Lecture 2, Part 1: Yields

R. BELLWIED

Basic Idea of Statistical Hadronic Models

- Assume thermally (constant T_{ch}) and chemically (constant n_i) equilibrated system
- Given T_{ch} and μ 's (+ system size), n_i's can be calculated in a grand canonical ensemble (μVT ensemble)

Chemical freeze-out

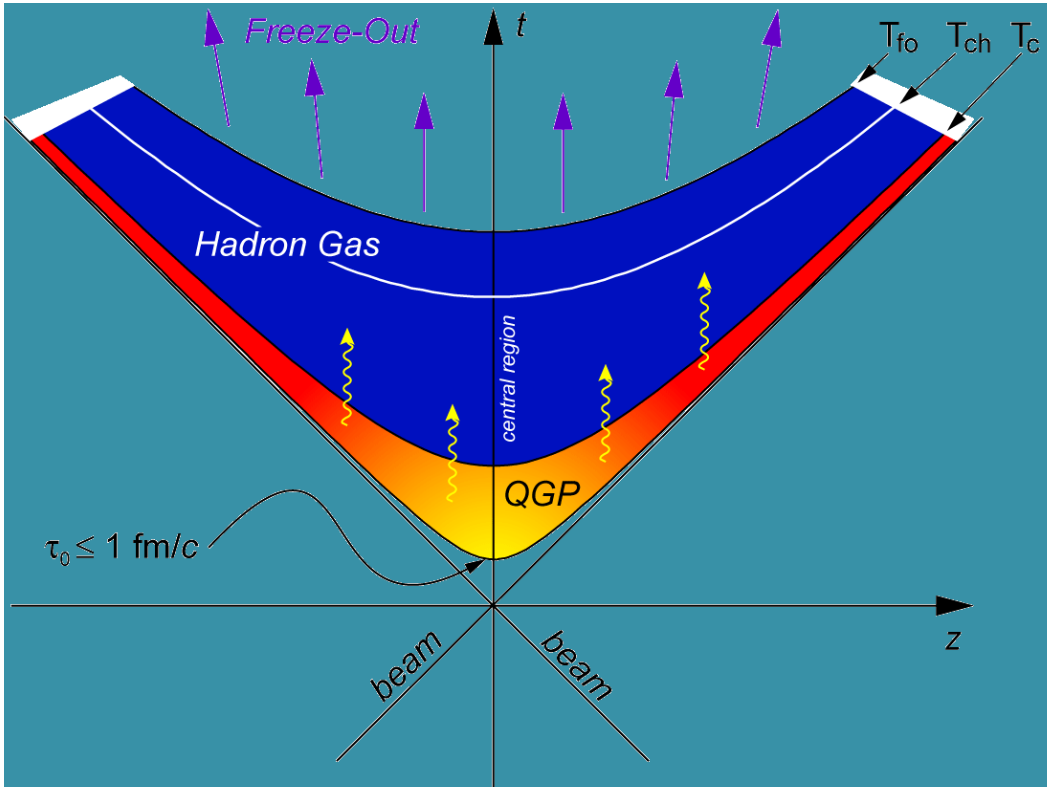
(yields & ratios)

- inelastic interactions cease
- particle abundances fixed (except maybe resonances)

Thermal freeze-out

(*shapes* of p_T,m_T spectra):

- elastic interactions cease
- particle dynamics fixed



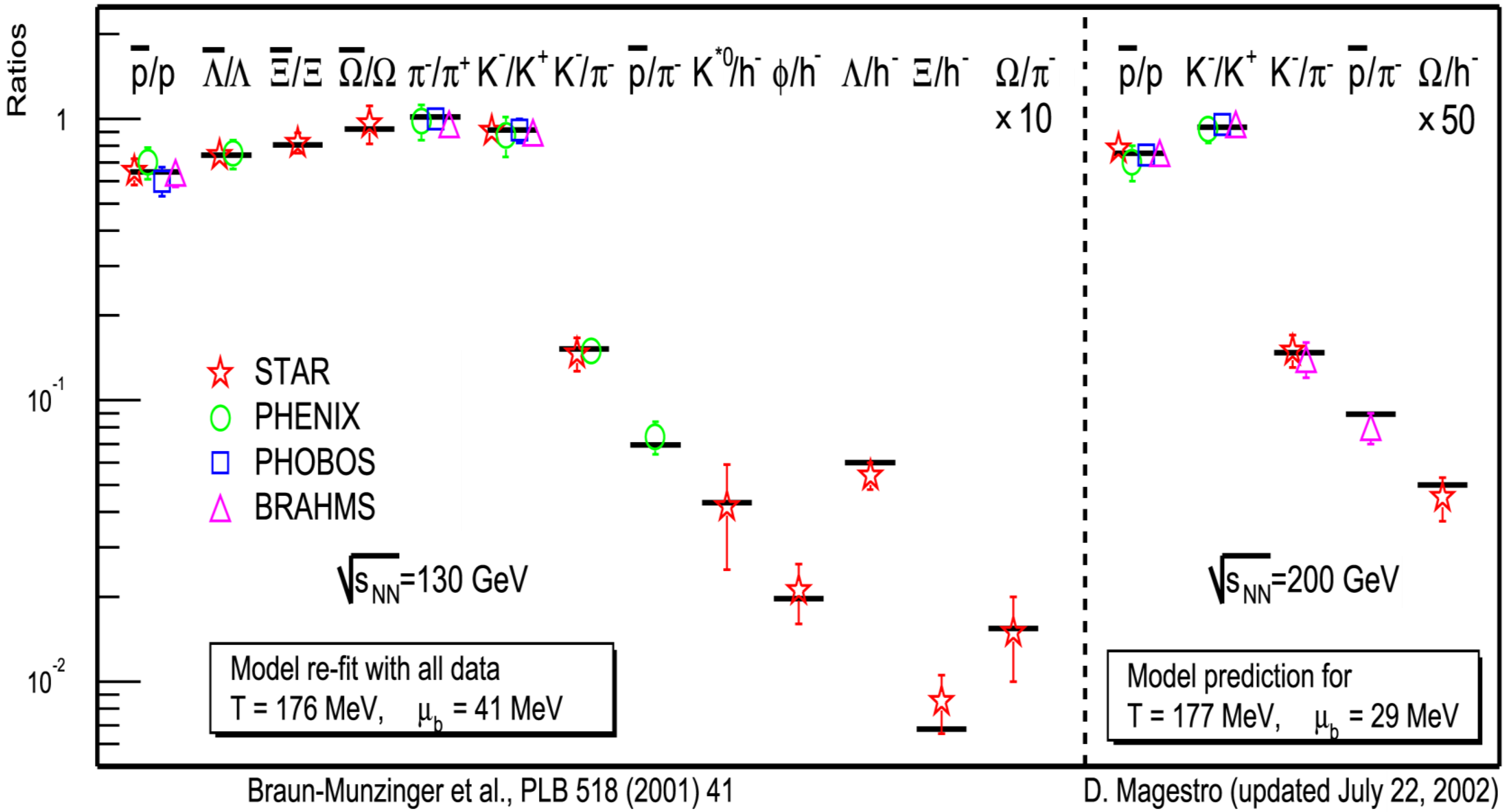
Ensembles in statistical mechanics

- Microcanonical ensemble is a concept used to describe the thermodynamic properties of an isolated system. Possible states of the system have the same energy and the probability for the system to be in any given state is the same. So, it describes a system with a fixed number of particles ("N"), a fixed volume ("V"), and a fixed energy ("E").

- Canonical ensemble describes a system where the number of particles ("N") and the volume ("V") is constant, and it has a well defined temperature ("T"), which specifies fluctuation of energy.

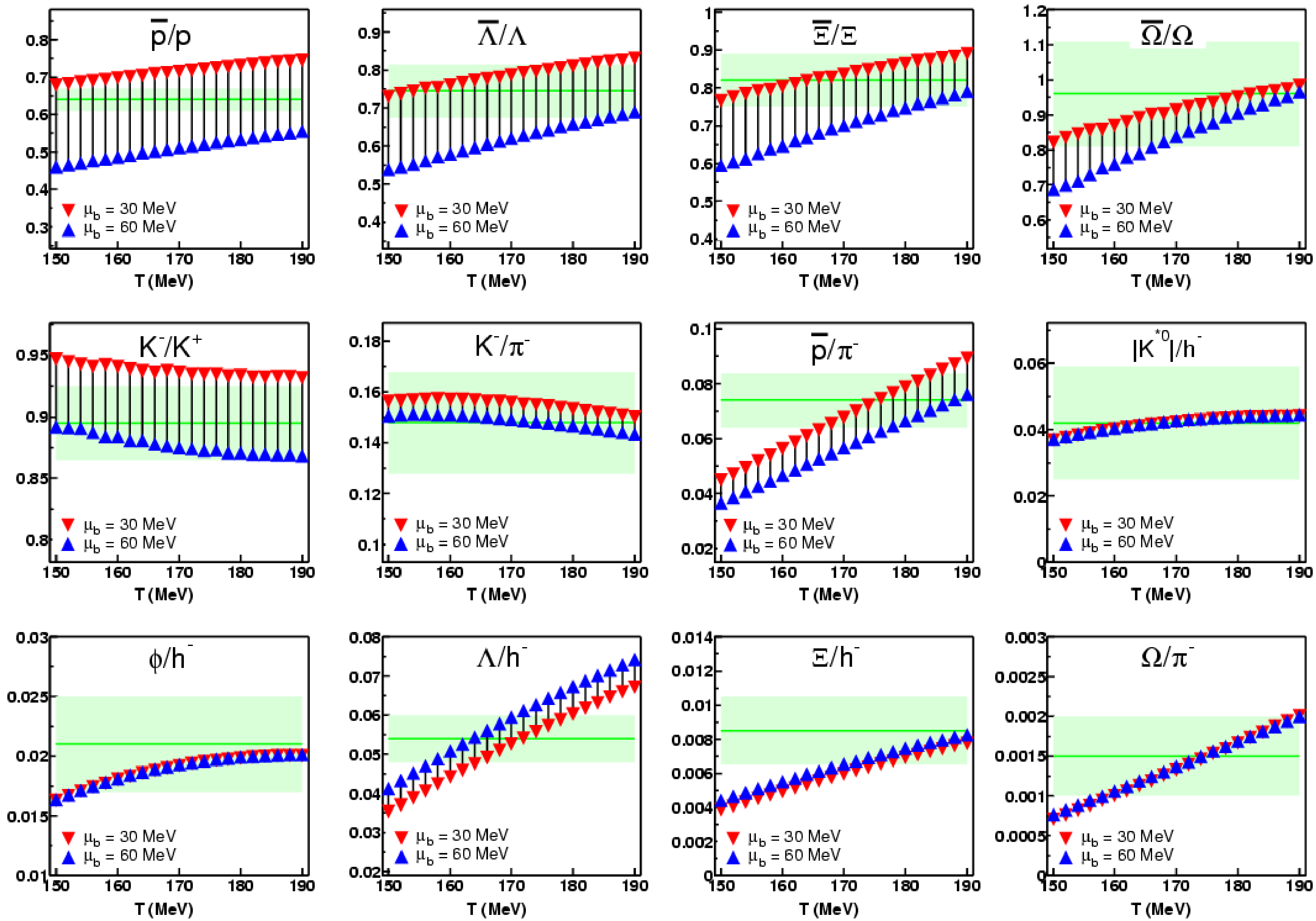
- Grand canonical ensemble describes a system with fixed volume ("V") which is in thermal and chemical equilibrium with a reservoir. Both, energy ("T") and particles ("N") are allowed to fluctuate. To specify the ("N") fluctuation it introduces a chemical potential ("μ").

Particle production at relativistic energies: statistical models do well



We get a chemical freeze-out temperature and a baryochemical potential from the fit

Ratios that constrain model parameters



D. Magestro, J. Phys G28 (2002) 1745; updated July 21

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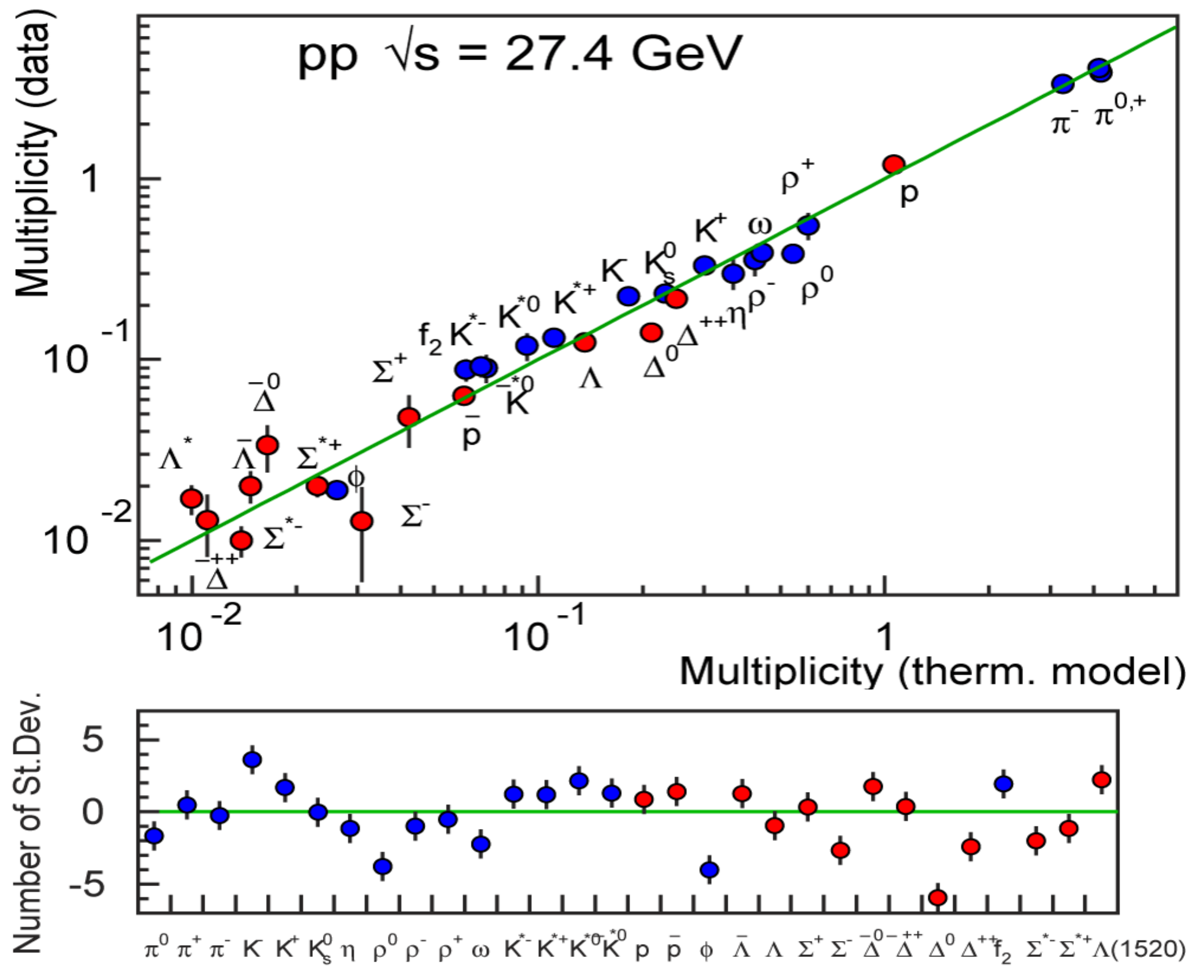
Statistical Hadronic Models : Misconceptions

- Model says <u>nothing</u> about how system reaches chemical equilibrium
- Model says <u>nothing</u> about when system reaches chemical equilibrium
- Model makes no predictions of dynamical quantities
- Some models use a strangeness suppression factor, others not
- Model does not make assumptions about a partonic phase; However the model findings can complement other studies of the phase diagram (e.g. Lattice-QCD)

Continuing puzzle: model does well in pp
Thermalization in Elementary Collisions ?



Beccatini, Heinz, Z.Phys. C76 (1997) 269

Thermalization in Elementary Collisions ?

- Is a process which leads to multiparticle production thermal?

➢ *Any* mechanism for producing hadrons which evenly populates the free particle phase space will mimic a microcanonical ensemble.

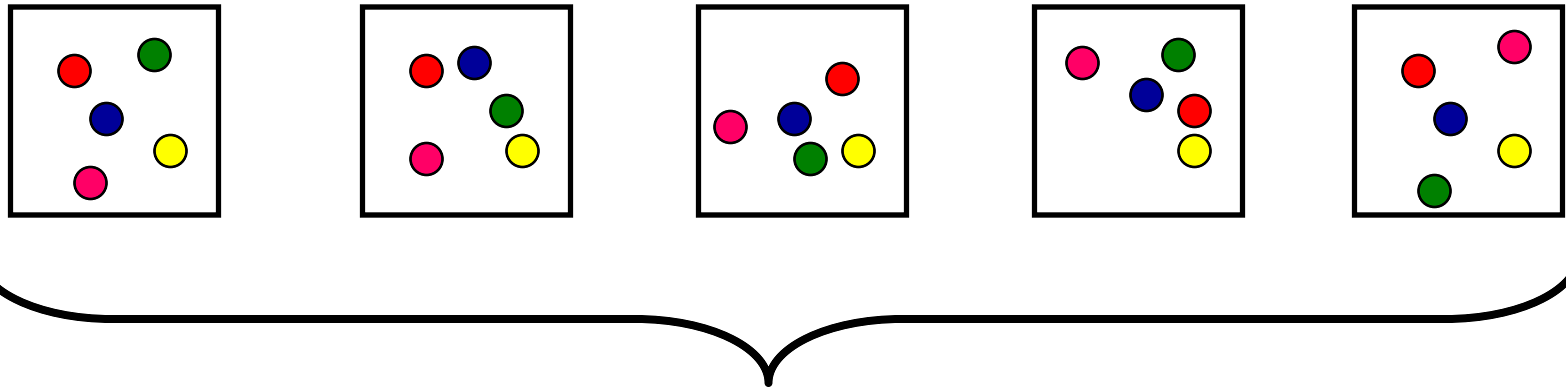
➢ Relative probability to find a given number of particles is given by the ratio of the phase-space volumes $P_n/P_{n'} = \phi_n(E)/\phi_{n'}(E) \Rightarrow$ given by statistics only. Difference between MCE and CE vanishes as the size of the system N increases.

