LINEAR MOMENTUM AND COLLISIONS

Chapter 9

Units of Chapter 9

- Linear Momentum
- Momentum and Newton’s Second Law
- Impulse
- Conservation of Linear Momentum
- Inelastic Collisions
- Elastic Collisions
- Center of Mass
- Systems with Changing Mass: Rocket Propulsion

9-1 Linear Momentum

Definition of Linear Momentum, $\vec{\mathbf{p}}$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$$

SI unit: $\mathrm{kg \cdot m/s}$

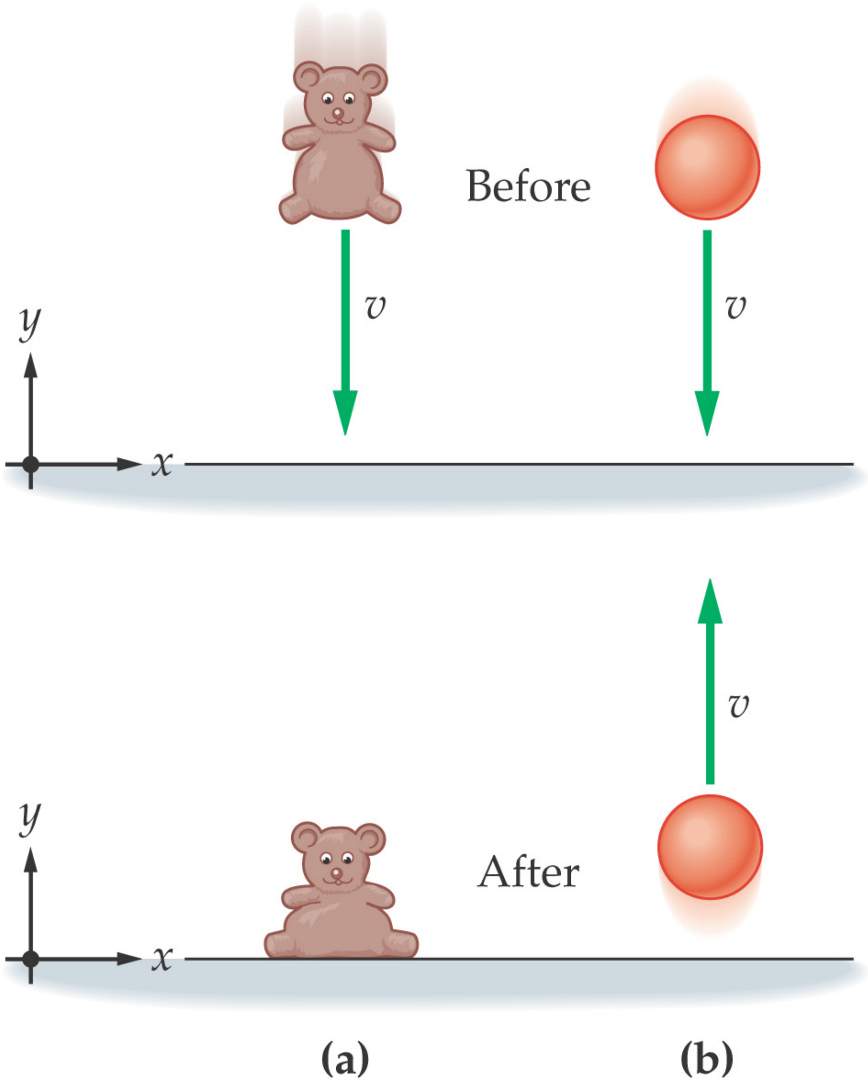
Momentum is a vector; its direction is the same as the direction of the velocity.

9-1 Linear Momentum

- Change in momentum:

(a) mv

(b) $2mv$



9-2 Momentum and Newton's Second Law

- Newton's second law, as we wrote it before:

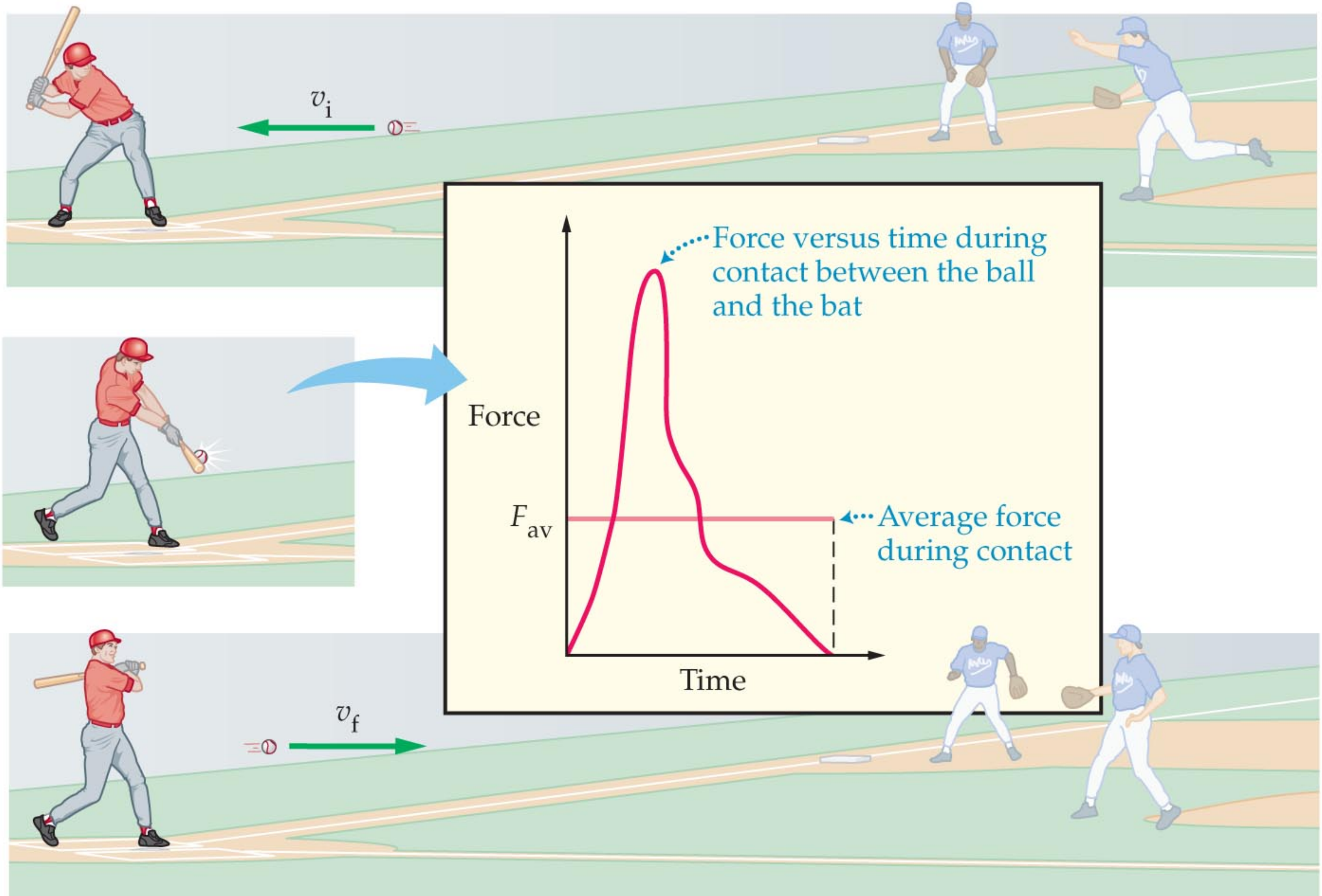
$$\sum \vec{F} = m\vec{a}$$

is only valid for objects that have constant mass. Here is a more general form, also useful when the mass is changing:

Newton's Second Law

$$\sum \vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

9-3 Impulse


v_i
Force
F_{av}
Force versus time during contact between the ball and the bat
Average force during contact
Time
v_f

9-3 Impulse

Definition of Impulse, $\vec{\mathbf{I}}$

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\mathrm{av}} \Delta t$$

SI unit: $\mathrm{N \cdot s = kg \cdot m/s}$

- Impulse is a vector, in the same direction as the average force.

