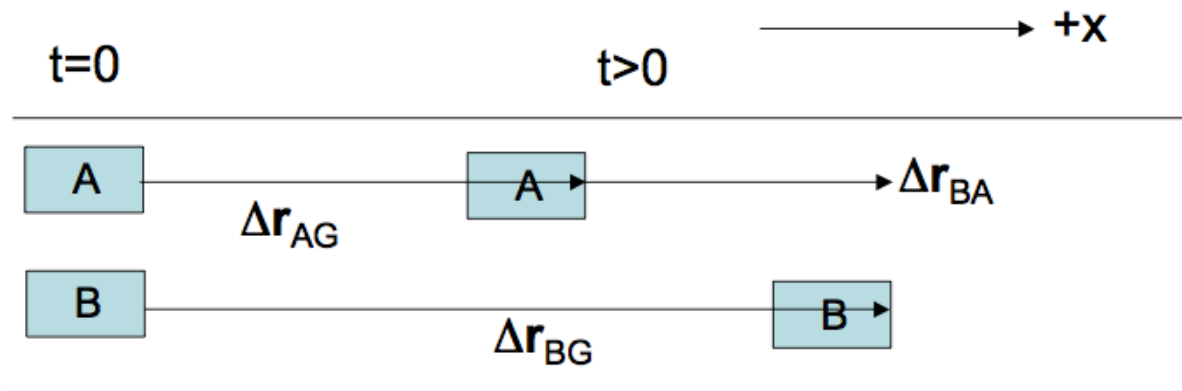

(a)
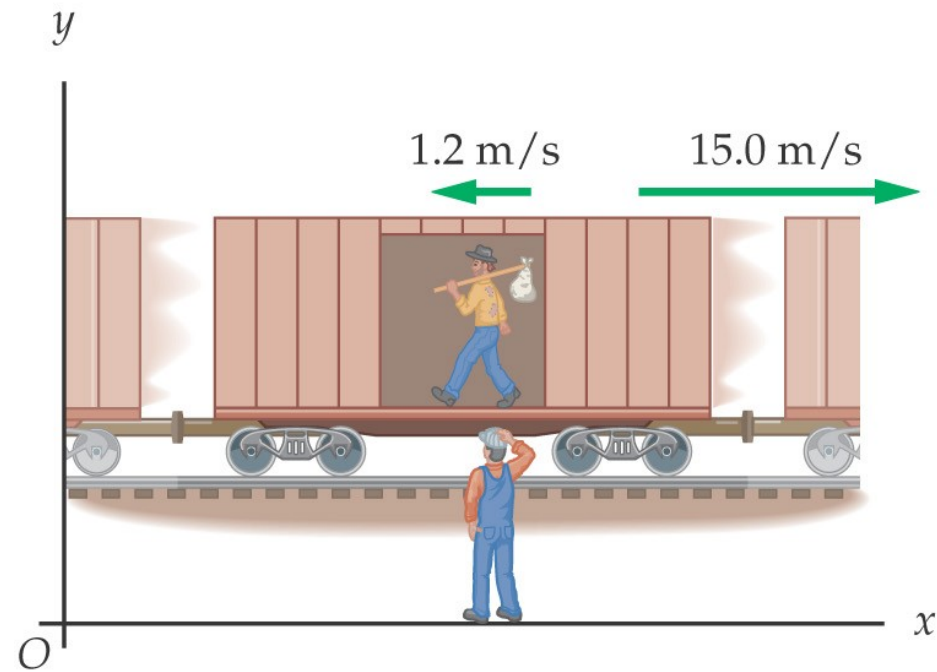
(b)

3-6 Relative motion

$$\vec{\mathbf{v}}_{pg} = \vec{\mathbf{v}}_{pt} + \vec{\mathbf{v}}_{tg}$$

- This also works in two dimensions:

Example

• You are traveling in a car (A) at 60 miles/hour east on a long straight road. The car (B) next to you is traveling at 65 miles/hour east. What is the speed of car B relative to car A?