This type of "thermal" behavior requires no rescattering and no interactions. The collisions simply serve as a mechanism to populate phase space without ever reaching thermal or chemical equilibrium

In RHI we are looking for large collective effects.

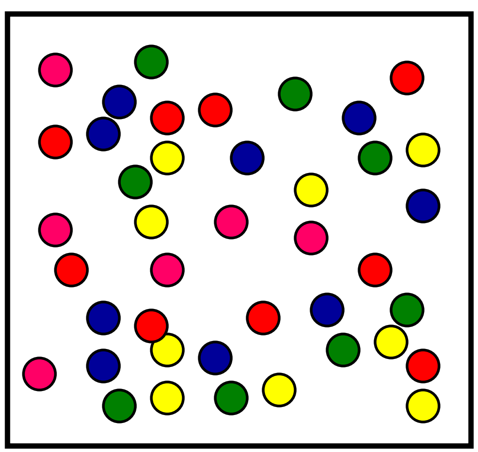
Statistics ≠ Thermodynamics

p+p



Ensemble of events constitutes a statistical ensemble
T and μ are simply Lagrange multipliers
“Phase Space Dominance”

A+A



We can talk about pressure

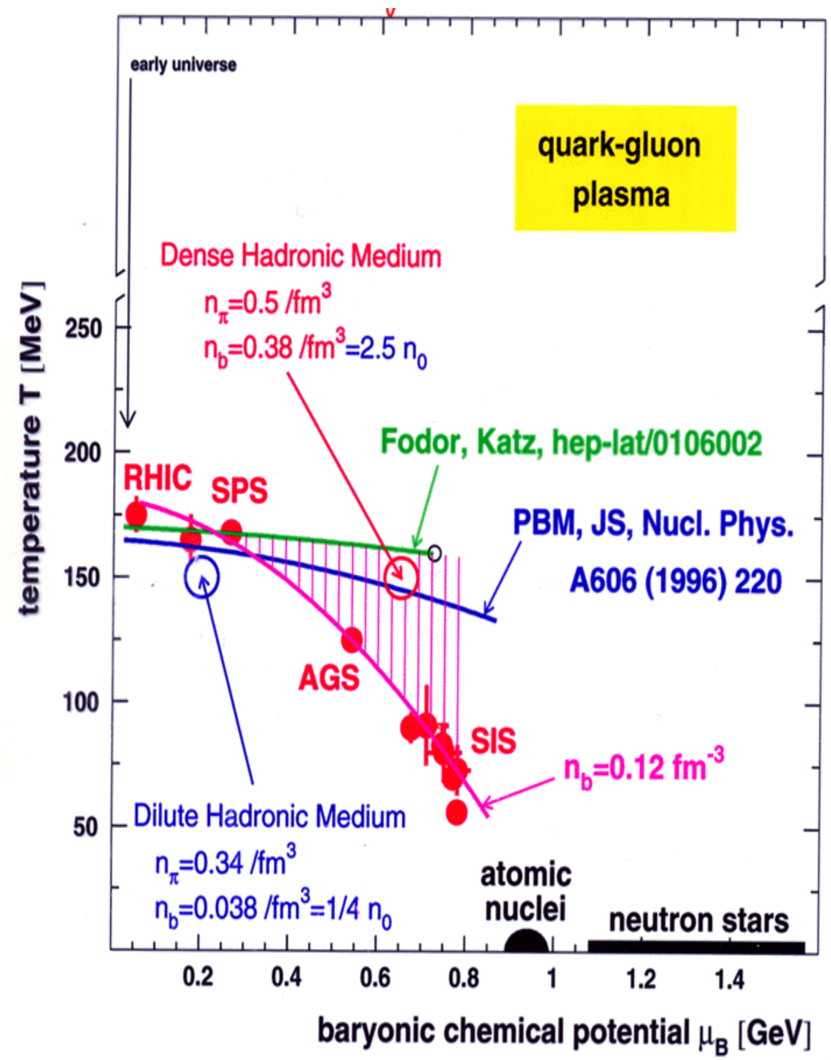
- **T and μ are more than Lagrange multipliers**

Are statistical hadronization models relevant ?

Since there is no information about the dynamics prior to freeze-out, thermal models generally will tell us nothing about QGP, but (e.g. PBM et al., nucl-th/0112051):

Elementary particle collisions:
canonical description, i.e. local quantum number conservation (e.g.strangeness) over small volume. Just Lagrange multipliers, not indicators of thermalization.

Heavy ion collisions:
grand-canonical description, i.e. percolation of strangeness over large volumes, most likely in deconfined phase if chemical freeze-out is close to phase boundary.



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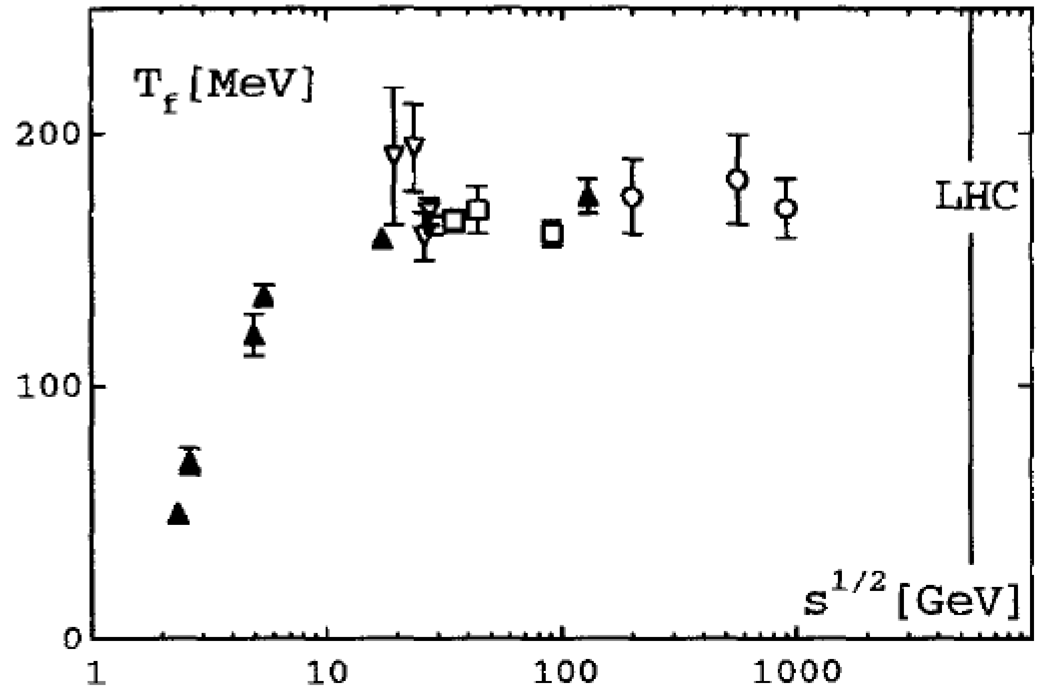
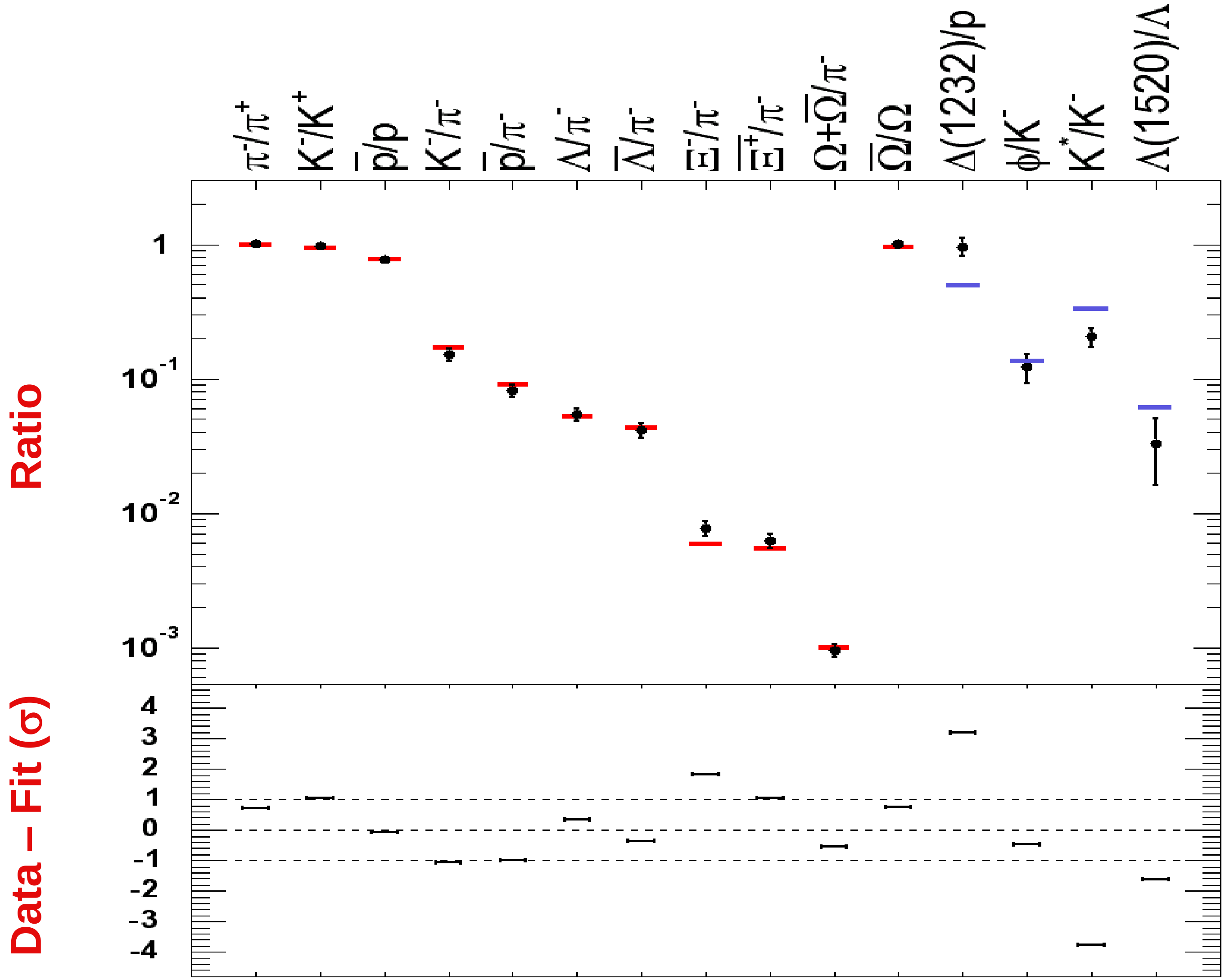
T_{ch} systematics



- it looks like Hagedorn was right!
  - if the resonance mass spectrum grows exponentially (and this seems to be the case), there is a maximum possible temperature for a system of hadrons
  - indeed, we do not seem to be able to produce a system of hadrons with a temperature beyond T_{max} ~ 170 MeV!

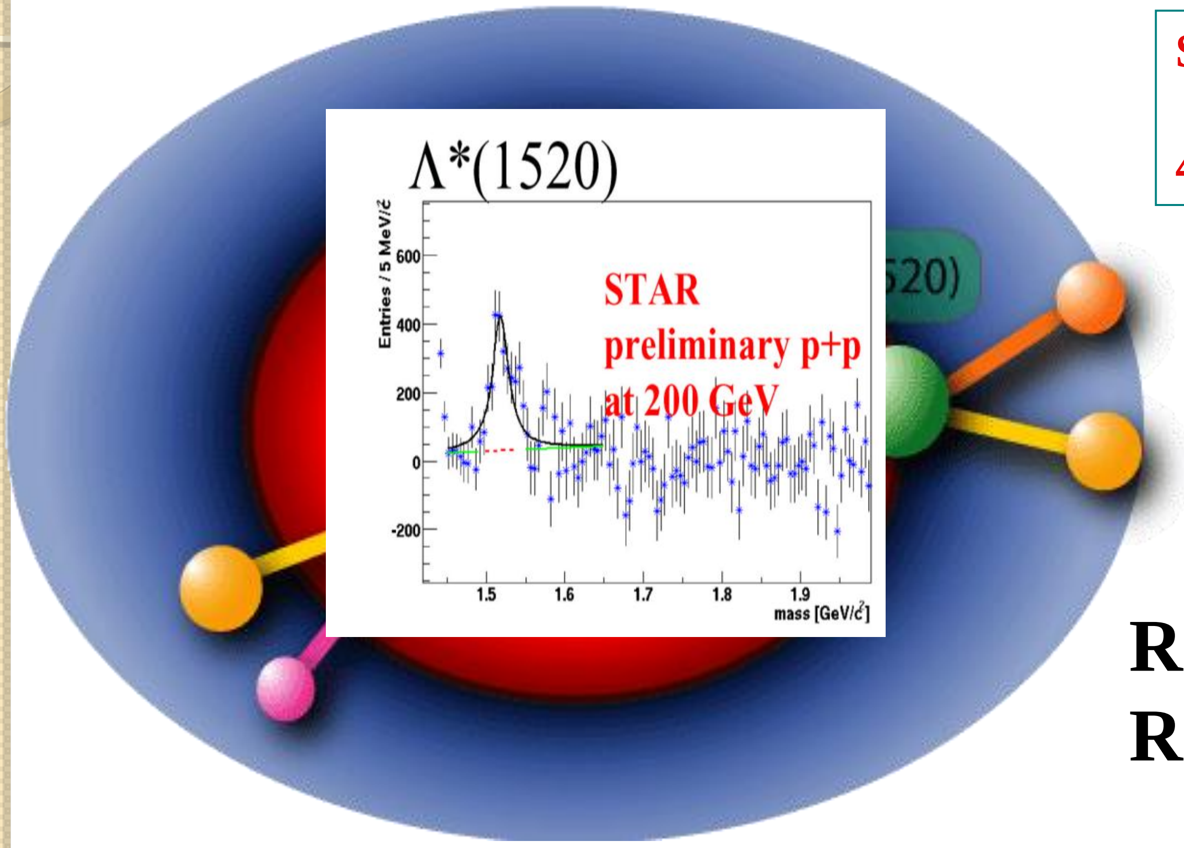
Does the thermal model always work ?



Resonance ratios deviate → Hadronic rescattering & regeneration

Strange resonances in medium

Short life times [fm/c]:
K* < Σ*< Λ(1520) < ϕ
4 < 6 < 13 < 40

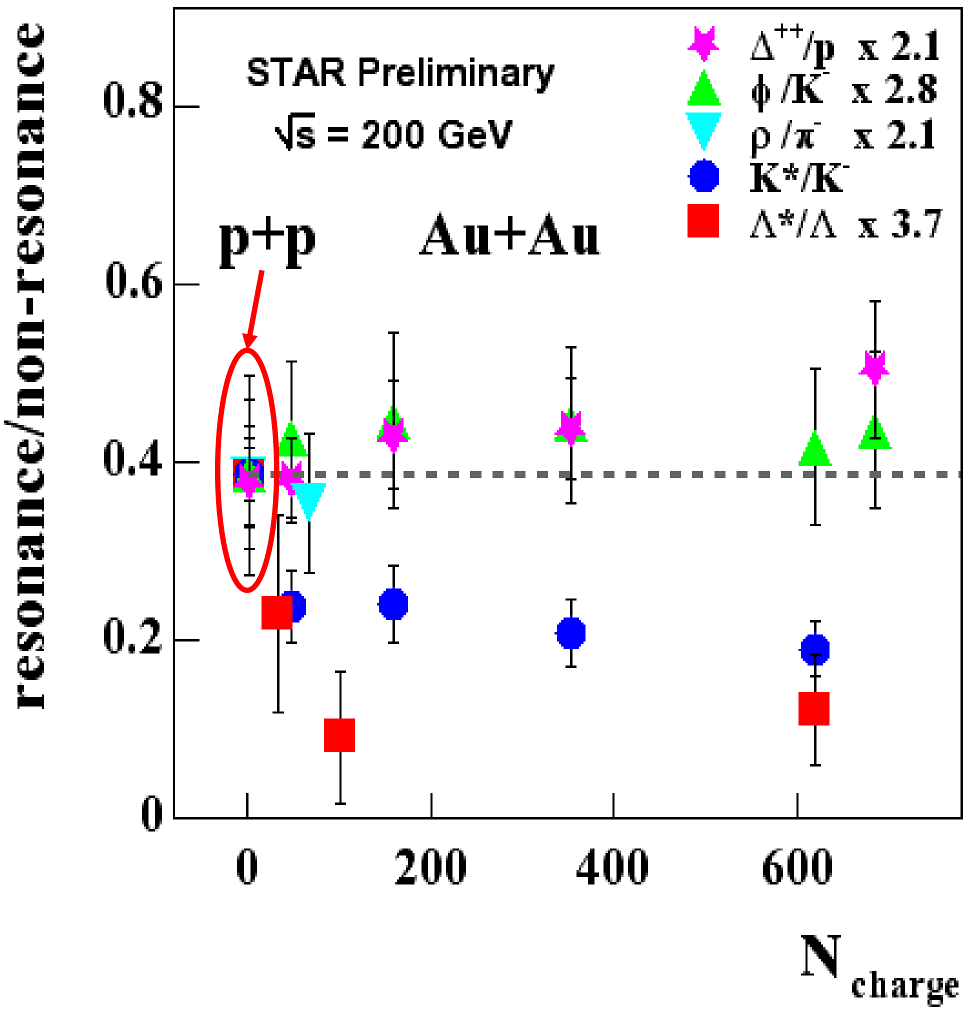


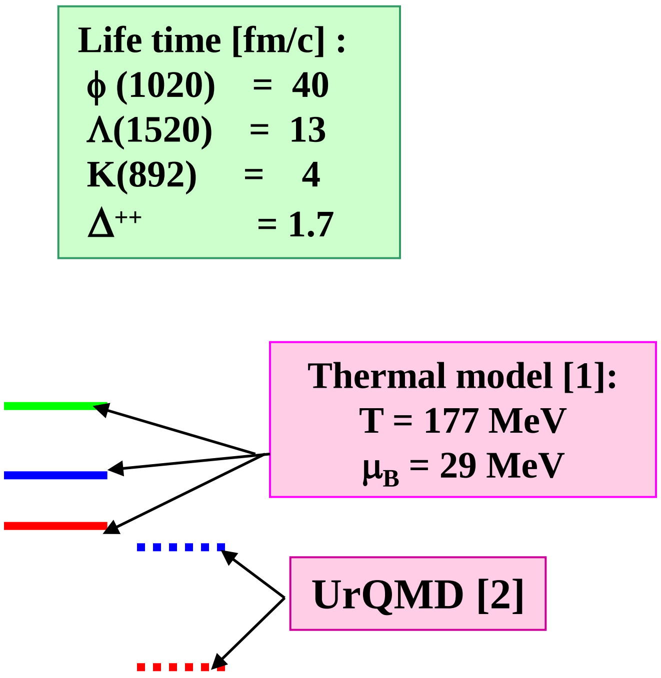
Rescattering vs. Regeneration ?