9-3 Impulse

- We can rewrite $$\vec{\mathbf{F}}_{\text{av}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t}$$

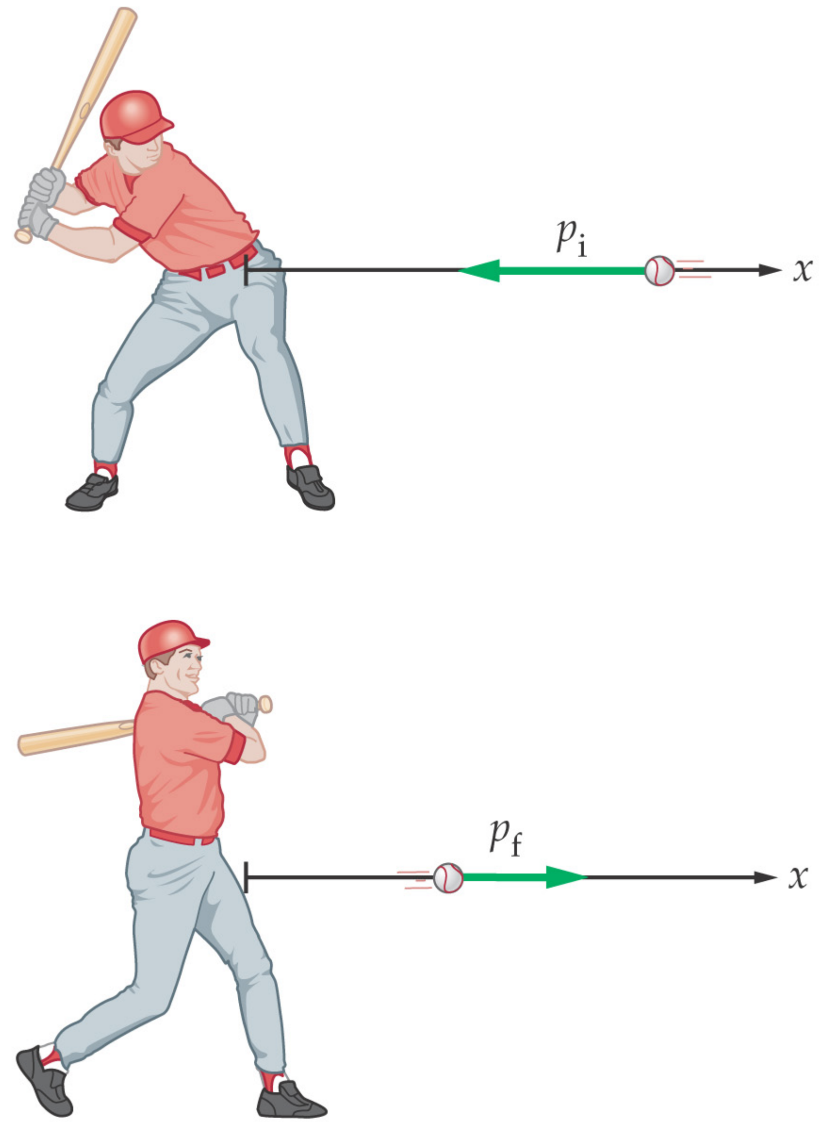
  as $$\vec{\mathbf{F}}_{\text{av}} \Delta t = \Delta \vec{\mathbf{p}}$$

  So we see that $$\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\text{av}} \Delta t = \Delta \vec{\mathbf{p}}$$

- The impulse is equal to the change in momentum.

9-3 Impulse

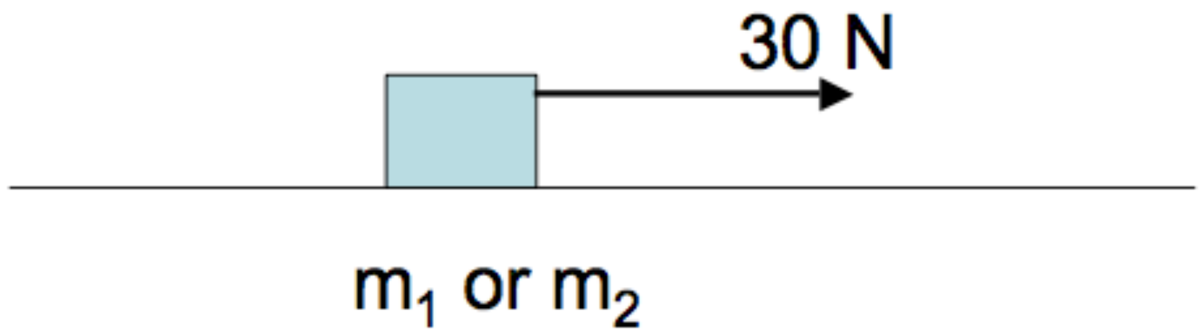
- Therefore, the same change in momentum may be produced by a large force acting for a short time, or by a smaller force acting for a longer time.



Example

- A force of 30 N is applied for 5 sec to each of two bodies of different masses.

Take $m_1 < m_2$



(a) Which mass has the greater momentum change?

$$\Delta \mathbf{p} = \mathbf{F}\Delta t$$

Since the same force is applied to each mass for the same interval, $\Delta \mathbf{p}$ is the same for both masses.

Example continued

(b) Which mass has the greatest velocity change?

$$\Delta \mathbf{v} = \frac{\Delta \mathbf{p}}{m}$$

Since both masses have the same $\Delta\mathbf{p}$, the smaller mass (mass 1) will have the larger change in velocity.

(c) Which mass has the greatest acceleration?

$$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Since $a \propto \Delta v$ the mass with the greater velocity change will have the greatest acceleration (mass 1).

Example

- An object of mass 3.0 kg is allowed to fall from rest under the force of gravity for 3.4 seconds. What is the change in momentum? Ignore air resistance.

Want $\Delta \mathbf{p} = m\Delta \mathbf{v}$.

$$\Delta \mathbf{v} = \mathbf{a}\Delta t$$

$$\Delta \mathbf{v} = -g\Delta t = -33.3\ \text{m/sec}$$

$$\Delta \mathbf{p} = m\Delta \mathbf{v} = \text{-}100\ \text{kg m/s (downward)}$$

Example

- What average force is necessary to bring a 50.0-kg sled from rest to 3.0 m/s in a period of 20.0 seconds? Assume frictionless ice.

$$\Delta \mathbf{p} = \mathbf{F}_{av} \Delta t$$

$$\mathbf{F}_{av} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{m \Delta \mathbf{v}}{\Delta t}$$

$$F_{av} = \frac{(50.0\ \text{kg})(3.0\ \text{m/s})}{20.0\ \text{s}} = 7.5\ \text{N}$$

The force will be in the direction of motion.