From the picture: $\Delta \mathbf{r}_{BG} = \Delta \mathbf{r}_{AG} + \Delta \mathbf{r}_{BA}$

$$\Delta \mathbf{r}_{BA} = \Delta \mathbf{r}_{BG} - \Delta \mathbf{r}_{AG}$$

Divide by Δt: $\mathbf{v}_{BA} = \mathbf{v}_{BG} - \mathbf{v}_{AG}$

$\mathbf{v}_{BA} = 65$ miles/hr east $- 60$ miles/hr east

$= 5$ miles/hour east

Example

- You are traveling in a car (A) at 60 miles/hour east on a long straight road. The car (B) next to you is traveling at 65 miles/hour west. What is the speed of car B relative to car A?

From the picture: $\Delta \mathbf{r}_{BA} = \Delta \mathbf{r}_{BG} - \Delta \mathbf{r}_{AG}$
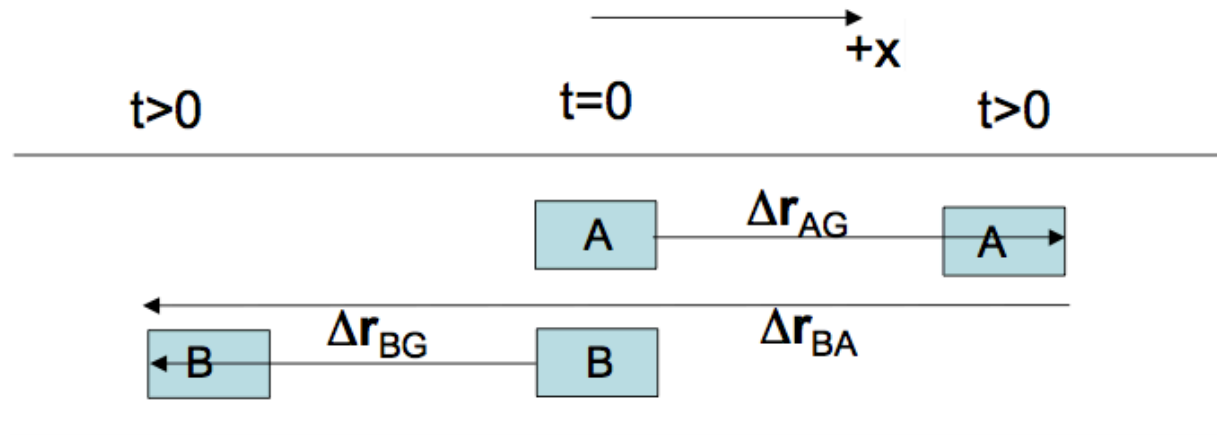
Divide by Δt: $\mathbf{v}_{BA} = \mathbf{v}_{BG} - \mathbf{v}_{AG}$

$$= 65 \text{ miles/hr west} - 60 \text{ miles/hr east}$$

$$= 125 \text{ miles/hr west}$$

Summary of Chapter 3

- Scalar: number, with appropriate units

- Vector: quantity with magnitude and direction

- Vector components: $A_x = A \cos \theta$, $B_y = B \sin \theta$

- Magnitude: $A = (A_x^2 + A_y^2)^{1/2}$

- Direction: $\theta = \tan^{-1} (A_y / A_x)$

- Graphical vector addition: Place tail of second at head of first; the sum points from tail of first to head of last

Summary of Chapter 3

- Component method: add components of individual vectors, then find magnitude and direction

- Unit vectors are dimensionless and of unit length

- Position vector points from origin to location

- Displacement vector points from original position to final position

- Velocity vector points in direction of motion

- Acceleration vector points in direction of change of motion

- Relative motion: $\mathbf{v}_{13} = \mathbf{v}_{12} + \mathbf{v}_{23}$