Red: before chemical freeze out
Blue: after chemical freeze out

Medium effects on resonance and their decay products before (inelastic) and after chemical freeze out (elastic).

Resonance Production in p+p and Au+Au





[1] P. Braun-Munzinger et.al., PLB 518(2001) 41
D.Magestro, private communication
[2] Marcus Bleicher and Jörg Aichelin
Phys. Lett. B530 (2002) 81-87.
M. Bleicher, private communication

Rescattering and regeneration is needed !

Strength can determine hadronic/partonic lifetime

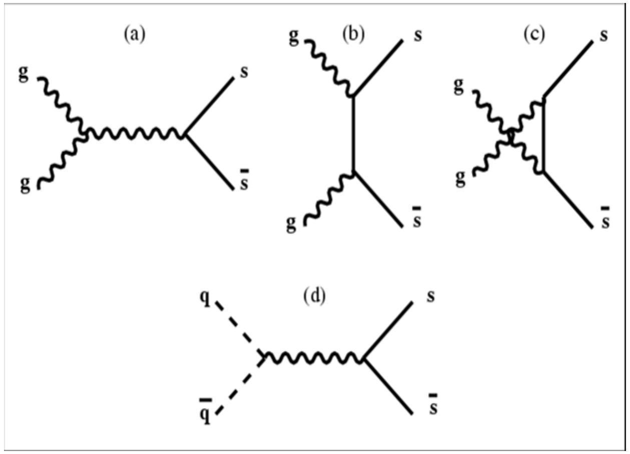
Strangeness: Two historic QGP predictions

- restoration of χ symmetry -> increased production of s
  - mass of strange quark in QGP expected to go back to current value (m_S ~ 150 MeV ~ T_c)

  → copious production of ss pairs, mostly by gg fusion

  [Rafelski: Phys. Rep. 88 (1982) 331]

  [Rafelski-Müller: P. R. Lett. 48 (1982) 1066]



- deconfinement → stronger effect for multi-strange baryons
  - by using uncorrelated s quarks produced in independent partonic reactions, faster and more copious than in hadronic phase

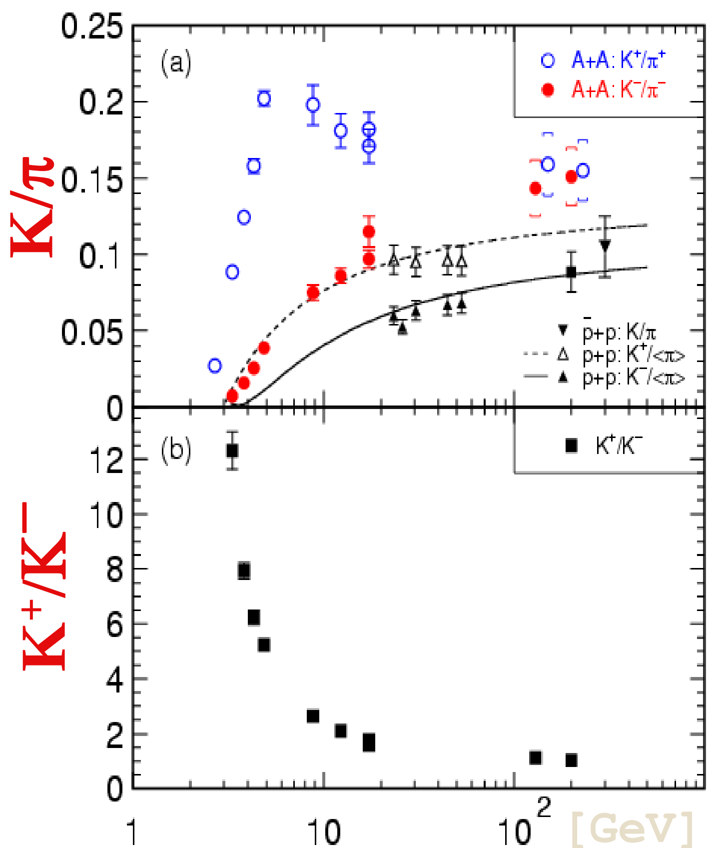
$$q+\bar{q}\rightarrow s+\bar{s} \qquad E_{thres}=2m_s\approx 300\ \mathrm{MeV}$$
$$g+g\rightarrow s+\bar{s}$$

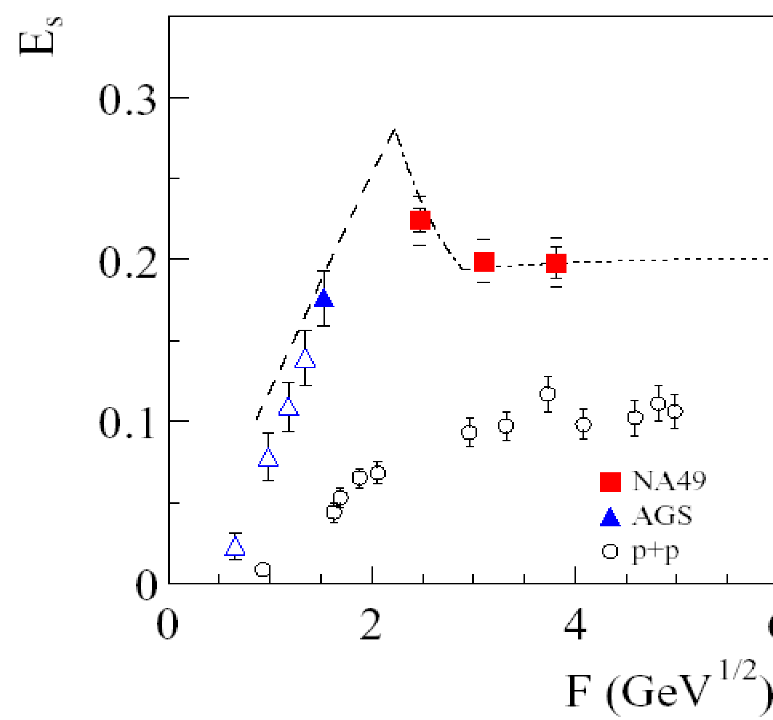
$$\pi+\mathrm{N}\rightarrow\Lambda+K \qquad E_{thres}\approx 530\ \mathrm{MeV}$$
$$K+\pi\rightarrow\bar{\Lambda}+\mathrm{N} \qquad E_{thres}\approx 1420\ \mathrm{MeV}$$

→ strangeness enhancement increasing with strangeness content [Koch, Müller & Rafelski: Phys. Rep. 142 (1986) 167]

Strangeness enhancement

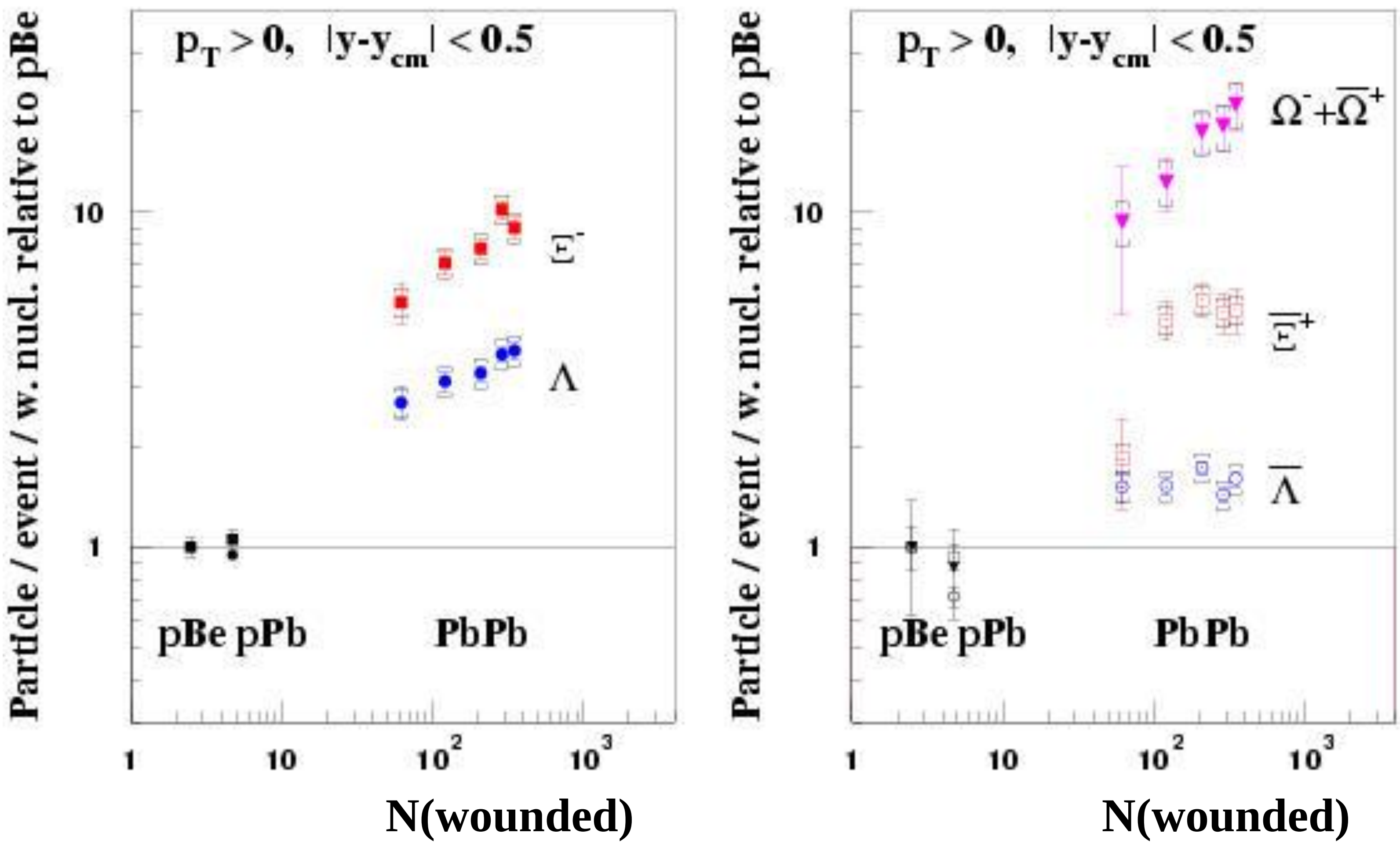
- K/π – the benchmark for abundant strangeness production:





line: M.Gazdzicki, M.I. Gorenstein, Acta Phys. Pol. B 9, 2705

The SPS 'discovery plot' (WA97/NA57) Unusual strangeness enhancement



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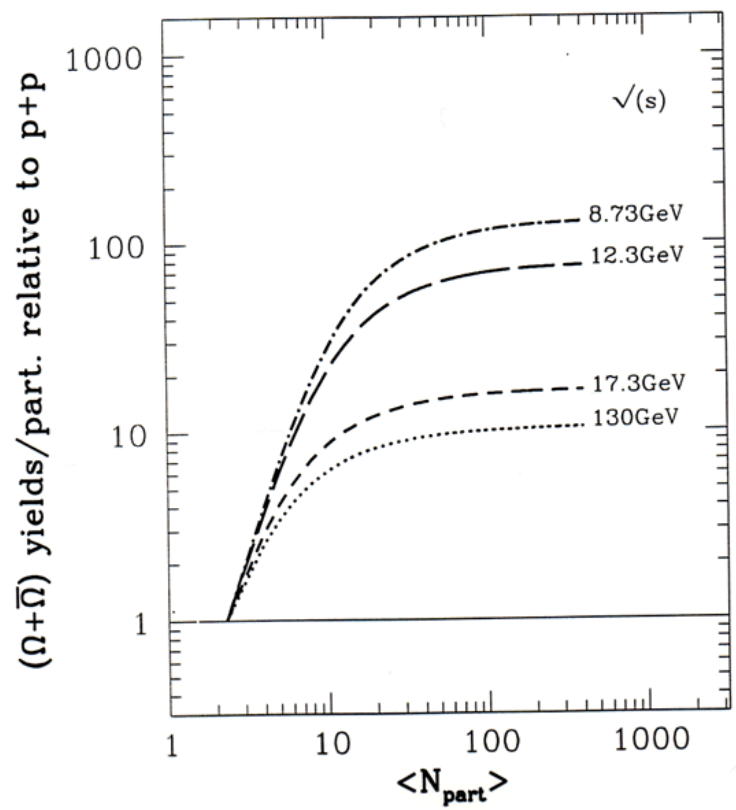
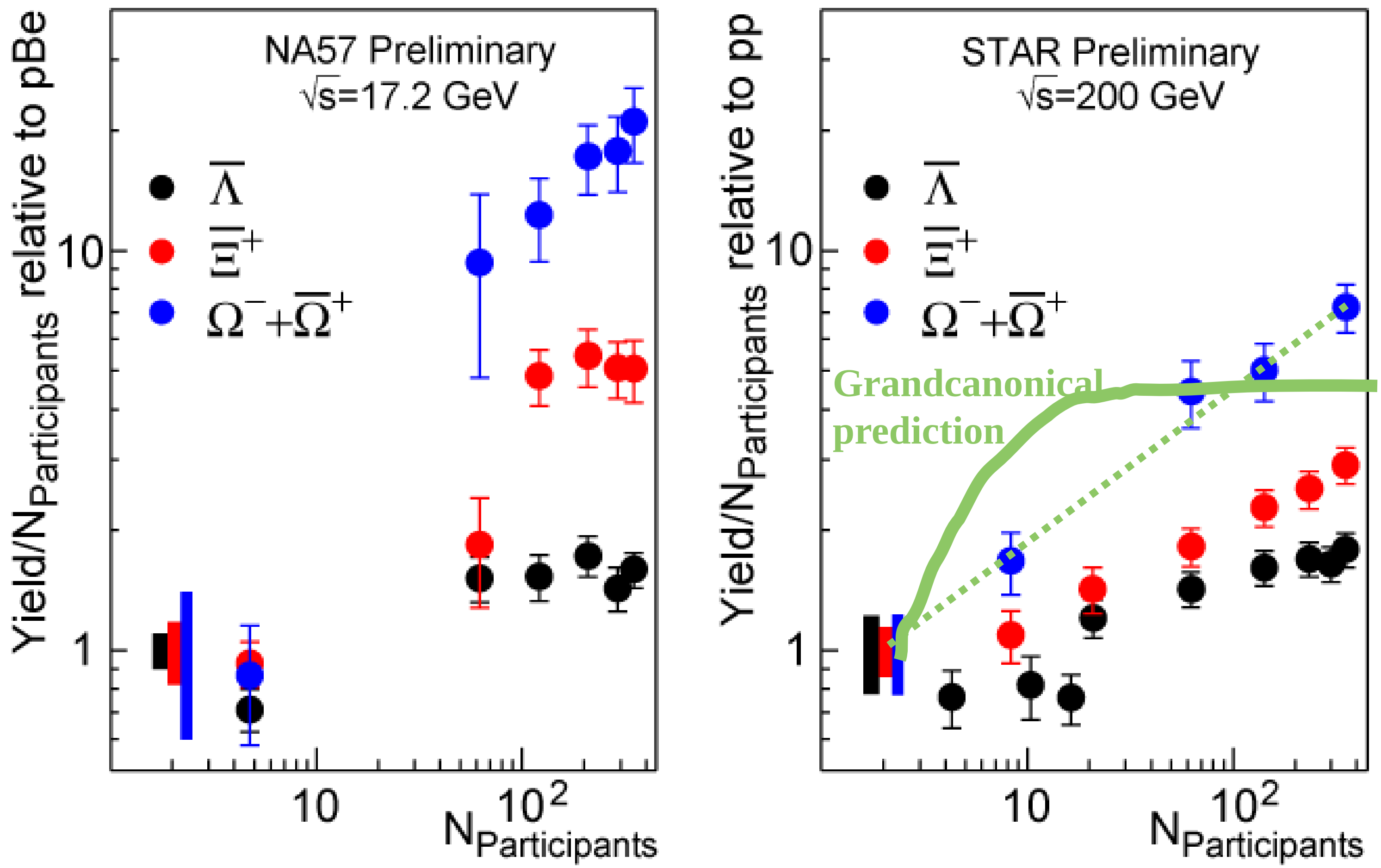
The switch from canonical to grand-canonical
(Tounsi,Redlich, hep-ph/0111159, hep-ph/0209284)

The strangeness enhancement factors at the SPS (WA97) can be explained not as an enhancement in AA but a suppression in pp.