9-4 Conservation of linear momentum

- The net force acting on an object is the rate of change of its momentum:

$$\sum \vec{\mathbf{F}} = \frac{\Delta \vec{\mathbf{p}}}{\Delta t}$$

- If the net force is zero, the momentum does not change:

Conservation of Momentum

If the net force acting on an object is zero, its momentum is conserved; that is, $\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i$

9-4 Conservation of linear momentum

- **Internal Versus External Forces**:
- **Internal forces** act between objects within the system.
- As with all forces, they occur in action-reaction pairs. As all pairs act between objects in the system, **the internal forces always sum to zero**:

$$\sum \vec{\mathbf{F}}_{\text{int}} = 0$$

- Therefore, the net force acting on a system is the sum of the external forces acting on it.

9-4 Conservation of linear momentum

- Furthermore, internal forces cannot change the momentum of a system.

Conservation of Momentum for a System of Objects

Internal forces have absolutely no effect on the net momentum of a system.

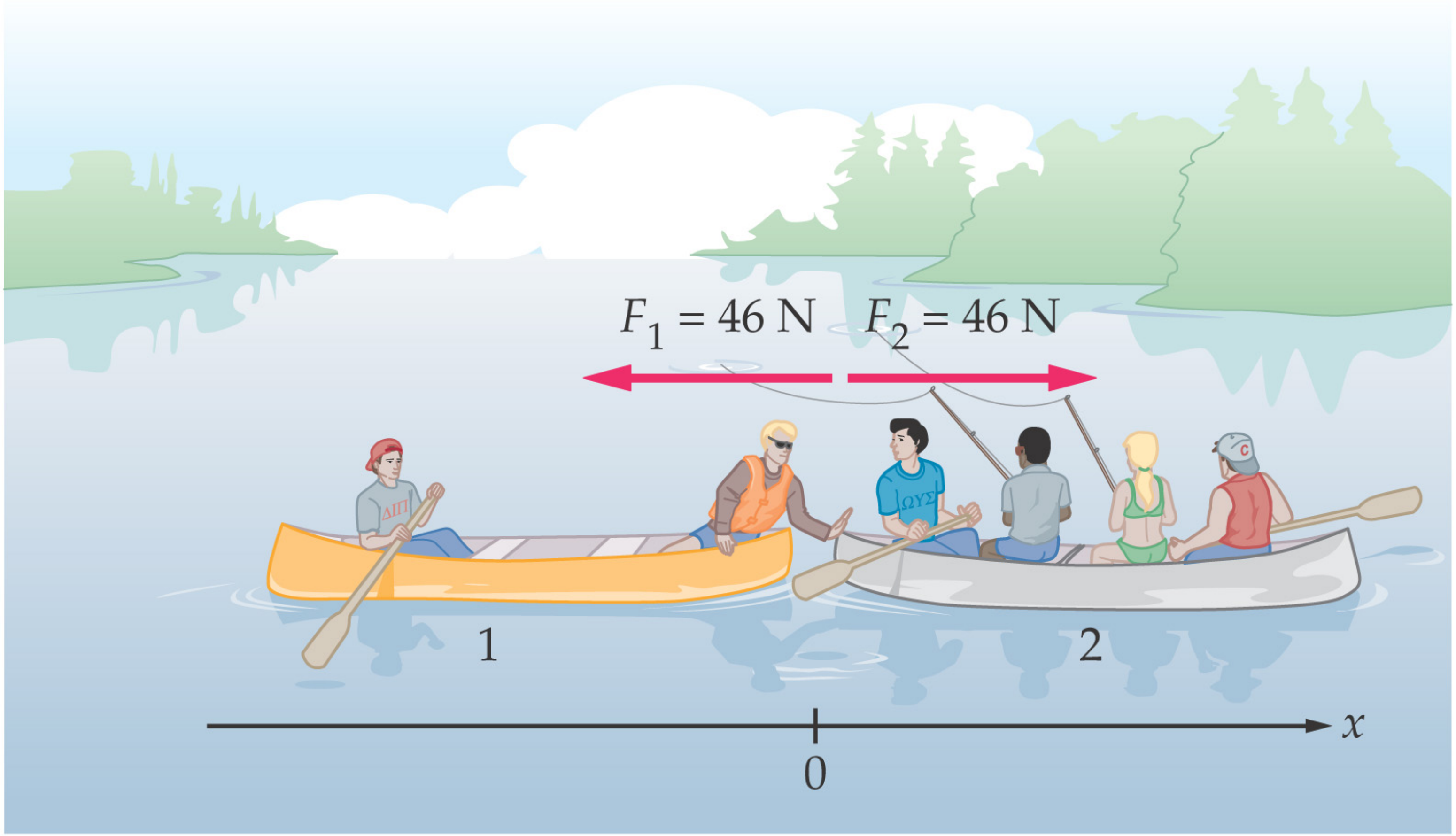
- If the *net external* force acting on a system is zero, its net momentum is conserved. That is,

$$\vec{p}_{1,f} + \vec{p}_{2,f} + \vec{p}_{3,f} + \ldots = \vec{p}_{1,i} + \vec{p}_{2,i} + \vec{p}_{3,i} + \ldots$$

- However, the momenta of components of the system may change.

9-4 Conservation of linear momentum

- An example of internal forces moving components of a system:



Example

- A rifle has a mass of 4.5 kg and it fires a bullet of 10.0 grams at a muzzle speed of 820 m/s. What is the recoil speed of the rifle as the bullet leaves the barrel?

As long as the rifle is horizontal, there will be no net external force acting on the rifle-bullet system and momentum will be conserved.