The pp phase space for particle production is small. The volume is small and the volume term will dominate the ensemble (canonical (local)). The grand-canonical approach works for central AA collisions, but because the enhancements are quoted relative to pp they are due to a canonical suppression of strangeness in pp.

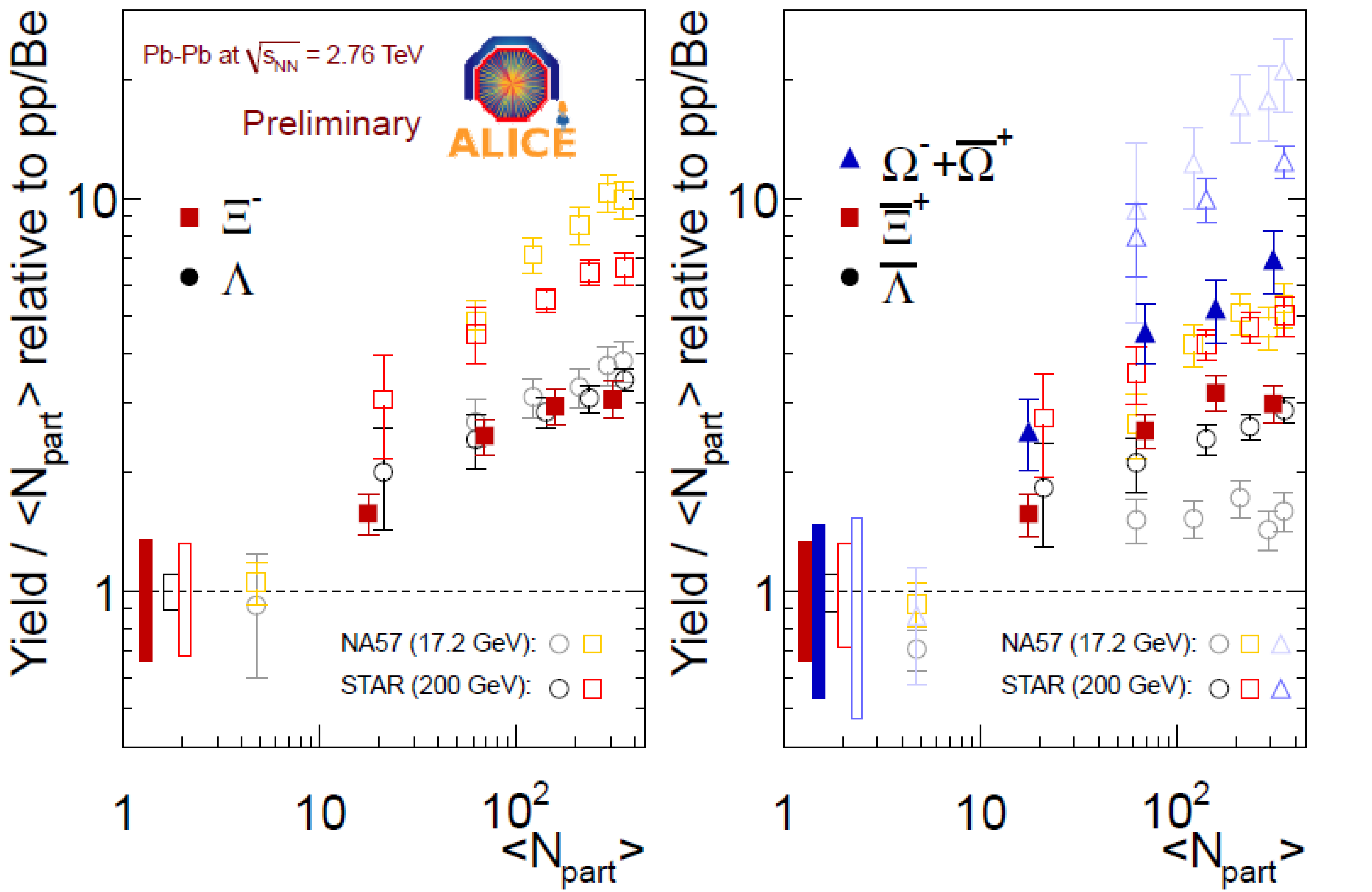


Does not really work at RHIC



Alternative: flavor hierarchy ?

…or the LHC



Alternative: flavor hierarchy ?

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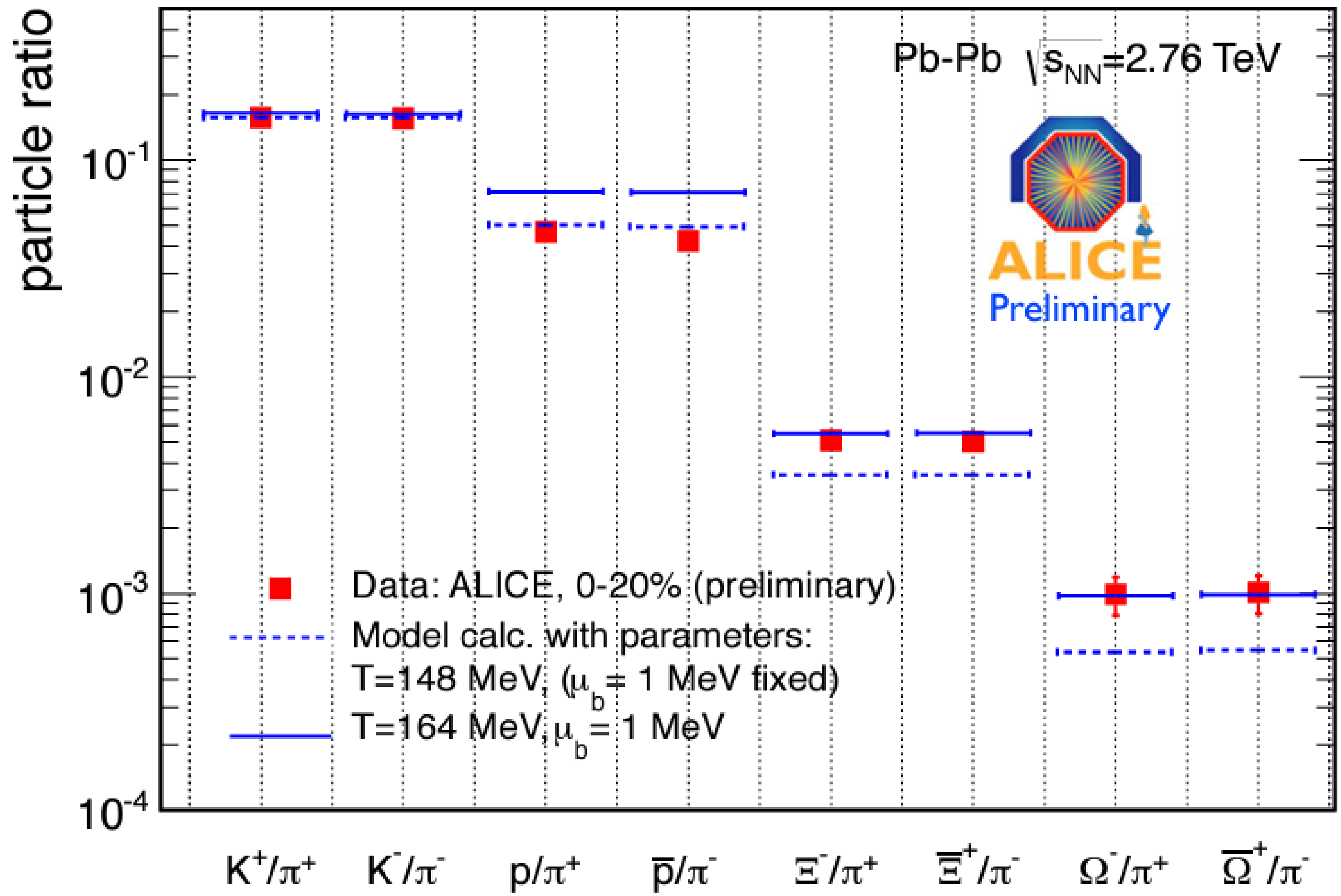
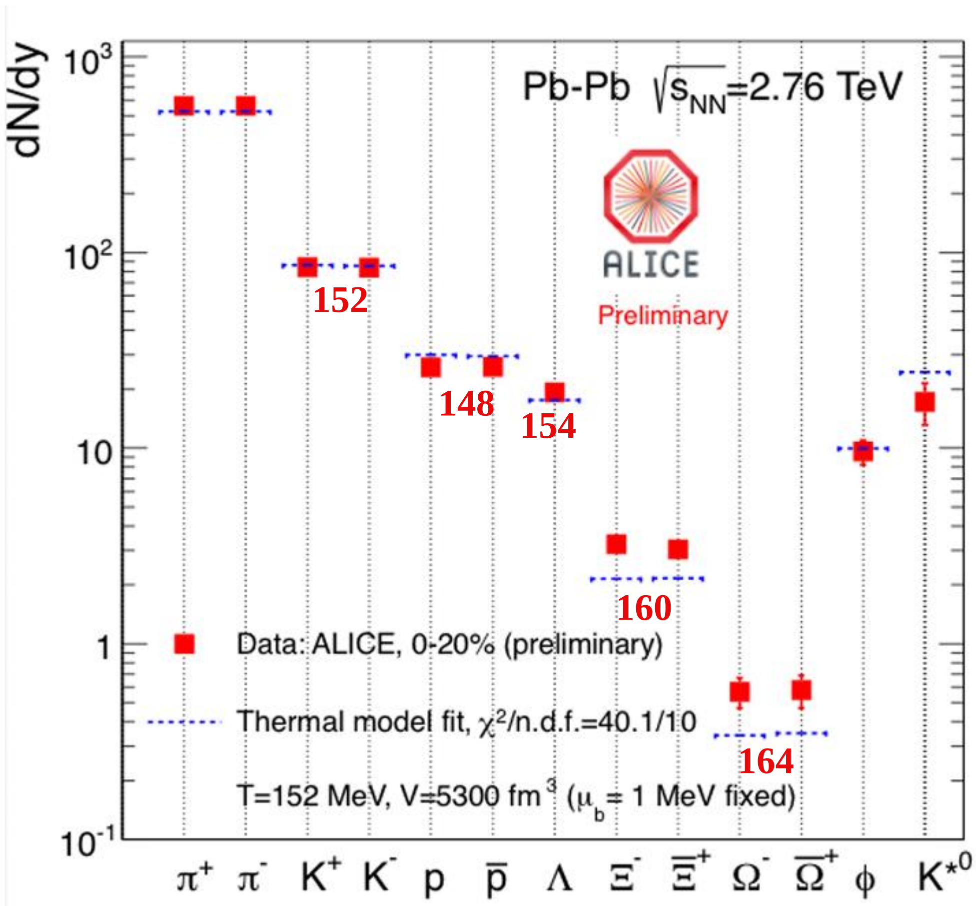
In addition: a peculiar flavor dependence



R. Preghenella for ALICE (SQM 2012)
arXiv:1111.7080, Acta Phys. Pol.

SHM model comparison based on yields including multi-strange baryons



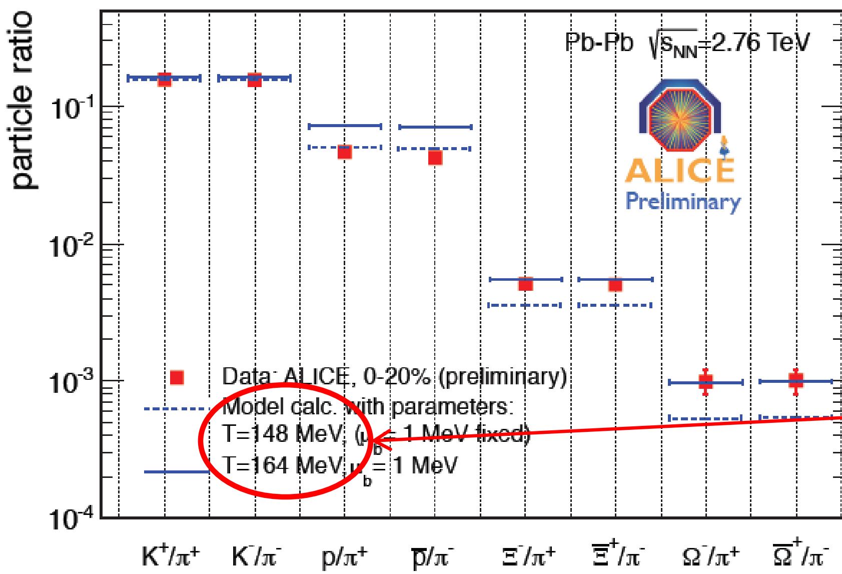
Either a bad fit with a common freeze-out…..

..or a good fit with a flavor specific freez-out

Data: L.Milano for ALICE (QM 2012)
Fit: R. Bellwied

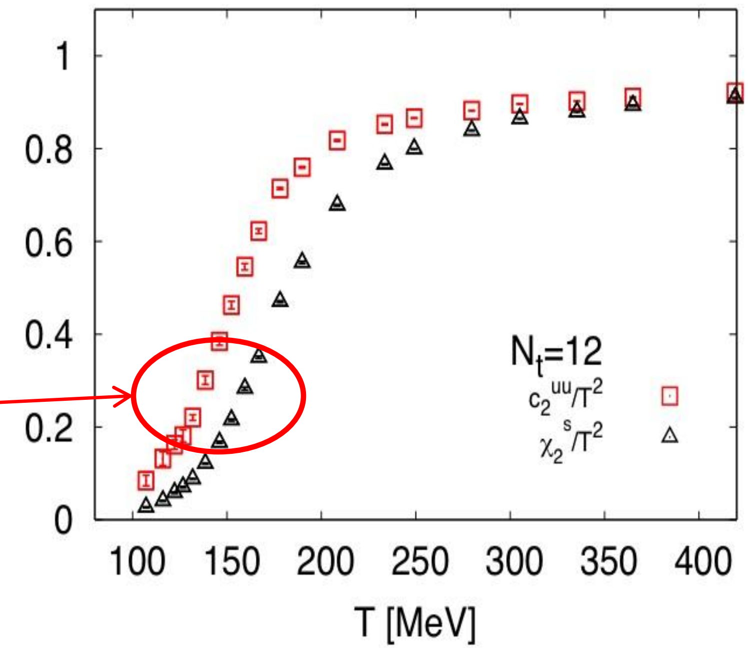
Potential evidence of flavor dependence in equilibrium freeze-out

Data: ALICE, SQM 2011



Model: A. Andronic et al., Phys. Lett. B 673:142-145,2009

Theory: Ratti et al., QM 2011

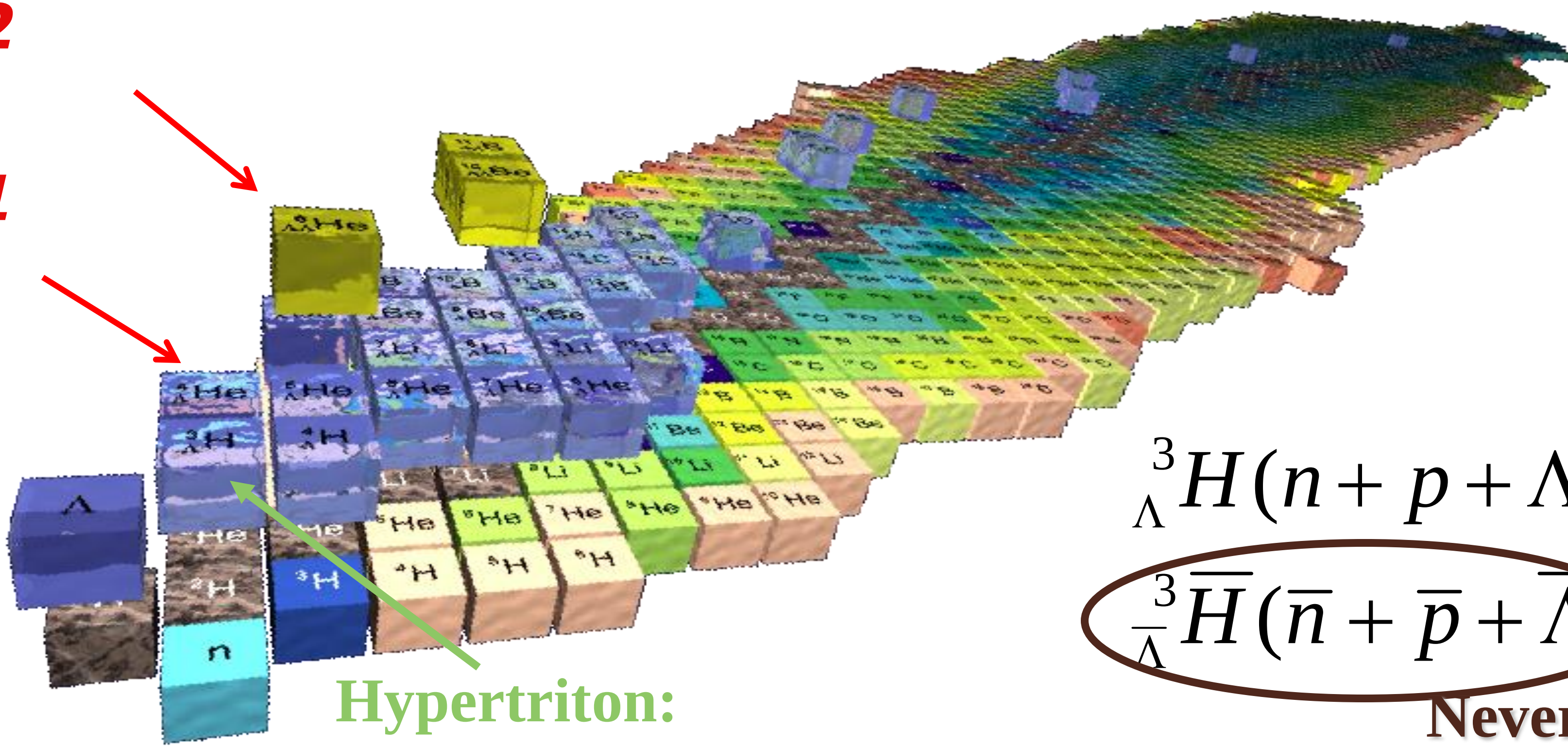


Observation of ${}^{3}_{\Lambda}\mathrm{H}$ and ${}^{3}_{\bar{\Lambda}}\overline{\mathrm{H}}$ @ RHIC