$$\mathbf{p}_i = \mathbf{p}_f$$

$$0 = m_b v_b + m_r v_r$$

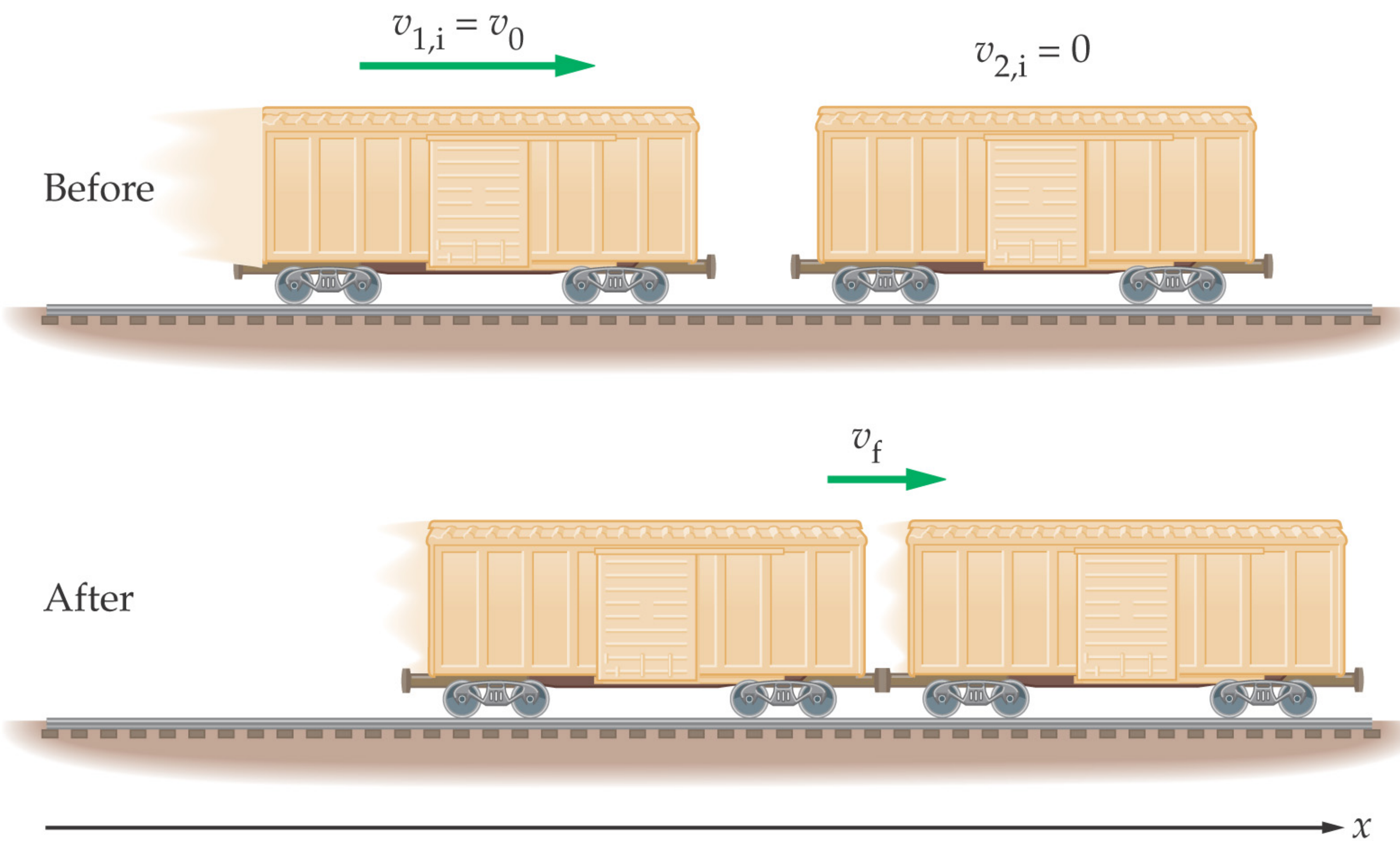
$$v_r = -\frac{m_b}{m_r} v_b = -\left(\frac{0.01\,\text{kg}}{4.5\,\text{kg}}\right) 820\,\text{m/s} = -1.82\,\text{m/s}$$

9-5 Inelastic collisions

- **Collision**: two objects striking one another
- Time of collision is short enough that external forces may be ignored
- **Inelastic collision**: momentum is conserved but kinetic energy is not
- **Completely inelastic collision**: objects stick together afterwards

9-5 Inelastic collisions

- A completely inelastic collision



9-5 Inelastic collisions

- Solving for the final momentum in terms of the initial momenta and masses:

$$p_i = m_1 v_{1,i} + m_2 v_{2,i}$$

$$p_f = (m_1 + m_2) v_f$$

$$v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2}$$

9-5 Inelastic collisions

- Ballistic pendulum: the height h can be found using conservation of mechanical energy after the object is embedded in the block.
- **Momentum conservation** in the collision:

$$mv_0 = (M + m)v_f \quad \rightarrow \quad v_f = \left(\frac{m}{M+m}\right)v_0$$

- **Kinetic energy** just after the collision:

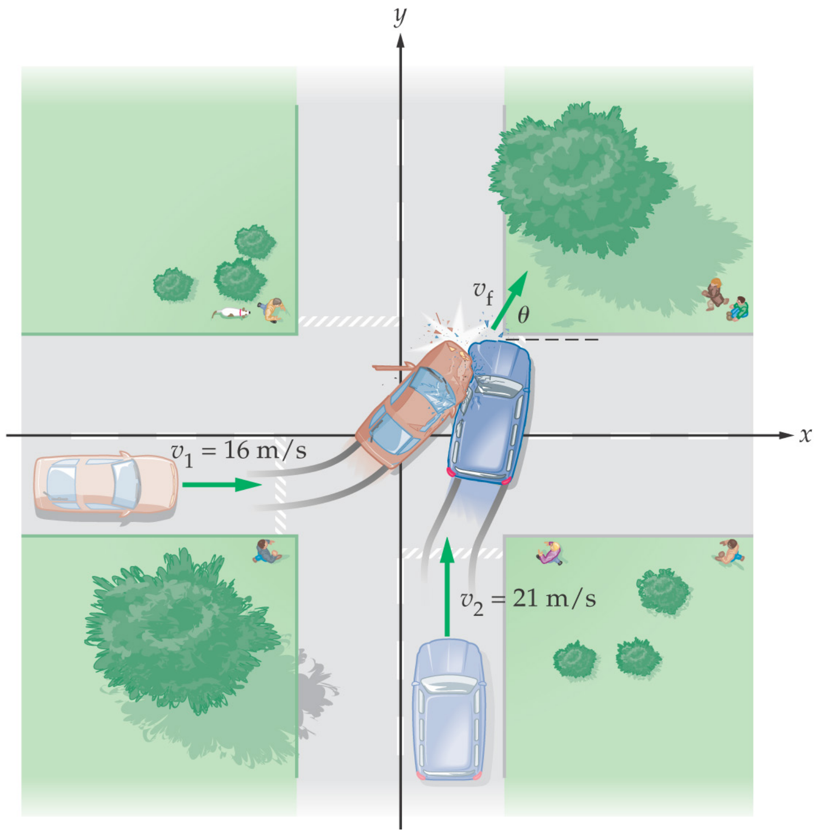
$$K_f = \frac{1}{2}(M + m)v_f^2 = \frac{1}{2}mv_0^2\left(\frac{m}{M+m}\right)$$

- Conservation of **mechanical energy**:

$$\frac{1}{2}mv_0^2\left(\frac{m}{M+m}\right) = (M + m)gh \quad \rightarrow \quad h = \left(\frac{m}{M+m}\right)^2\left(\frac{v_0^2}{2g}\right)$$

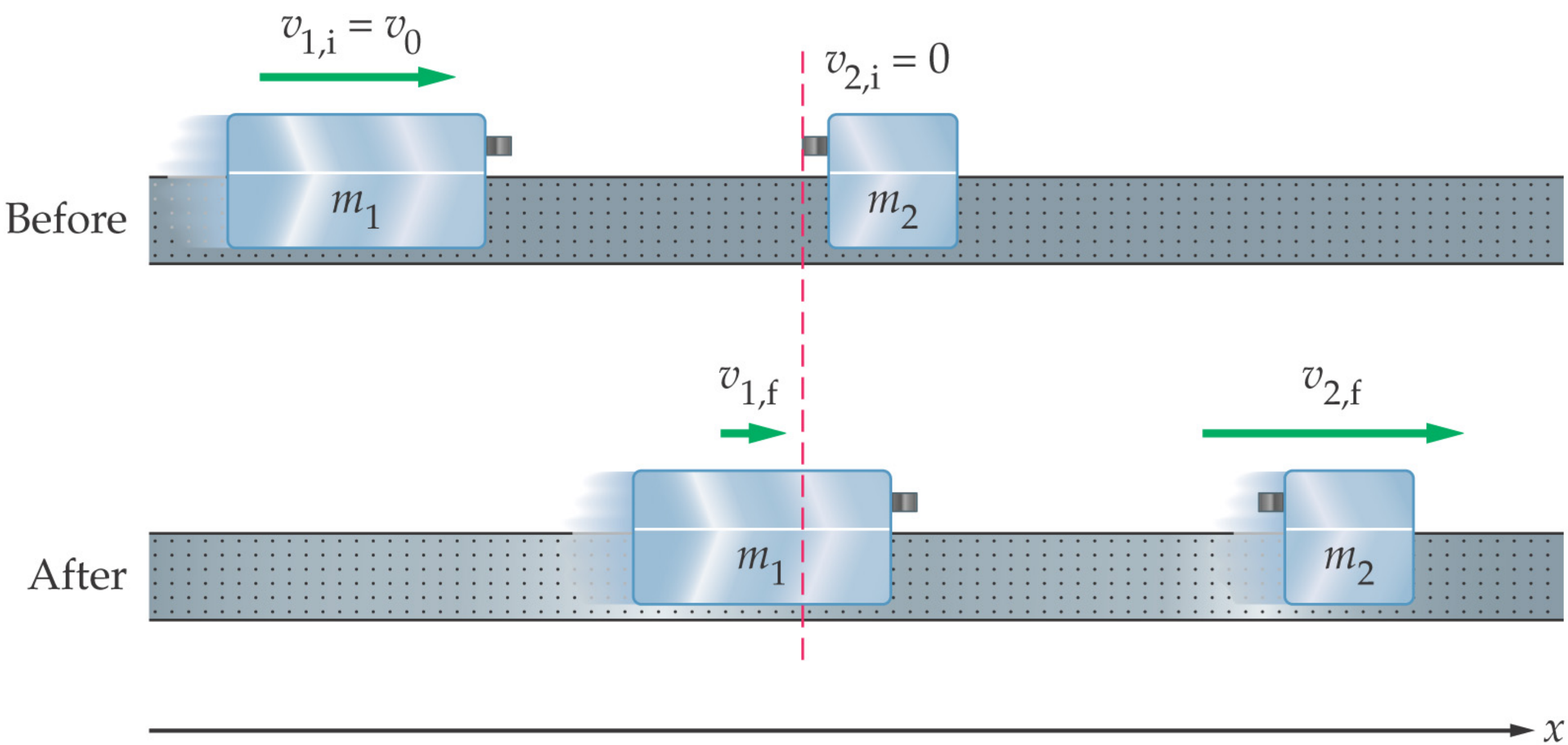
9-5 Inelastic collisions

- For collisions in two dimensions, conservation of momentum is applied separately along each axis:



9-6 Elastic collisions

- In elastic collisions, both kinetic energy and momentum are conserved.
- One-dimensional elastic collision:



9-6 Elastic collisions

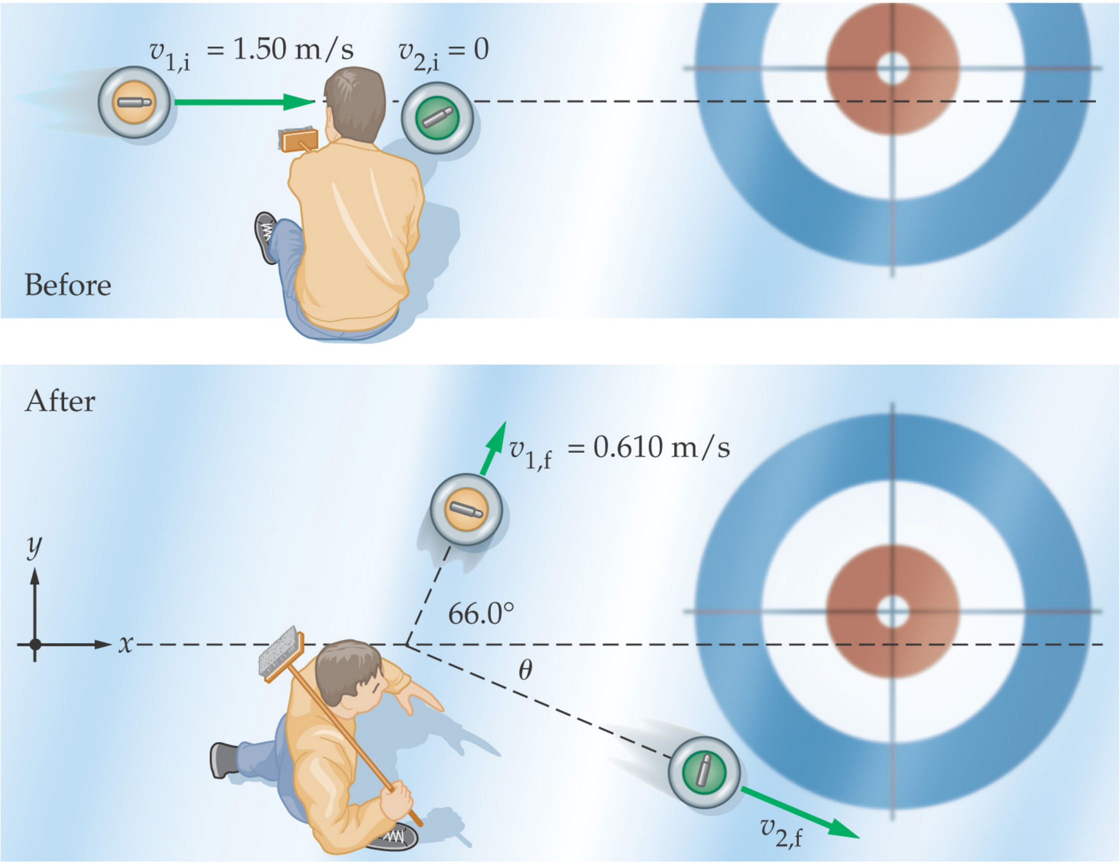
- We have two equations (conservation of momentum and conservation of kinetic energy) and two unknowns (the final speeds). Solving for the final speeds:

$$v_{1,\mathrm{f}} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_0$$

$$v_{2,\mathrm{f}} = \left(\frac{2m_1}{m_1 + m_2}\right)v_0$$

9-6 Elastic collisions

- Two-dimensional collisions can only be solved if some of the final information is known, such as the final velocity of one object:



Example

- In a railroad freight yard, an empty freight car of mass m rolls along a straight level track at 1.0 m/s and collides with an initially stationary, fully loaded, boxcar of mass 4.0m. The two cars couple together upon collision.

(a) What is the speed of the two cars after the collision?

$$p_i = p_f$$

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$m_1 v_1 + 0 = m_1 v + m_2 v = (m_1 + m_2) v$$

$$v = \left(\frac{m_1}{m_1 + m_2} \right) v_1 = 0.2 \text{ m/s}$$

Example continued

(b) Suppose instead that both cars are at rest after the collision. With what speed was the loaded boxcar moving before the collision if the empty one had v_{1i} = 1.0 m/s.

$$p_i = p_f$$

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$m_1 v_{1i} + m_2 v_{2i} = 0 + 0$$

$$v_{2i} = -\left(\frac{m_1}{m_2}\right) v_{1i} = -0.25\ \mathrm{m/s}$$

Example

- A projectile of 1.0 kg mass approaches a stationary body of 5.0 kg mass at 10.0 m/s and, after colliding, rebounds in the reverse direction along the same line with a speed of 5.0 m/s. What is the speed of the 5.0 kg mass after the collision?