S=-2

S=-1

S=0



Hypertriton:

hypernucleus with lowest A

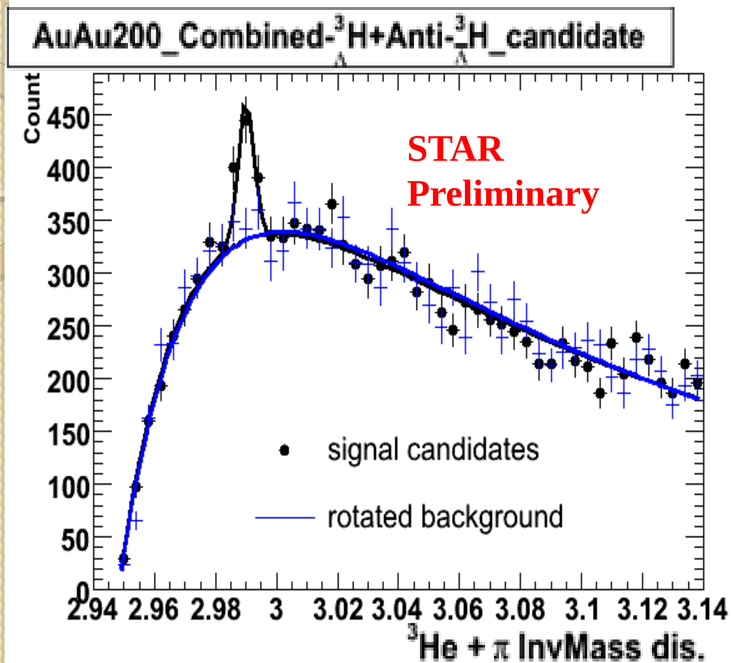
$${}^{3}_{\Lambda}H(n+p+\Lambda)$$

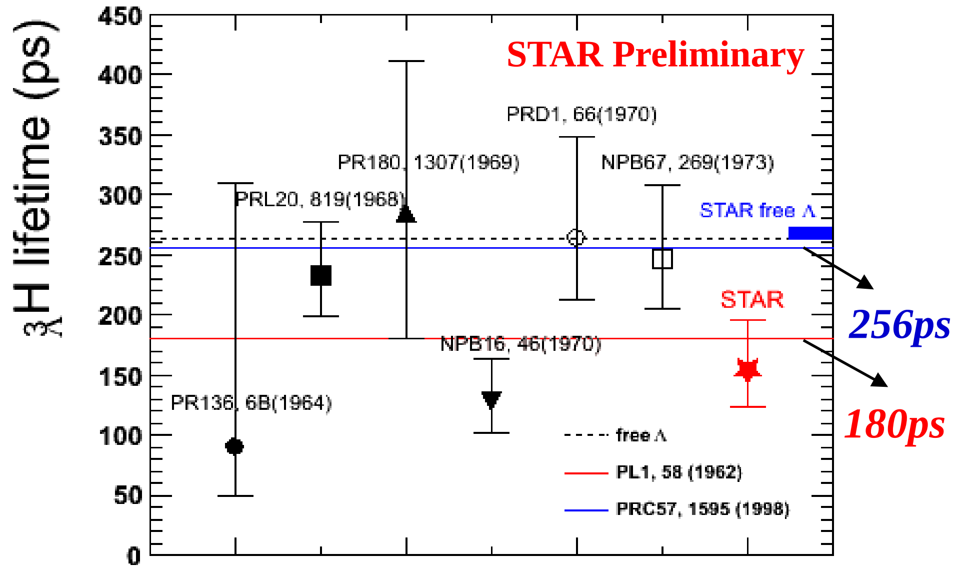
$${}^{3}_{\bar{\Lambda}}\overline{H}(\bar{n}+\bar{p}+\bar{\Lambda})$$

Never observed before

- strangeness production dominance generates exotic states
- produced through coalescence in the hadronic phase

Observation of ${}^{3}_{\Lambda}H$ and ${}^{3}_{\bar{\Lambda}}\overline{H}$ @ RHIC



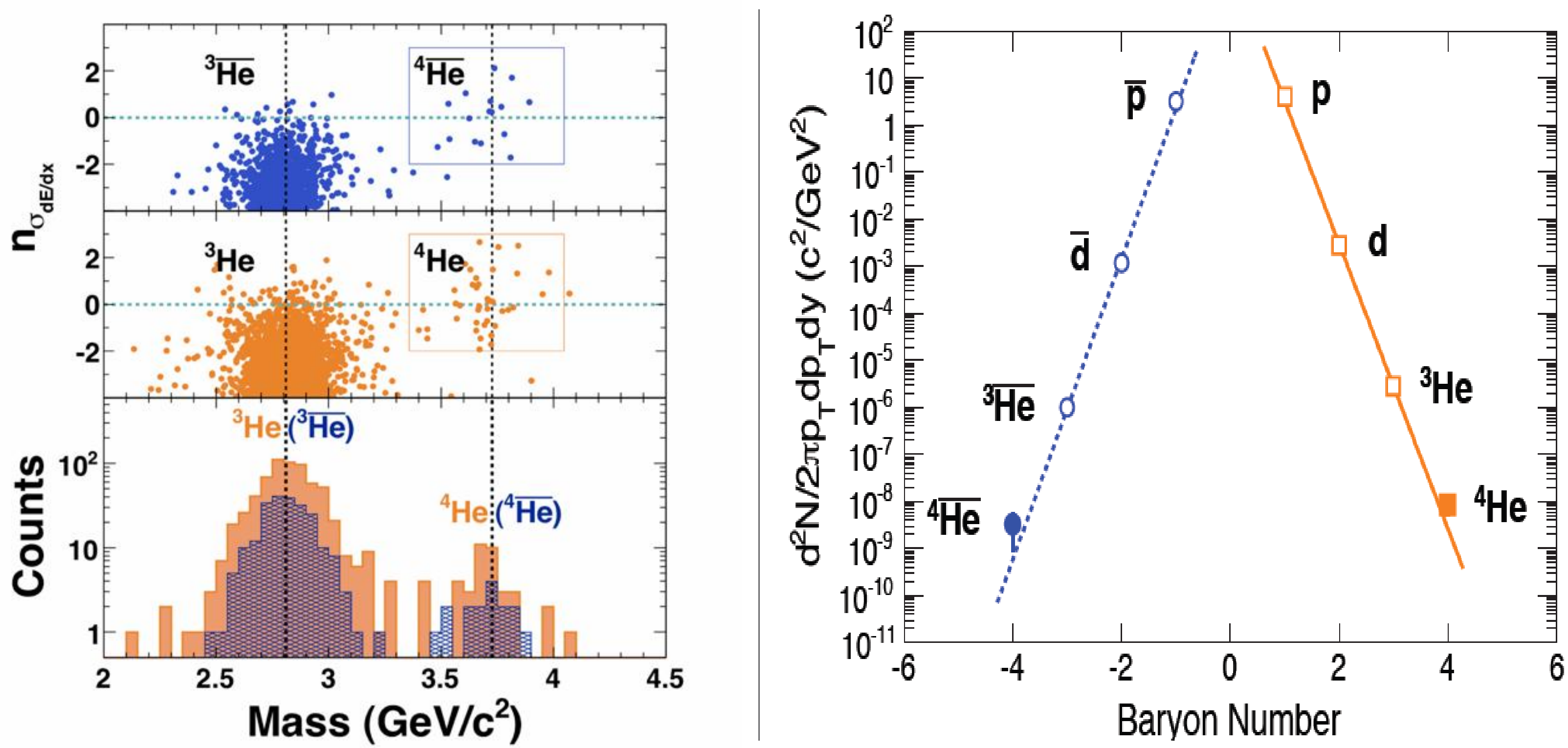


World data

- **First ever observation of an anti-hypernucleus (4σ signal of ${}^{3}_{\bar{\Lambda}}\overline{H}$)**
- **The hypertriton and anti-hypertriton signal : 244±35**
- **The hypertriton and anti-hypertriton lifetime:** $\tau = 153 \pm^{43}_{30}\ ps$

[1] R. H. Dalitz, *Nuclear Interactions of the Hyperons* (1965).
[2] R.H. Dalitz and G. Rajasekharan, Phys. Letts. 1, 58 (1962).
[3] H. Kamada, W. Glockle at al., Phys. Rev. C 57, 1595(1998).

A dense deconfined medium also produce exotic non-strange matter



Discovery of Anti-Helium-4 (Nature 473, 353 (2011))

1977: two distinctly different hadronization processes

Independent jet fragmentation: a[parton] ↣ h[hadron]

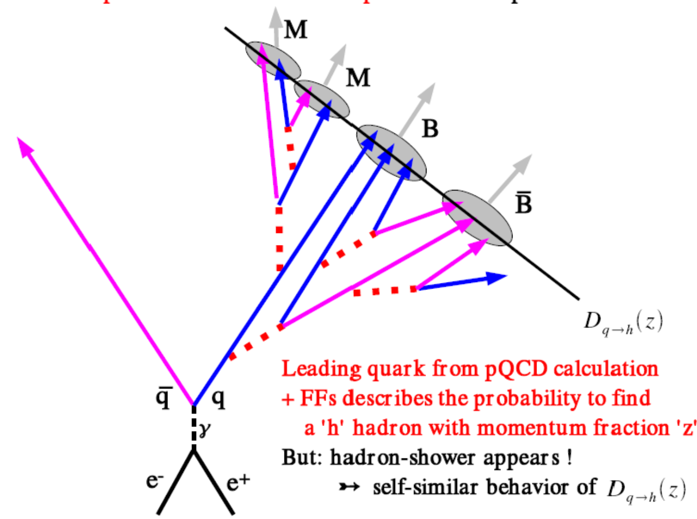
R.D. Field, R.P. Feynman, PRD15(1977)2590, ...

$$E\frac{d\sigma_h}{d^3p}=\sum_a\int\frac{dz}{z^2}D_{a\to h}(z)E\frac{d\sigma_a}{d^3p_a}$$

$D_{a\to h}(z)$: FFs are determined from e^+-e^- collisions

More likely in vacuum ?

Hadron production at the microscopical level: FF picture



Parton recombination/coalescence/clustering: a+b ↣ h

K.P. Das, R.C. Hwa, PLB68(1977)459, ...

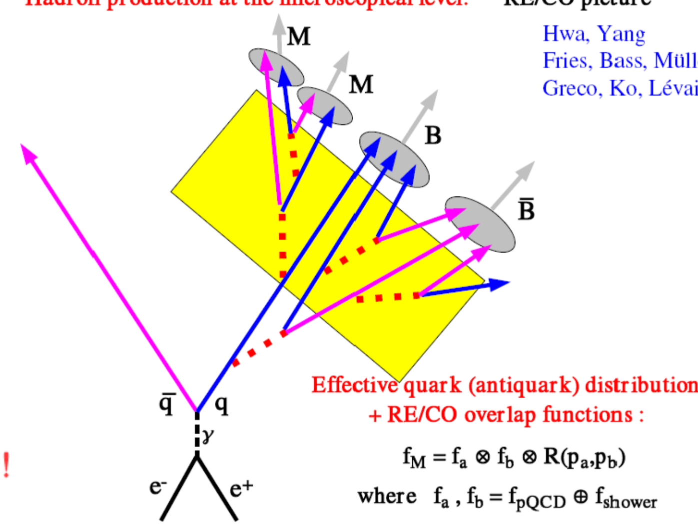
$$E\frac{d\sigma_h}{d^3p}=\sum_a\int d^3p_a d^3p_b E\frac{d\sigma_a}{d^3p_a}E\frac{d\sigma_b}{d^3p_b}R(\vec{p}_a,\vec{p}_b,\vec{p}_h)\delta^{(3)}(\vec{p}_a+\vec{p}_b-\vec{p}_h)$$

⟷ can be substituted by 'effective' FFs

Momentum distributions + momentum overlap functions. NO explicit interaction picture ?!

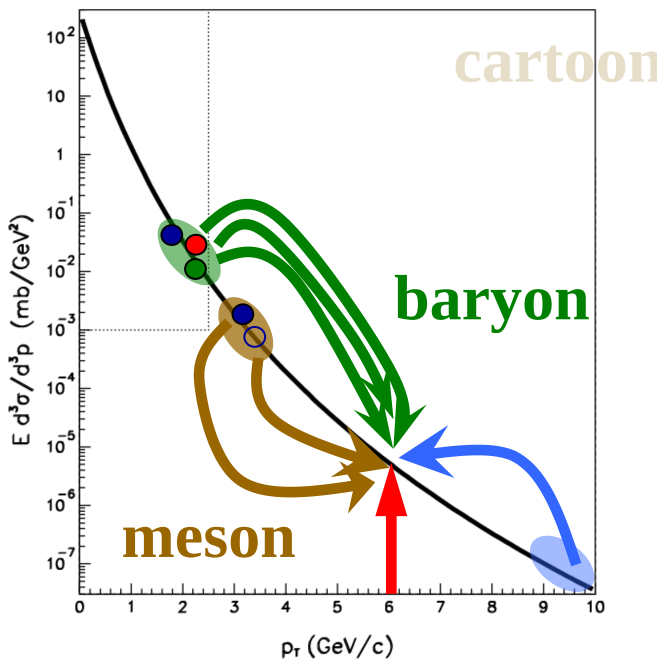
Hadron production at the microscopical level: RE/CO picture

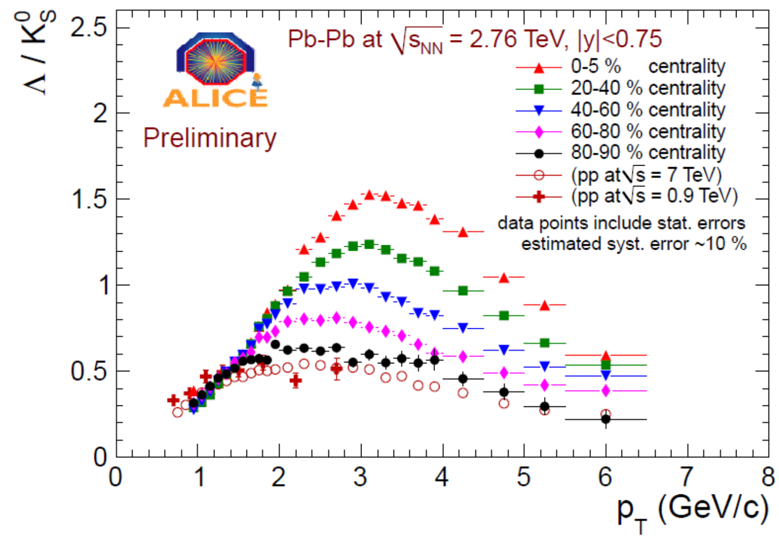
Hwa, Yang
Fries, Bass, Müller
Greco, Ko, Lévai



More likely in medium ?