$$p_i = p_f$$

$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

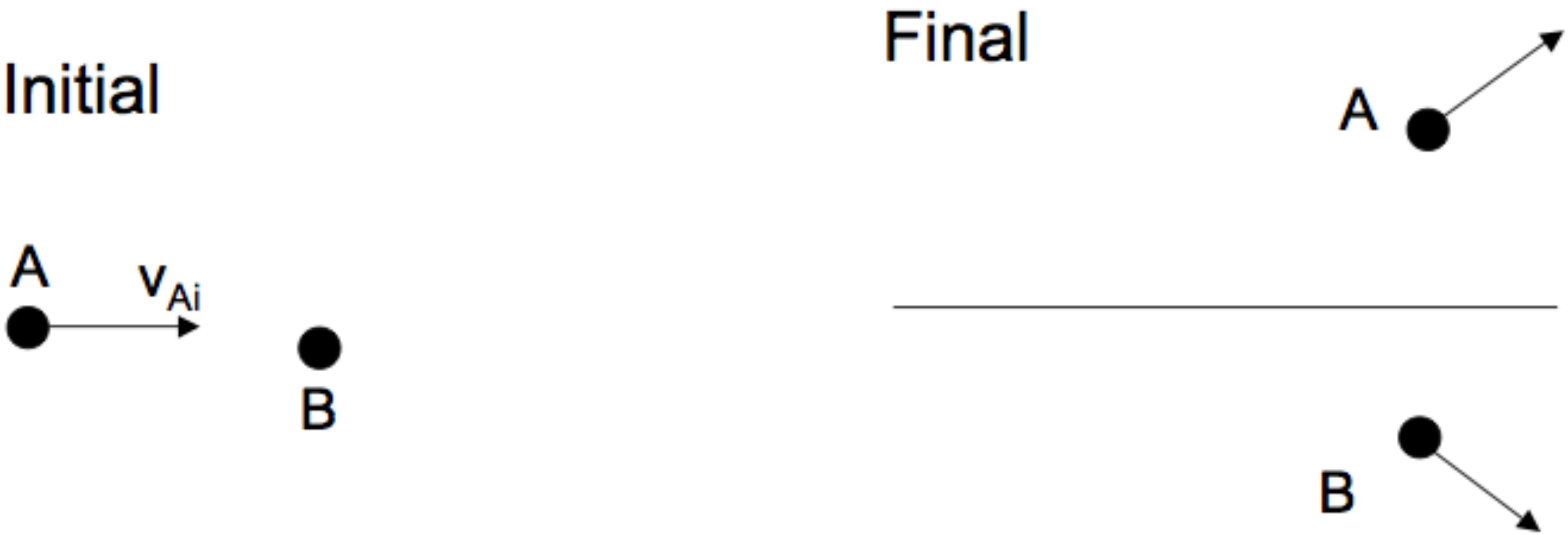
$$m_1 v_{1i} + 0 = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{2f} = \frac{m_1}{m_2}\left(v_{1i} - v_{1f}\right)$$

$$= \frac{1.0\ \text{kg}}{5.0\ \text{kg}}\left(10\ \text{m/s} - \left(-5.0\ \text{m/s}\right)\right) = 3.0\ \text{m/s}$$

Example

- Body A of mass M has an original velocity of 6.0 m/s in the +x-direction toward a stationary body (B) of the same mass. After the collision, body A has v_x = +1.0 m/s and v_y = +2.0 m/s. What is the magnitude of body B's velocity after the collision?



Example continued

x momentum:

$$p_{ix} = p_{fx}$$

$$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$$

$$m_1 v_{1ix} + 0 = m_1 v_{1fx} + m_2 v_{2fx}$$

Solve for v_{2fx}:

$$v_{2fx} = \frac{m_1 v_{1ix} - m_1 v_{1fx}}{m_2}$$

$$= v_{1ix} - v_{1fx}$$

$$= 5.00\ \text{m/s}$$

y momentum:

$$p_{iy} = p_{fy}$$

$$p_{1iy} + p_{2iy} = p_{1fy} + p_{2fy}$$

$$0 + 0 = m_1 v_{1fy} + m_2 v_{2fy}$$

Solve for v_{2fy}:

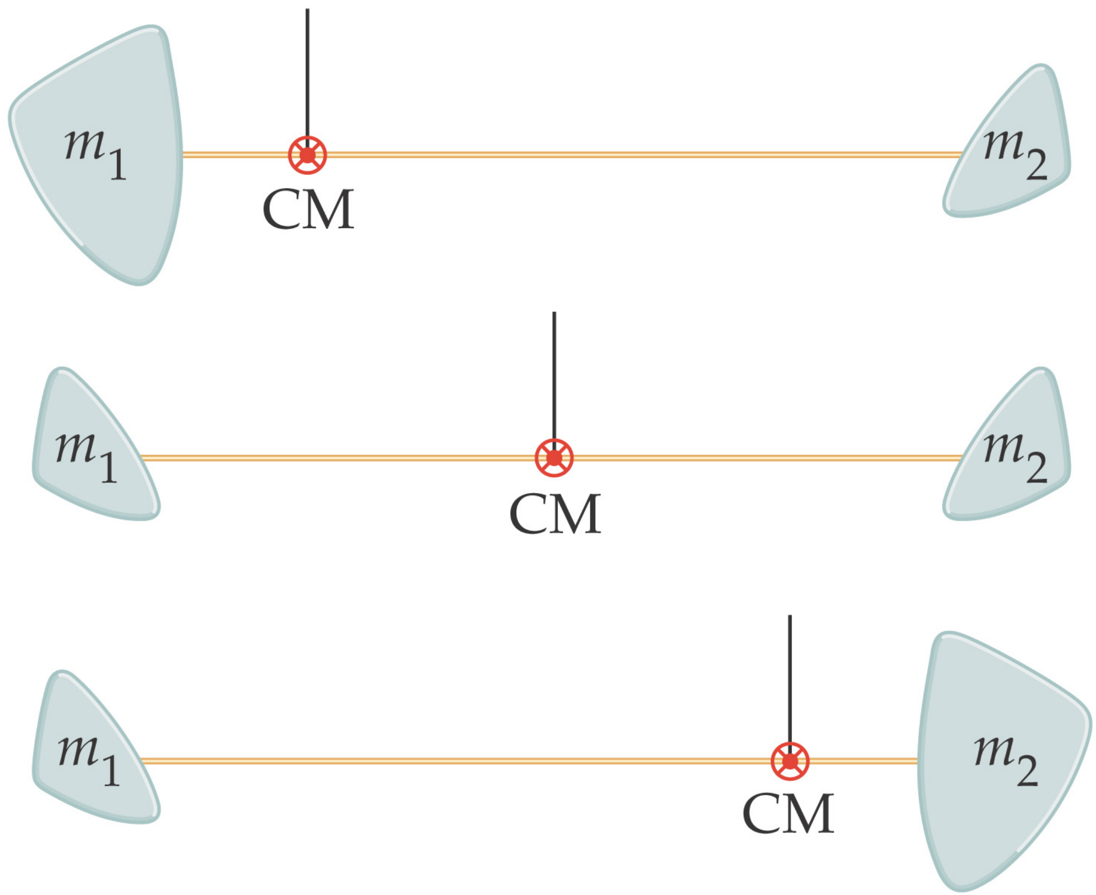
$$v_{2fy} = \frac{-m_1 v_{1fy}}{m_2} = -v_{1fy}$$

$$= -2.00\ \text{m/s}$$

The mag. of v_2 is $v_{2f} = \sqrt{v^2{}_{2fy} + v^2{}_{2fx}} = 5.40\ \text{m/s}$