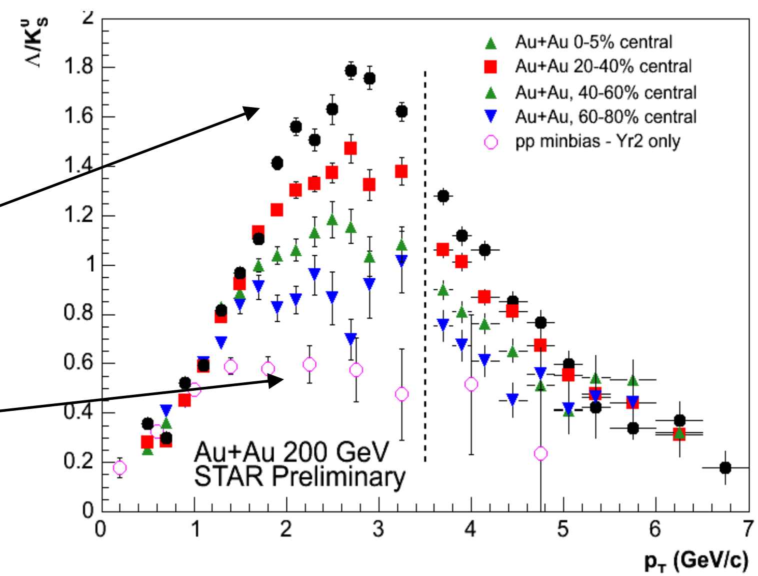
Evidence at RHIC & LHC: confounding result (more baryons than mesons at particular momentum) can be attributed to recombination





Recombination in medium

Fragmentation in vacuum



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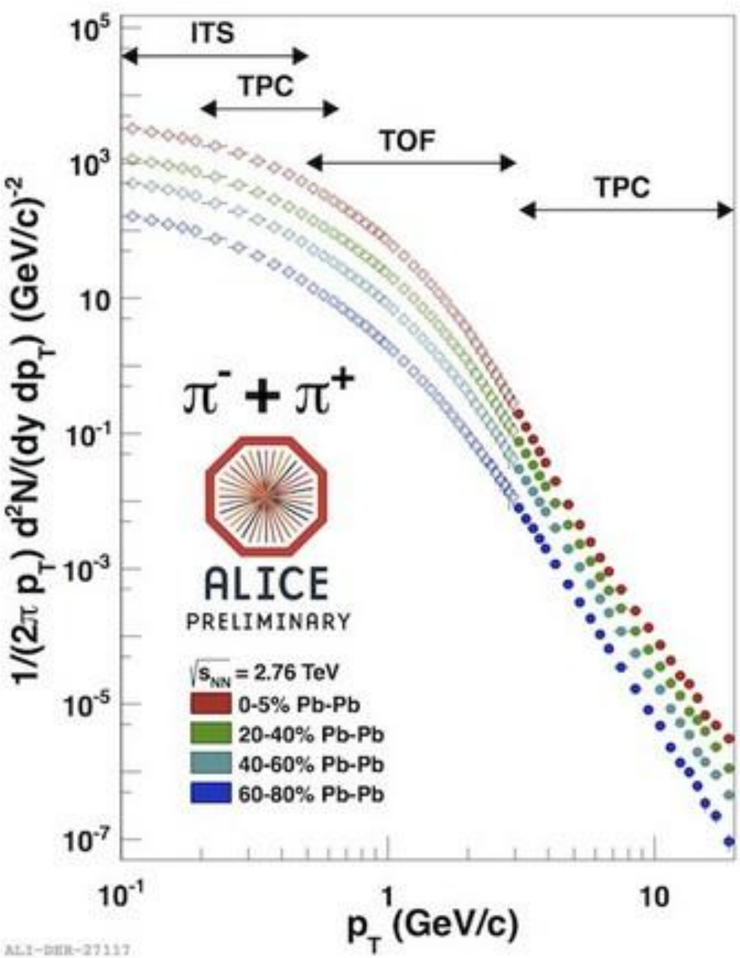
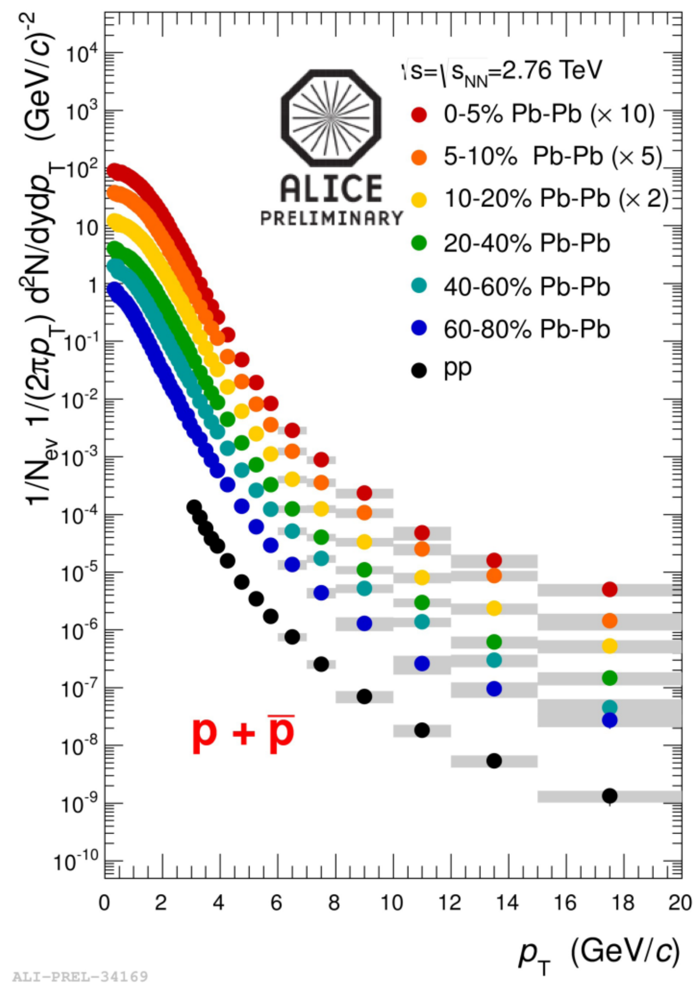
Lecture 2, Part 2: Spectra

Identified particle spectra : p, $\bar{p}$, $K^{-,+}$, $\pi^{-,+}$, K^0_s , Λ. Ξ, Ω, ϕ, K*, ρ, etc.





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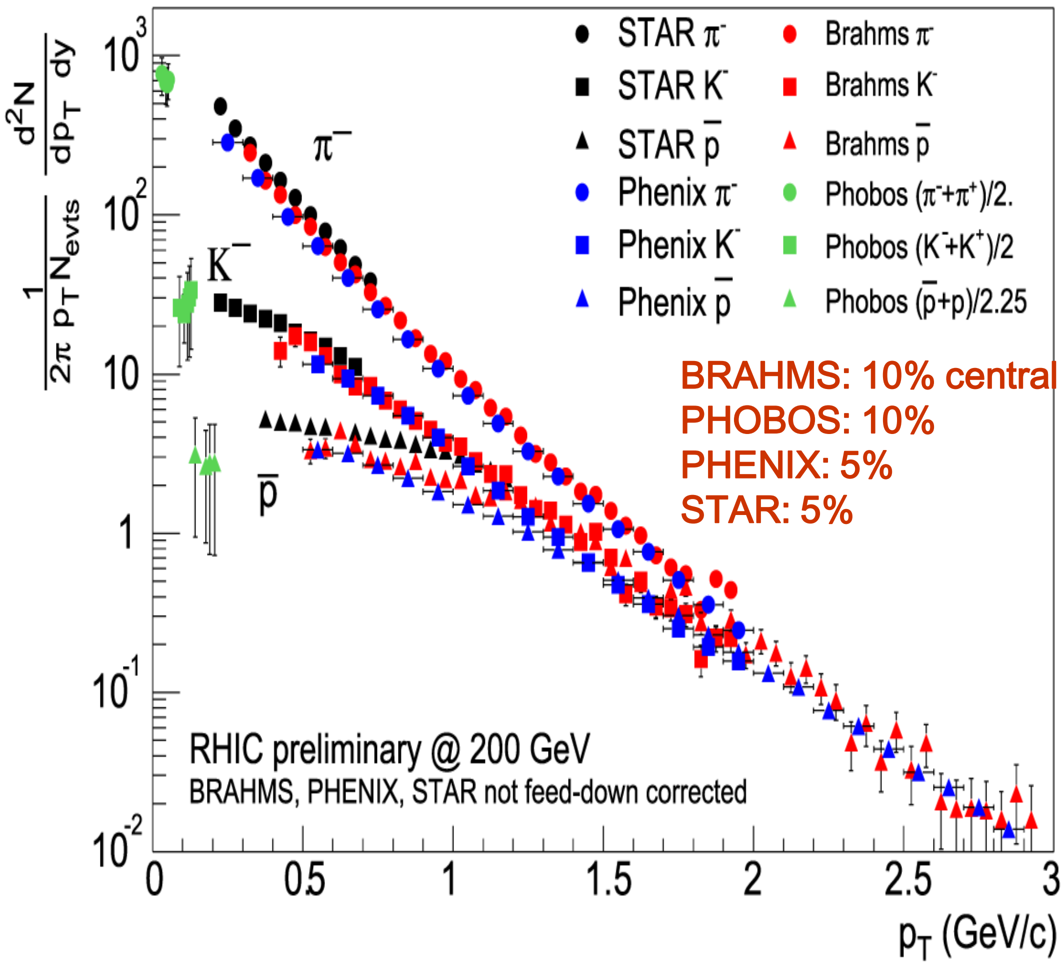
Identified Particle Spectra for Au-Au @ 200 GeV



- The spectral shape gives us:
  - Kinetic freeze-out temperatures
  - Transverse flow
- The stronger the flow the less appropriate are simple exponential fits:
  - Hydrodynamic models (e.g. Heinz et al., Shuryak et al.)
  - Hydro-like parameters (Blastwave)
- Blastwave parameterization e.g.:
  - E.Schnedermann et al, PRC48 (1993) 2462

Explains: spectra, flow & HBT

"Thermal" Spectra

Invariant spectrum of particles radiated by a thermal source:

$$E\frac{d^3N}{dp^3} = \frac{dN}{dy\, m_T\, dm_T\, d\phi} \propto E e^{-(E-\mu)/T}$$

where: $m_T = (m^2+p_T^2)^{½}$ **transverse mass (requires knowledge of mass)**
$\mu = b\,\mu_b + s\,\mu_s$ **grand canonical chem. potential**
T **temperature of source**

Neglect quantum statistics (small effect) and integrating over rapidity gives:

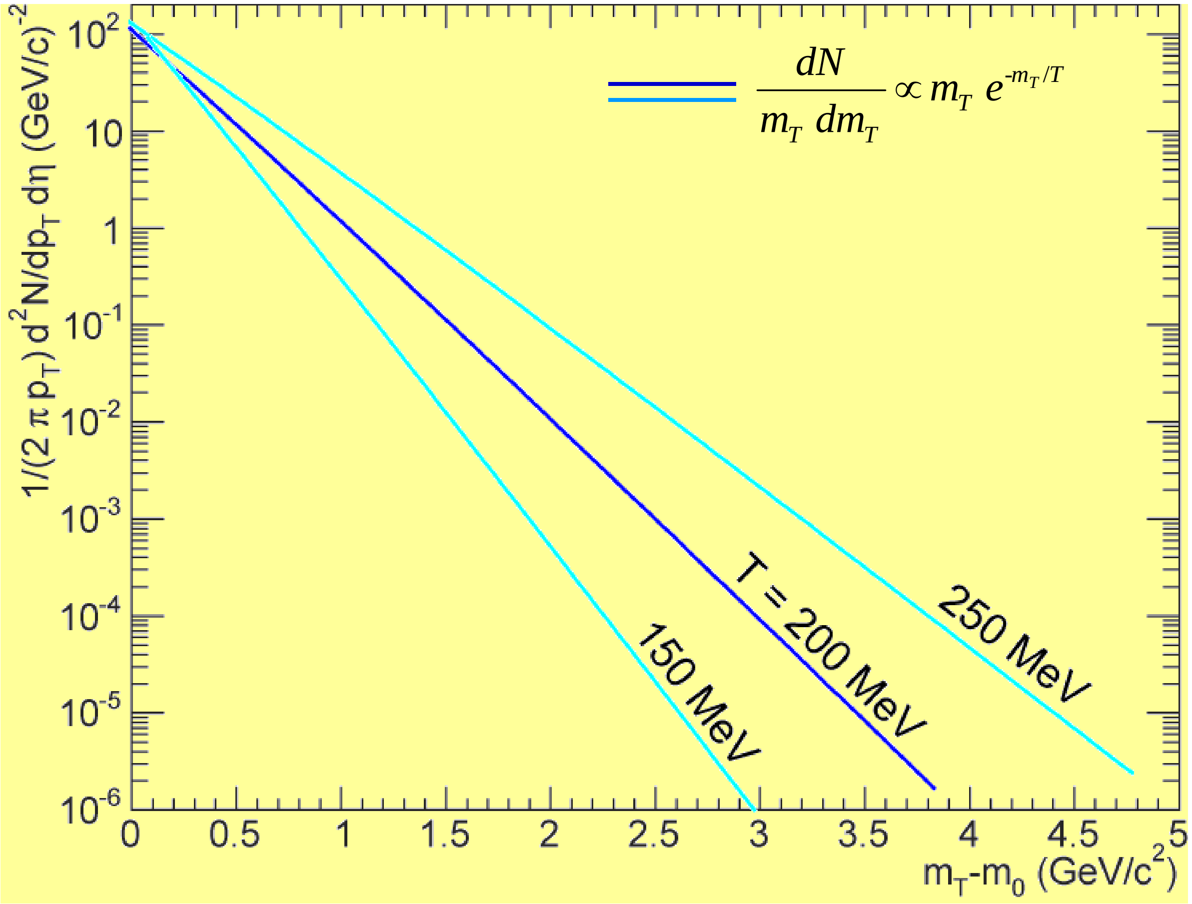
$$\frac{dN}{m_T\, dm_T} \propto m_T\, K_1(m_T/T) \xrightarrow{m_T >> T} \sqrt{m_T}\, e^{-m_T/T}$$

R. Hagedorn, Supplemento al Nuovo Cimento Vol. III, No.2 (1965)

At mid-rapidity $E = m_T \cosh y = m_T$ **and hence:** $$\frac{dN}{m_T\, dm_T} \propto m_T\, e^{-m_T/T}$$

"Boltzmann"

“Thermal” Spectra (radial flow aside)



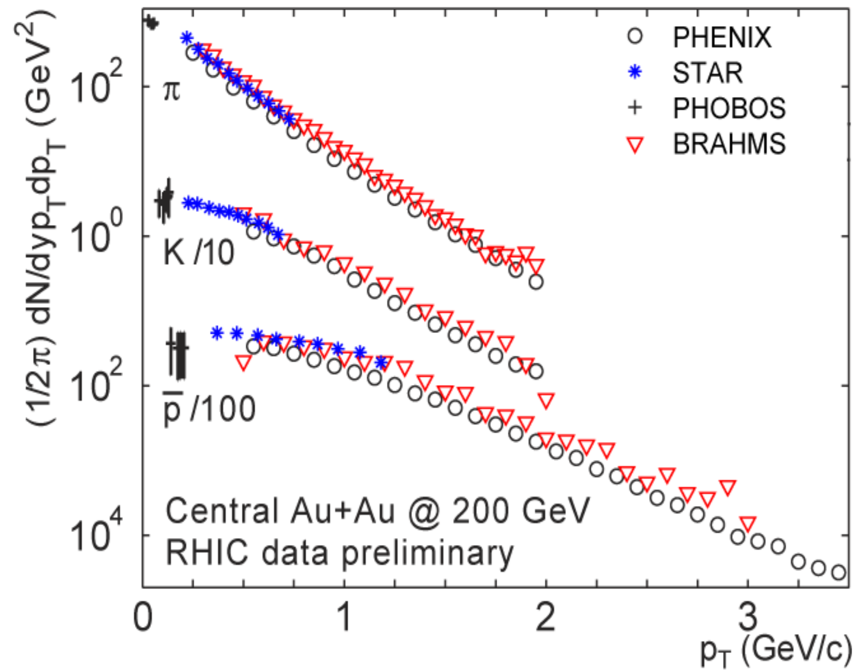
Describes many spectra well over several orders of magnitude with almost uniform slope 1/T

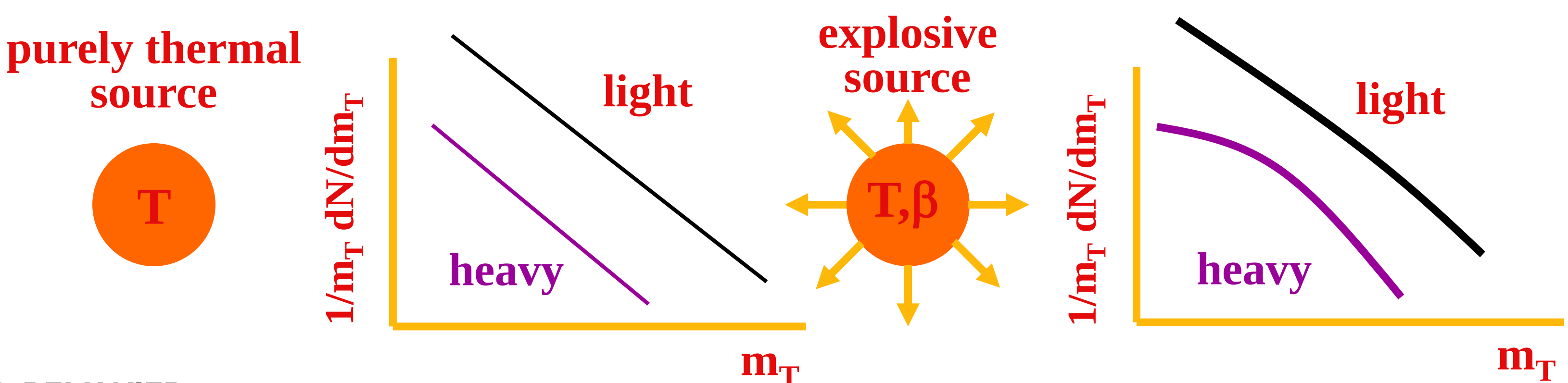
- **usually fails at low-p_T (⇒ flow)**
- **most certainly will fail at high-p_T (⇒ power-law)**

N.B. Constituent quark and parton recombination models yield exponential spectra with partons following a pQCD power-law distribution. (Biro, Müller, hep-ph/0309052) ⇒ in this case T is not related to actual “temperature” but reflects pQCD parameter p_0 and n (similar to Tsallis function).

“Thermal” spectra and radial expansion (flow)

- Different spectral shapes for particles of differing mass
  → strong collective radial flow
- Spectral shape is determined by more than a simple T
  → at a minimum T, β_T



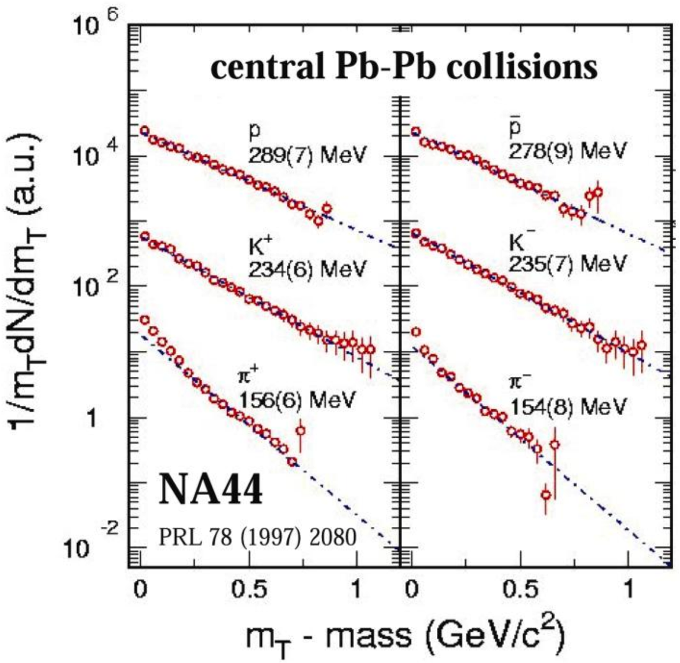


Thermal + Flow: "Traditional" Approach

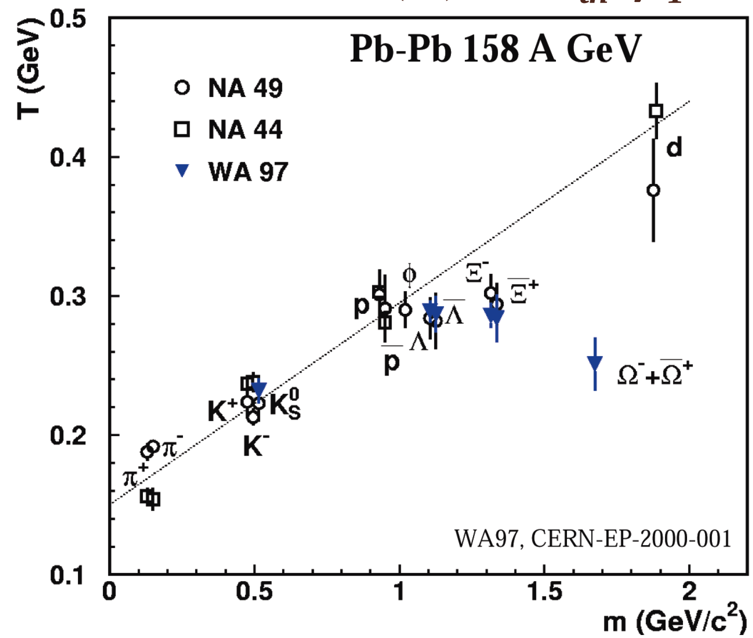
$$T_{measured} = \begin{cases} T_{th} + m\langle\beta_T\rangle^2 & \text{for} \quad p_T \le m \\ T_{th}\sqrt{\dfrac{1+\langle\beta_T\rangle}{1-\langle\beta_T\rangle}} & \text{for} \quad p_T >> m \quad \text{(blue shift)} \end{cases}$$

Assume *common* flow pattern and *common* temperature T_{th}

1. Fit Data ⇒ *T*



2. Plot *T(m)* ⇒ T_{th}, β_T



β is the transverse expansion velocity. 2nd term = KE term (½ m β²) → common T_{th}, β.

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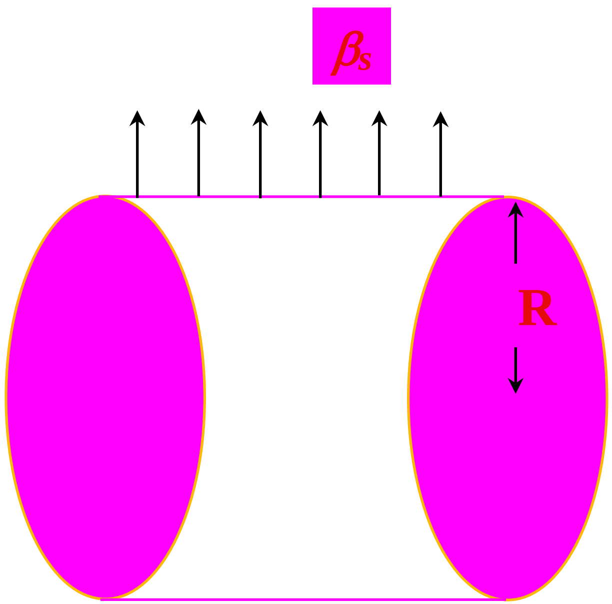
Blastwave: a hydro inspired description of spectra

Spectrum of longitudinal and transverse boosted thermal source:



$$\frac{dN}{m_T dm_T} \propto \int_0^R r\, dr\, m_T\, I_0\left(\frac{p_T \sinh\rho}{T}\right) K_1\left(\frac{m_T \cosh\rho}{T}\right)$$

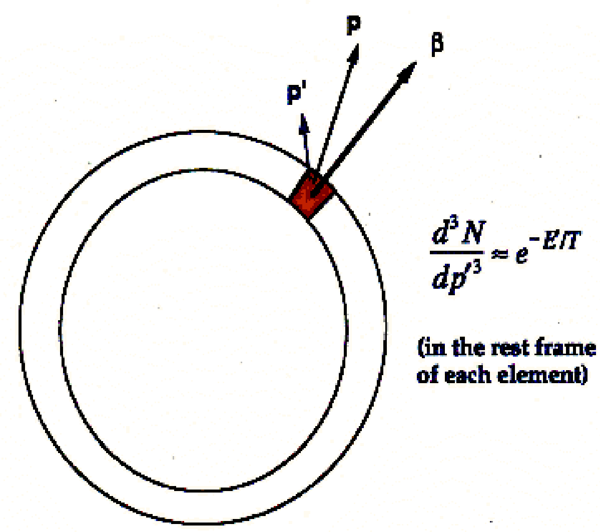
with

transverse velocity distribution $\beta_r(r) = \beta_s\left(\frac{r}{R}\right)^n$

and boost angle (boost rapidity) $\rho = \tanh^{-1} \beta_r$

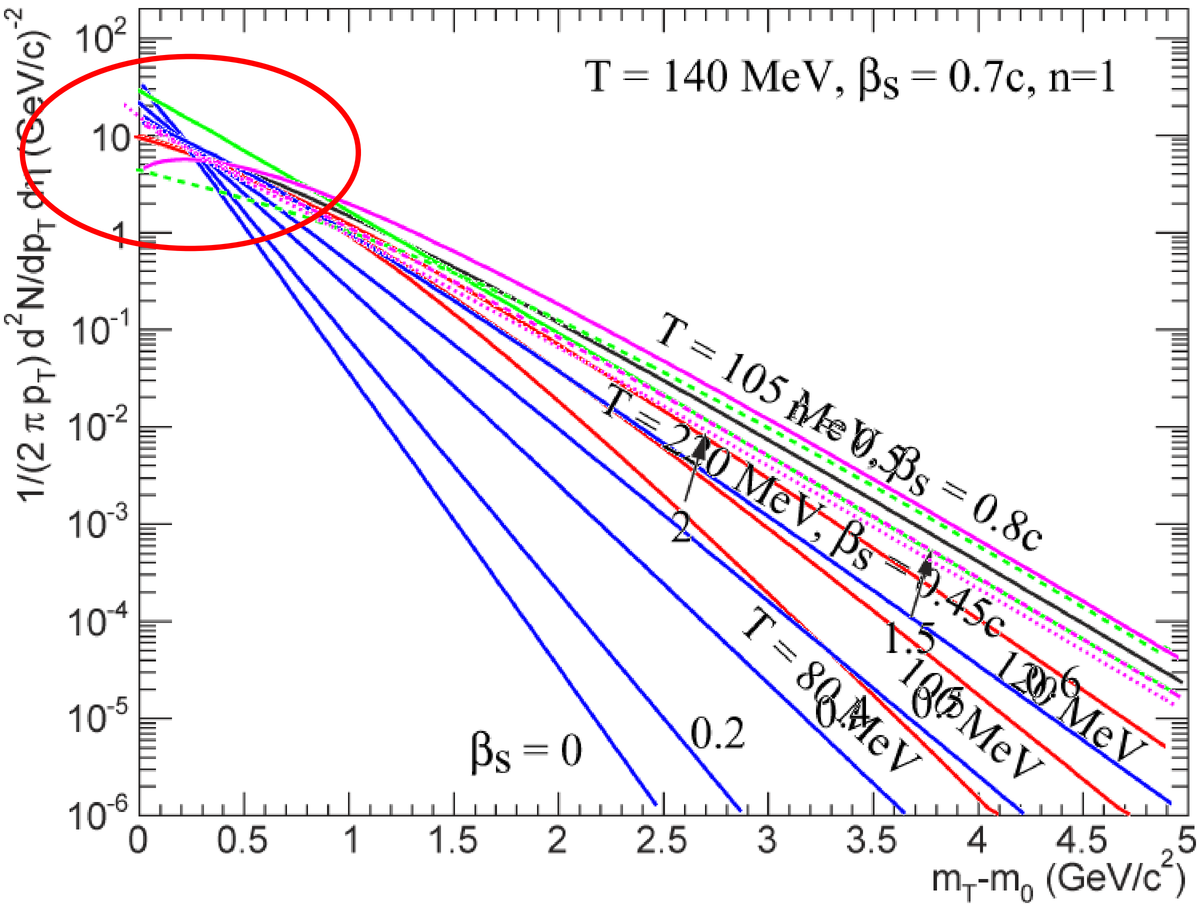
Ref. : Schnedermann, Sollfrank & Heinz, PRC48 (1993) 2462

Emission from a Thermal Expanding Source



Static Freeze-out picture,
No dynamical evolution to freeze-out

The Blastwave Function



• Increasing T has similar effect on a spectrum as increasing β_s

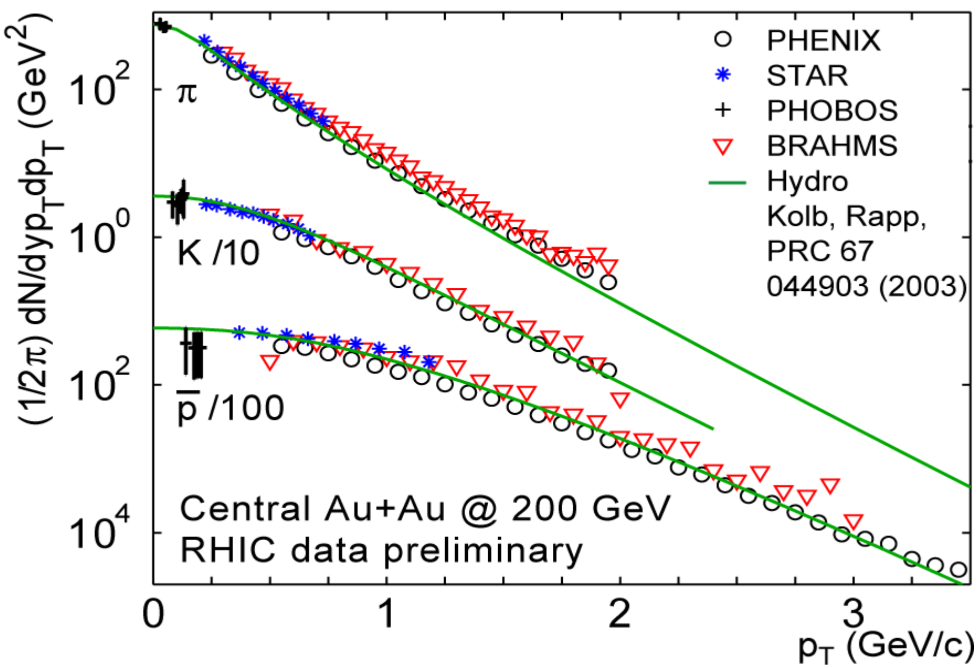
• Flow profile (n) matters at lower m_T!

• Need high quality data down to low-m_T

Hydrodynamics in High-Density Scenarios

- Assumes local thermal equilibrium (zero mean-free-path limit) and solves equations of motion (energy momentum tensor) for fluid elements (not particles)
- Equations given by continuity, conservation laws, and lattice QCD Equation of State (EOS)

Kolb, Sollfrank & Heinz, hep-ph/0006129

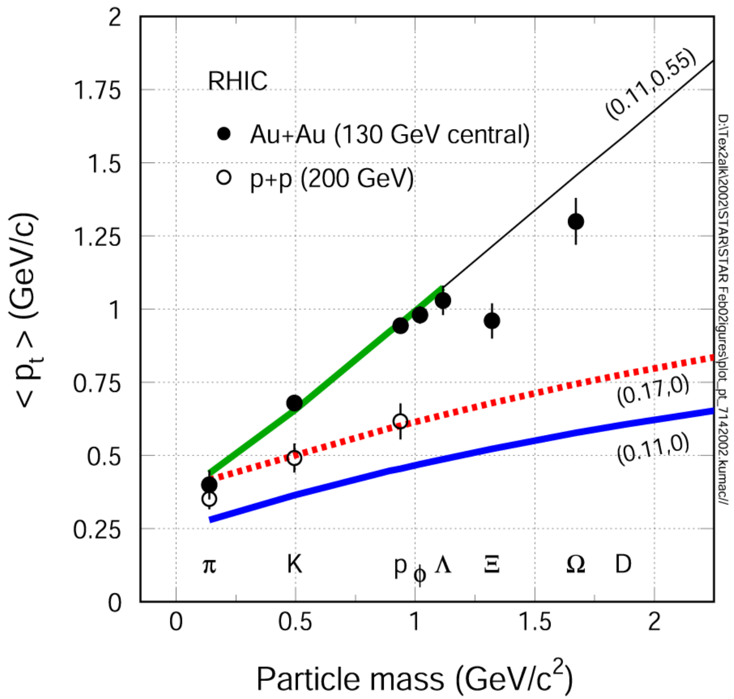


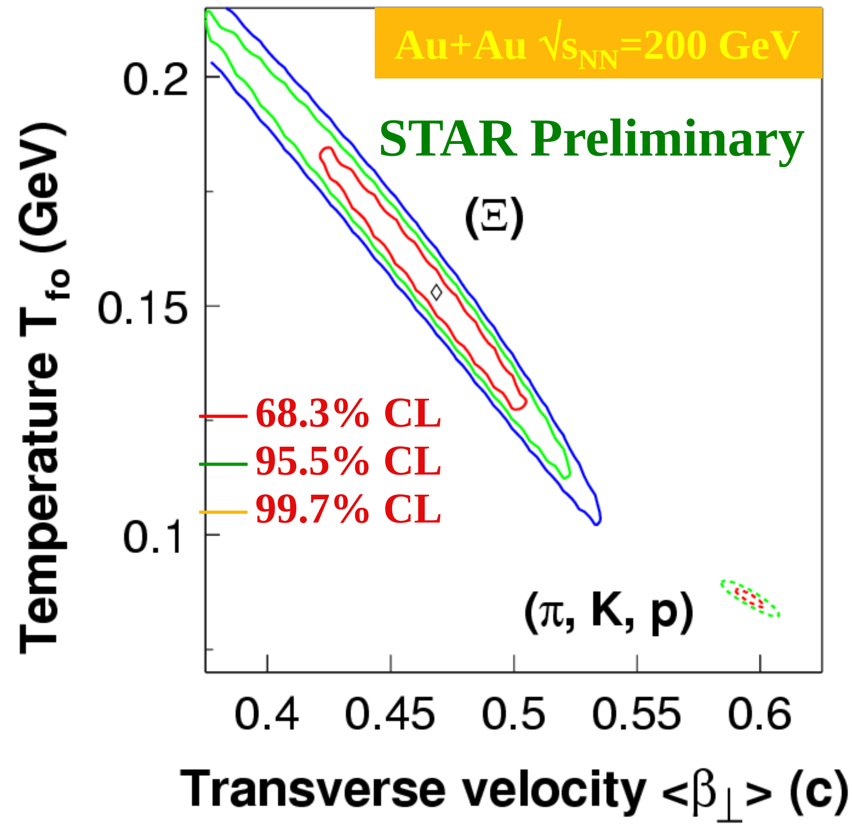
Does well with spectra

T_{th} ~ 100 MeV,

$\langle \beta_T \rangle$ ~ 0.55 c

Strange baryons show deviations in basic thermal parametrization





•π, K, p: Common thermal freeze-out at T~90 MeV and $<\beta_\perp>$~0.60 c

• Ξ, Ω: different thermal freeze-out at higher T and lower β

→ less hadronic re-interaction earlier freeze-out (close to T_{ch})