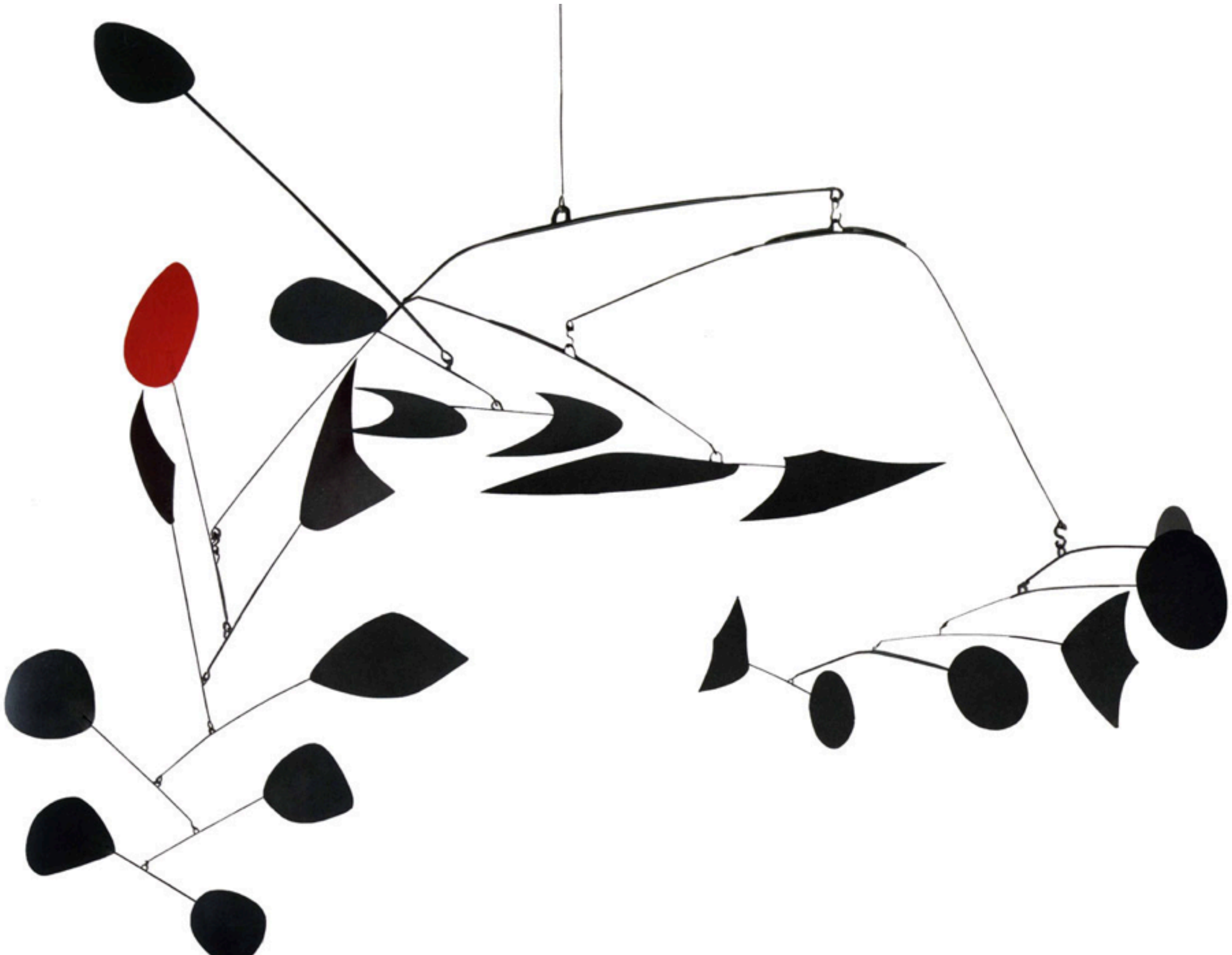
9-7 Center of Mass

- The center of mass of a system is the point where the system can be balanced in a uniform gravitational field.



9-7 Center of Mass

- Alexander Calder:



9-7 Center of Mass

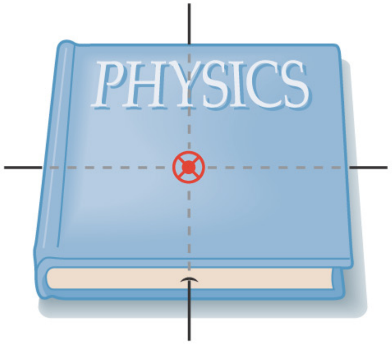
- For two objects:

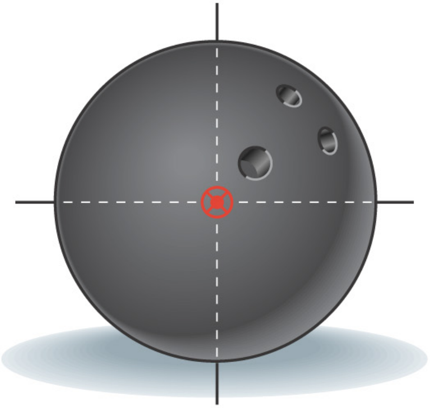
$$X_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{m_1x_1 + m_2x_2}{M}$$

- The center of mass is closer to the more massive object.

9-7 Center of Mass

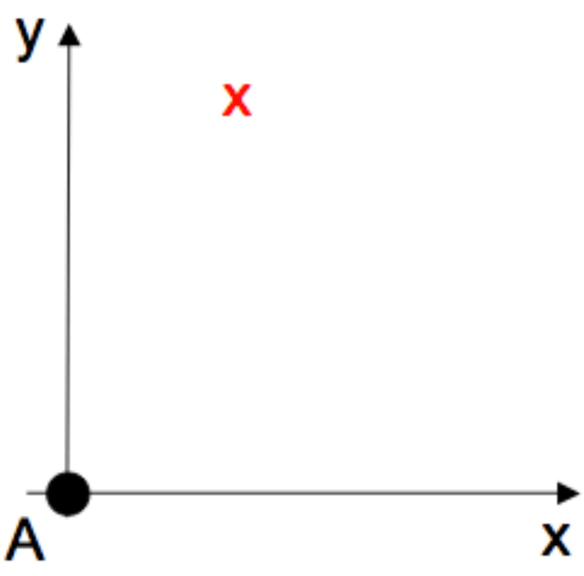
- The center of mass need not be within the object:





Example

- Particle A is at the origin and has a mass of 30.0 grams. Particle B has a mass of 10.0 grams. Where must particle B be located so that the center of mass (marked with a red x) is located at the point (2.0 cm, 5.0 cm)?



$$x_{cm} = \frac{m_a x_a + m_b x_b}{m_a + m_b} = \frac{m_b x_b}{m_a + m_b}$$

$$y_{cm} = \frac{m_a y_a + m_b y_b}{m_a + m_b} = \frac{m_b y_b}{m_a + m_b}$$

Example continued

$$x_{cm} = \frac{(10\ \text{g})x_b}{10\ \text{g} + 30\text{g}} = 2\ \text{cm}$$

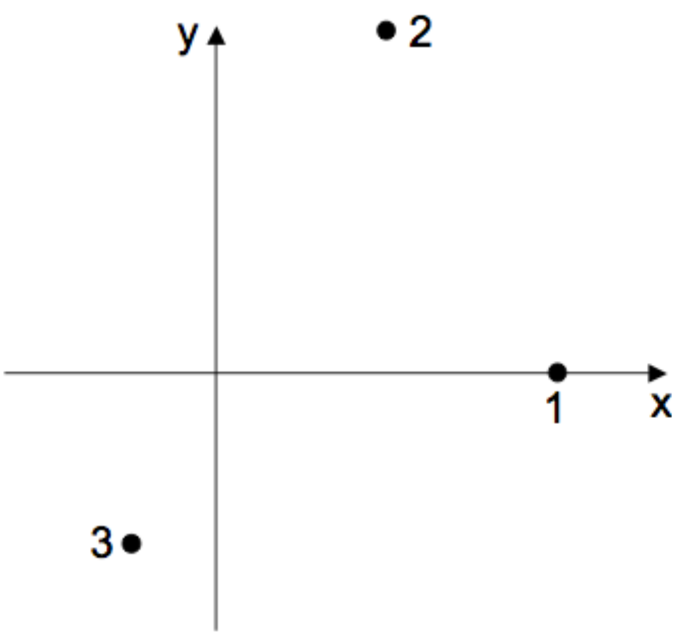
$$x_b = 8\ \text{cm}$$

$$y_{cm} = \frac{(10\ \text{g})y_b}{30\ \text{g} + 10\ \text{g}} = 5\ \text{cm}$$

$$y_b = 20\ \text{cm}$$

Example

- The positions of three particles are (4.0 m, 0.0 m),(2.0 m, 4.0 m), and (−1.0 m, −2.0 m). The masses are 4.0 kg,6.0 kg, and 3.0 kg respectively. What is the location of the center of mass?



$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{(4\text{ kg})(4\text{ m}) + (6\text{ kg})(2\text{ m}) + (3\text{ kg})(-1\text{ m})}{(4+6+3)\text{kg}}$$

$$= 1.92\text{ m}$$

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$= \frac{(4\text{ kg})(0\text{ m}) + (6\text{ kg})(4\text{ m}) + (3\text{ kg})(-2\text{ m})}{(4+6+3)\text{kg}}$$

$$= 1.38\text{ m}$$

9-7 Center of Mass

- **Motion of the center of mass:**

Velocity of the Center of Mass

$$\vec{\mathbf{V}}_{cm} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum m\vec{\mathbf{v}}}{M}$$

Acceleration of the Center of Mass

$$\vec{\mathbf{A}}_{cm} = \frac{m_1\vec{\mathbf{a}}_1 + m_2\vec{\mathbf{a}}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum m\vec{\mathbf{a}}}{M}$$

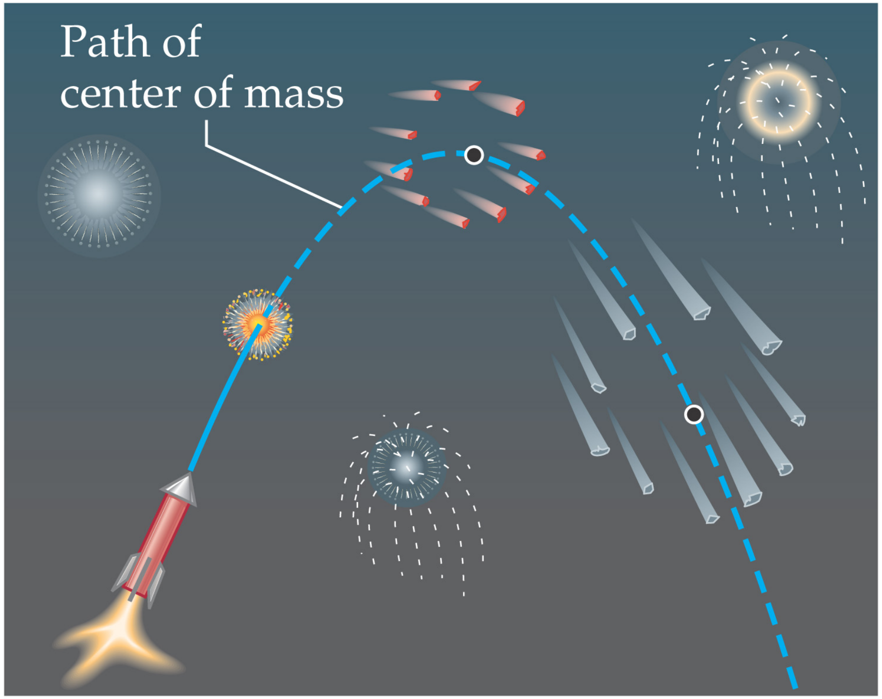
9-7 Center of Mass

- The total mass multiplied by the acceleration of the center of mass is equal to the net external force:

Newton's Second Law for a System of Particles

$$M\vec{\mathbf{A}}_{\mathrm{cm}} = \vec{\mathbf{F}}_{\mathrm{net,ext}}$$

The center of mass accelerates just as though it were a point particle of mass M acted on by $\vec{\mathbf{F}}_{\mathrm{net,ext}}$



Example

- Body a has a mass of 3.0 kg and v_x= +14.0 m/s. Body b has a mass of 4.0 kg and has v_y= −7.0 m/s. What is the velocity of the center of mass of the two bodies?

$$v_{cm,x} = \frac{m_a v_{a,x} + m_b v_{b,x}}{m_a + m_b}$$

$$= \frac{(3\text{ kg})(14\text{ m/s}) + (4\text{ kg})(0\text{ m/s})}{(3+4)\text{kg}} = 6\text{ m/s}$$

$$v_{cm,y} = \frac{m_a v_{a,y} + m_b v_{b,y}}{m_a + m_b}$$

$$= \frac{(3\text{ kg})(0\text{ m/s}) + (4\text{ kg})(-7\text{ m/s})}{(3+4)\text{kg}} = -4\text{ m/s}$$

Summary of Chapter 9

- Linear momentum: $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
- Momentum is a vector
- Newton's second law: $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$
- Impulse: $\vec{\mathbf{I}} = \vec{\mathbf{F}}_{\text{av}}\Delta t = \Delta\vec{\mathbf{p}}$
- Impulse is a vector
- The impulse is equal to the change in momentum
- If the time is short, the force can be quite large

Summary of Chapter 9

- Momentum is conserved if the net external force is zero
- Internal forces within a system always sum to zero
- In collision, assume external forces can be ignored
- Inelastic collision: kinetic energy is not conserved
- Completely inelastic collision: the objects stick together afterward

Summary of Chapter 9

- A one-dimensional collision takes place along a line
- In two dimensions, conservation of momentum is applied separately to each
- Elastic collision: kinetic energy is conserved
- Center of mass:

$$X_{\mathrm{cm}} = \frac{m_1 x_1 + m_2 x_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum mx}{M}$$

Summary of Chapter 9

- Center of mass:

$$Y_{\text{cm}} = \frac{m_1 y_1 + m_2 y_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum my}{M}$$

$$\vec{\mathbf{V}}_{\text{cm}} = \frac{m_1 \vec{\mathbf{v}}_1 + m_2 \vec{\mathbf{v}}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum m\vec{\mathbf{v}}}{M}$$

$$\vec{\mathbf{A}}_{\text{cm}} = \frac{m_1 \vec{\mathbf{a}}_1 + m_2 \vec{\mathbf{a}}_2 + \ldots}{m_1 + m_2 + \ldots} = \frac{\sum m\vec{\mathbf{a}}}{M}$$

- Motion of center of mass: $M\vec{\mathbf{A}}_{\text{cm}} = \vec{\mathbf{F}}_{\text{net,ext}}$