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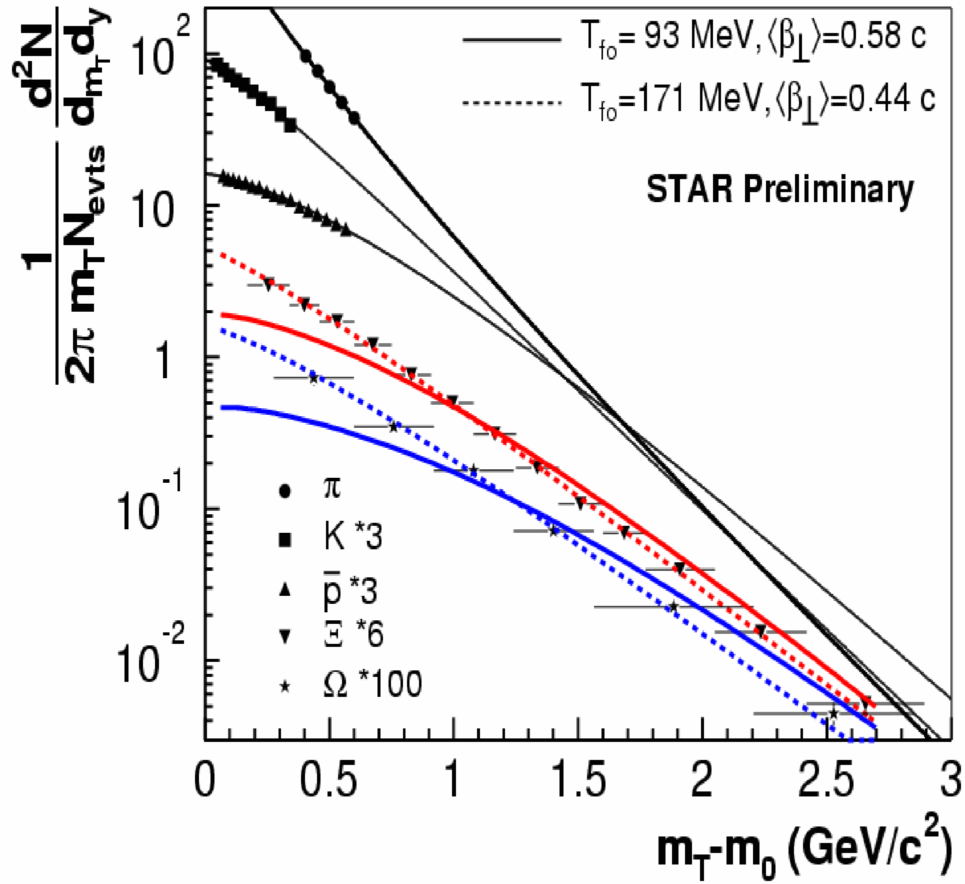
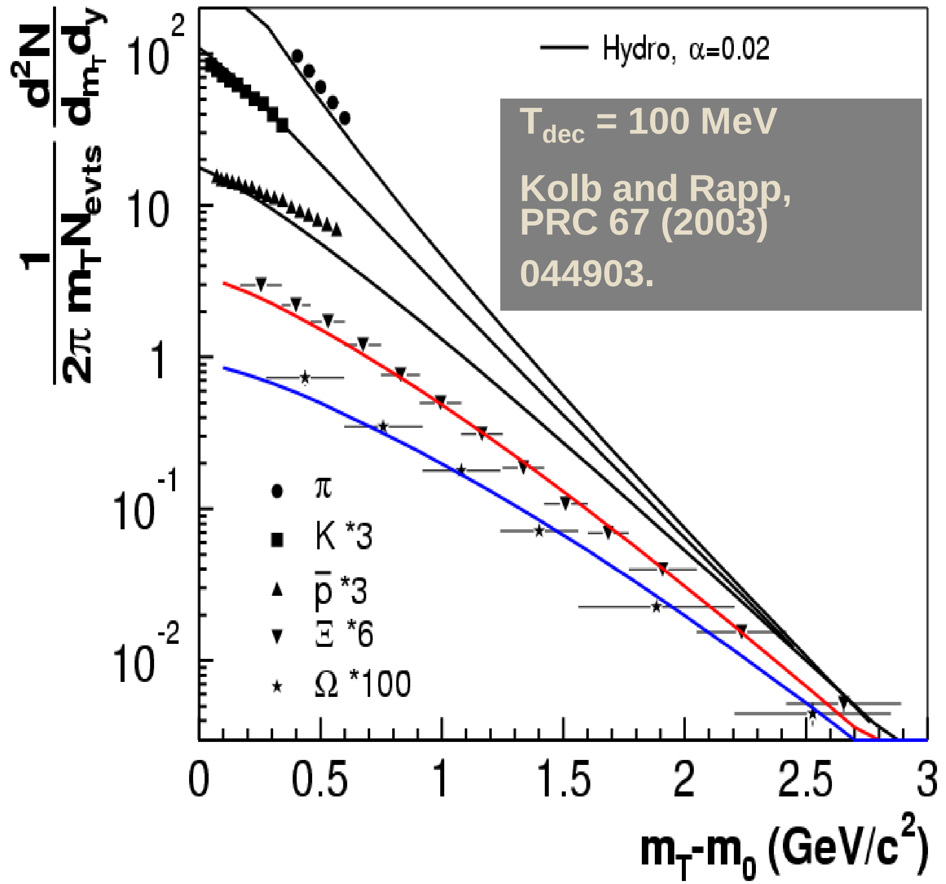
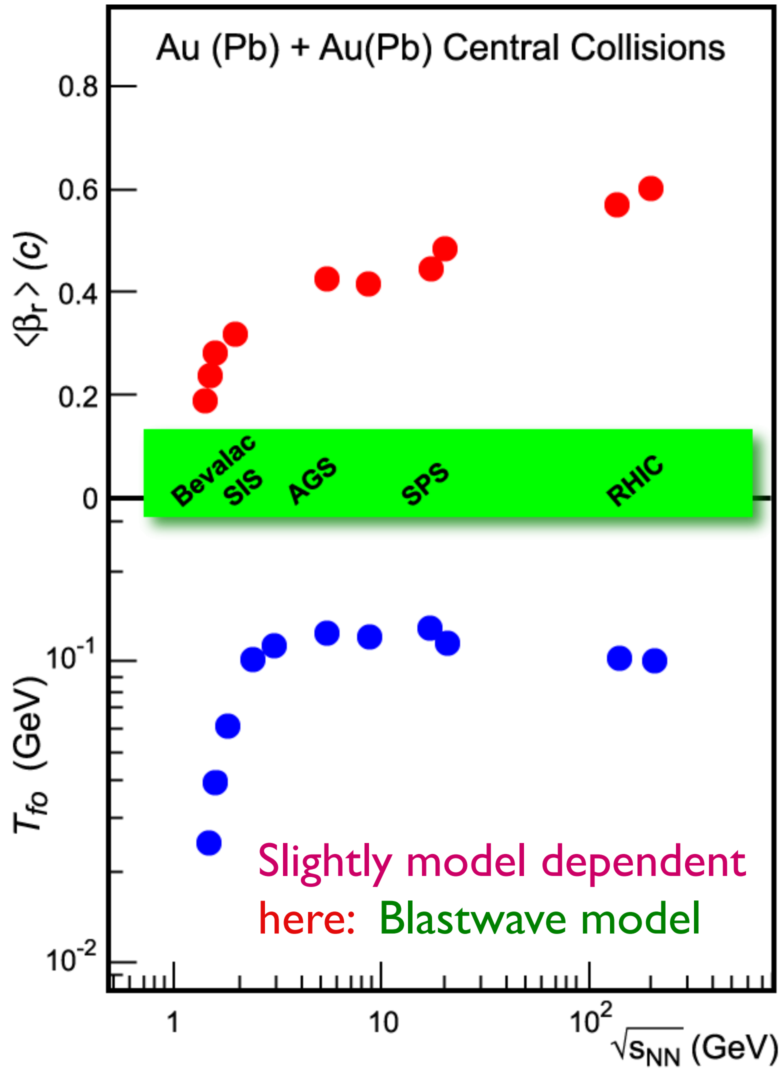
Blastwave vs. Hydrodynamics





Mike Lisa (QM04): Use it don’t abuse it ! Only use a static parametrization when the dynamic model doesn’t work !!

Collective Radial Expansion

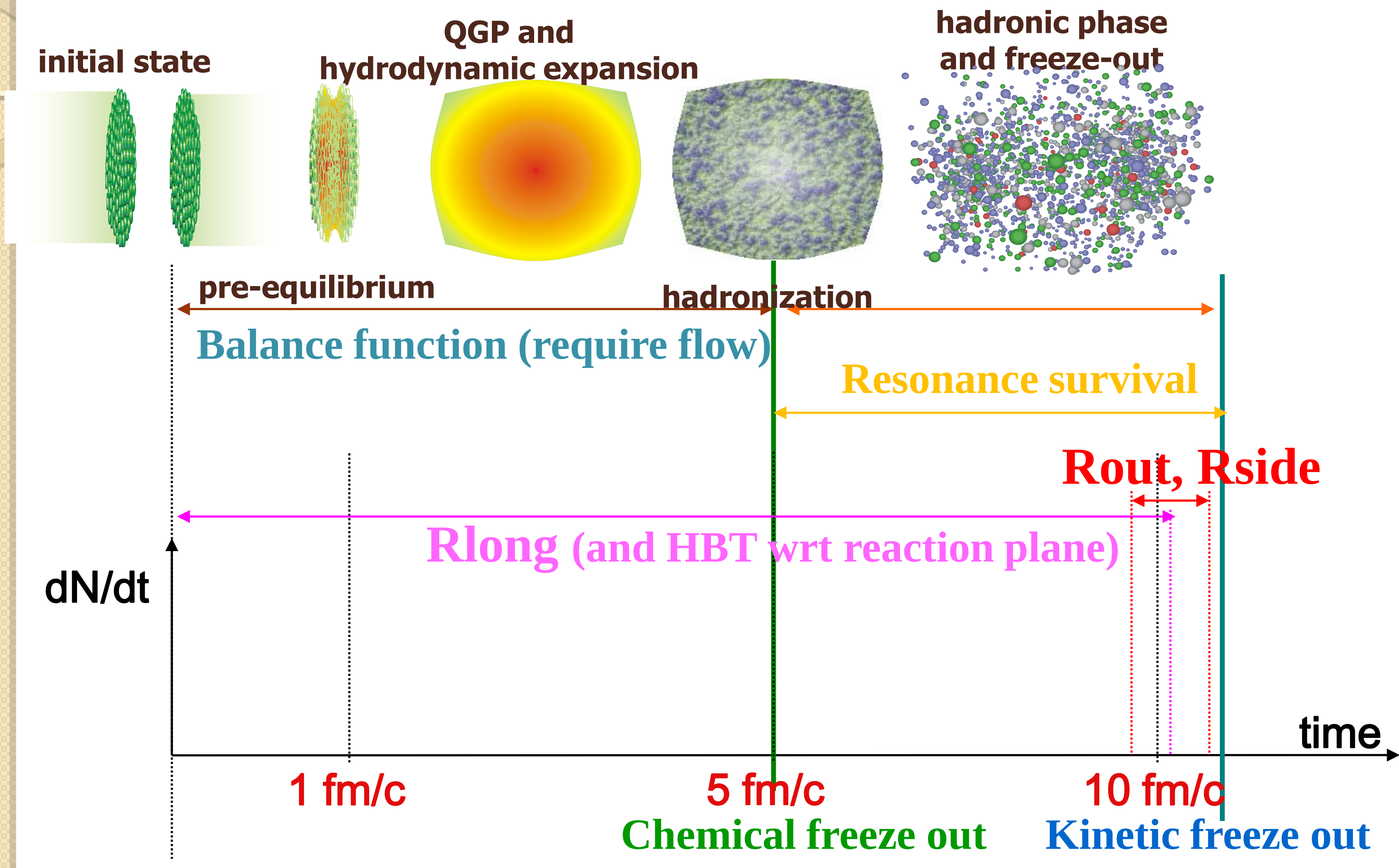


From fits to π, K, p spectra:

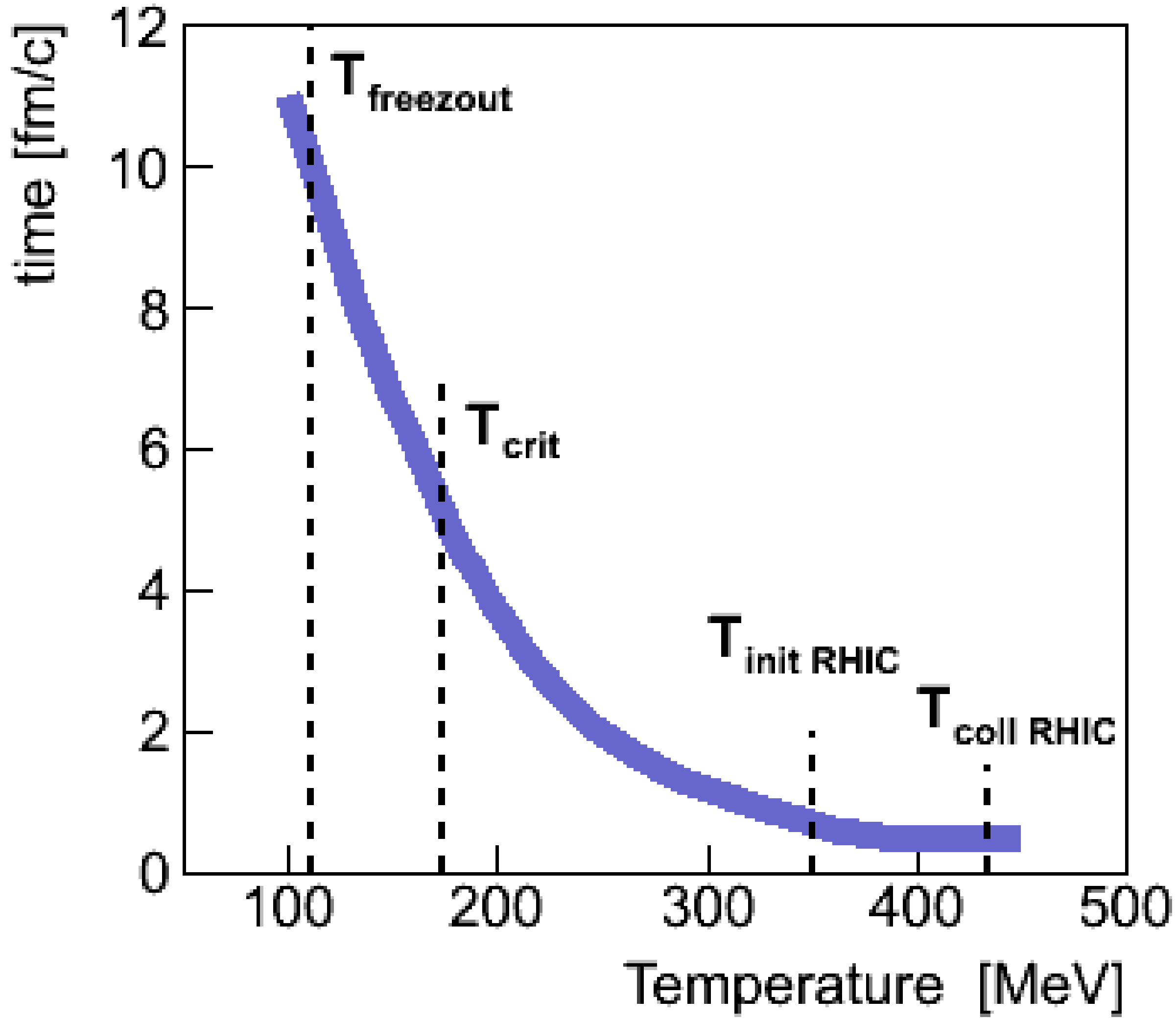
- $<\beta_r>$
  - increases continuously
- T_{th}
  - saturates around AGS energy

Strong collective radial expansion at RHIC
⇒ high pressure
⇒ high rescattering rate
⇒ Thermalization *likely*

Summary of basic PID observables



Time vs. Temperature



Evolution of a RHIC heavy ion collision

(as a function of temperature and time)

Model: lQCD | SHM | Blastwave

Effect: hadronization | chemical f.o. | kinetic f.o.

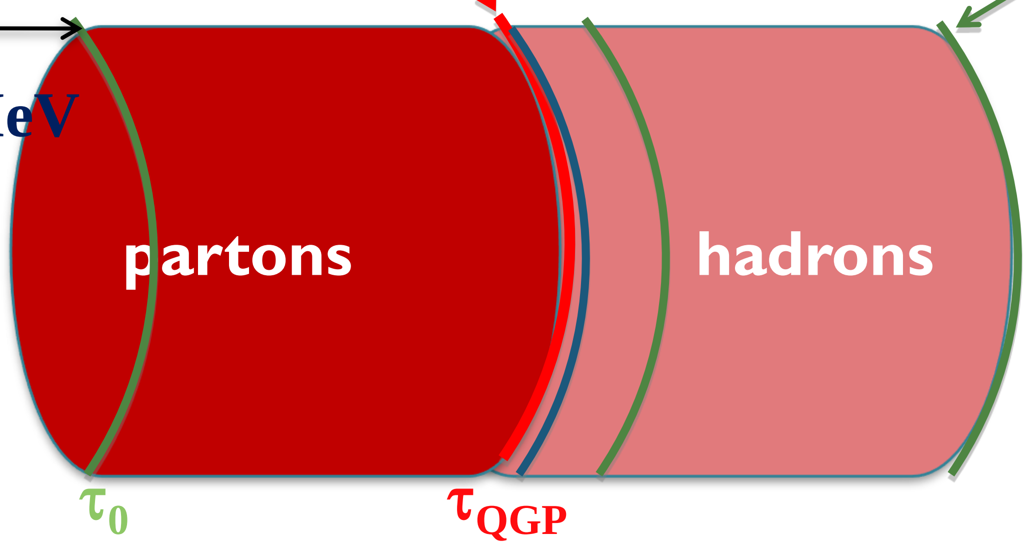
Freeze-out surface: T_{crit} | T_{ch} | $T_{kin}(\Xi,\Omega)$ | $T_{kin}(\pi,k,p,\Lambda)$

Temperature (MeV): 160 | 160 | 150 | 80

Expansion velocity (c): $\beta=0.45$ | $\beta=0.6$

T_{init}
370 MeV

Hydro
condition ?



Exp.: time: ~5 fm/c ~5 fm/c

(STAR, PRL 97:132301,(2006))

References:
Lattice QCD:
arXiv:1005.3508
arXiv:1107.5027

Statistical
Hadronization:
hep-ph/0511094
nucl-th/0511071

Blastwave:
nucl-ex/0307024
arXiv:0808.2041

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