

Alternating Current and RL Circuits

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(Dated: January 13, 2012)

When inductors are placed in series with a resistor and a voltage source, the potential difference and current in the circuit becomes time dependent. In this lab you will explore time varying voltages and RL circuits using an oscilloscope.

Preparatory Questions

1. Draw a diagram that illustrates how to use a multimeter to measure the voltage across a resistor in a circuit.
2. Draw a diagram that illustrates how to use a multimeter to measure the current in a circuit.
3. Calculate ϵ_{rms} for a square wave that varies from $-\epsilon_m$ to $+\epsilon_m$ with a period T.

1. INTRODUCTION

The object of this experiment is to become familiar with alternating voltages and currents, the oscilloscope, and the behavior of the voltage in a circuit containing an inductor and a resistor, driven by a time-varying emf.

Circuits in homes, offices, and laboratories receive energy from a power company in the form of an oscillating electromotive force (emf). The resulting current in the circuits is called alternating current (AC), as opposed to direct current (DC). Typical AC varies sinusoidally, reversing direction 120 times per second (in the U.S.), thus having a frequency of 60 Hz. Time dependent emf's can also be produced in the laboratory by a signal generator. Signal generators have variable frequency and amplitude, and can typically output voltage as a square wave, saw-tooth wave, or sinusoidal wave.

If you wish to measure a sinusoidal emf with a multimeter, you expect the meter to return a single value, not one that varies at 60 Hz. Let the emf, ϵ , be given by $\epsilon = \epsilon_m \sin(\omega t)$. (The subscript m refers to maximum.) We don't want the multimeter to tell us the average value of ϵ with time, because that would be zero. Instead, the multimeter typically reports the root-mean-square, or rms, value of ϵ , $\epsilon_{rms} = [\frac{1}{T} \int_0^T \epsilon^2(t) dt]^{1/2}$,

$$\epsilon_{rms} = \frac{\epsilon_m}{\sqrt{2}}. \quad (1)$$

Multimeters in the AC mode typically report the rms voltage, V_{rms} , current, I_{rms} , etc. Because the proportionality factor $1/\sqrt{2}$ is the same for all variables, the rms values can be used for all terms in an equation such as $V = IR$, giving $V_{rms} = I_{rms}R$, where R is the resistance in the circuit.

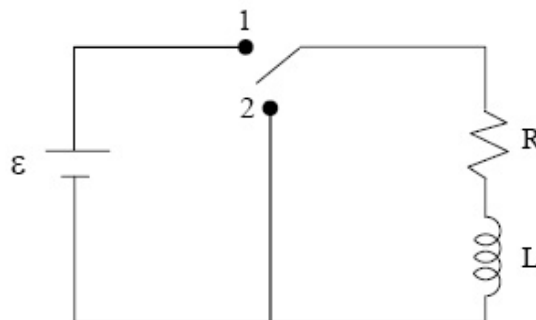


FIG. 1: A series RL circuit

Accompanying the current in any circuit is a magnetic field. If the current varies, so does the magnetic field. A time varying magnetic field in turn induces a potential difference, in a direction that opposes the change in the current in the circuit. This induced potential difference, often called the "back emf", exists only when the current in the circuit is changing its magnitude, and depends on the rate at which the current is changing. The induced potential difference, V_L , is described by $V_L = -L \frac{\Delta I}{\Delta t}$, where L is called the inductance. L depends on the physical properties of the circuit. Components called inductors are designed to enhance this induced voltage. They usually consist of a coil of wire, which would develop a large magnetic field in its interior. Introducing an inductor into a circuit alters the circuit's behavior.

If an inductor is placed in series with a potential source and a resistor as shown in Fig. 1 we have what is known as an RL circuit. If the switch is closed to position 1, the application of Kirchhoff's second law yields

$$\epsilon = IR + L \frac{\Delta I}{\Delta t}$$

where ϵ is the electromotive force of the source and I is the current at time t.

Solving this differential equation for I, we find $I = \frac{\epsilon}{R}(1 - e^{-Rt/L})$. The potential difference across the resistor, V_R , in an RL circuit is then $V_R = \epsilon(1 - e^{-Rt/L})$, and across the inductor is

$$V_L = \epsilon e^{-Rt/L}, \quad (2)$$

assuming the inductor's internal resistance is negligible.

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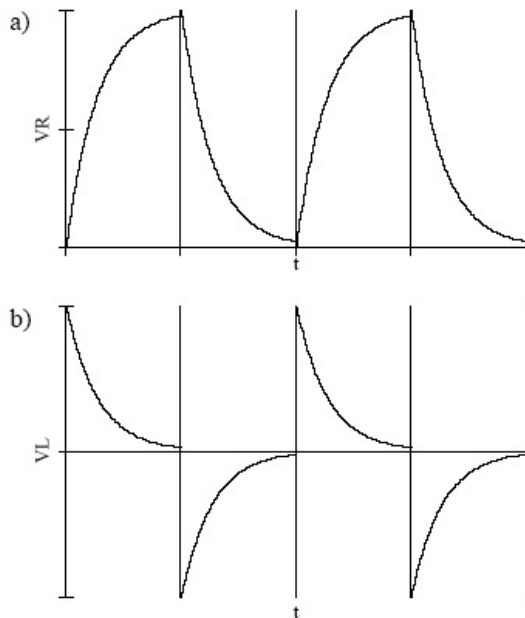


FIG. 2: The potential across the resistor, V_R , and the inductor, V_L , versus time in an RL circuit. The vertical lines indicate transitions from emf on/off.

Thus both V_L and V_R are time dependent, exponential functions. The value $\frac{L}{R}$ is called the time constant of the RL circuit, τ .

When the switch in Fig. 1 is opened, i.e. moved to position 2, the source is removed and the current decays. At a time t after the switch is opened, the potential difference across the inductor is $V_L = -\epsilon e^{-Rt/L}$, and the potential difference across the resistor is $V_R = \epsilon e^{-Rt/L}$. A plot of the potential differences across the inductor and resistor versus time for successive changes of the switch position are plotted in Fig. 2.

The time for V_L to fall to one half of its maximum value is called the half decay time, $t_{1/2}$, and is calculated by setting $V_L = \epsilon/2$ in Eq. 2, which yields

$$t_{1/2} = \frac{L}{R} \ln 2. \quad (3)$$

for an RC circuit. The same value of $t_{1/2}$ can be obtained from all four of the voltage equations for the RL circuit. $t_{1/2}$ is easier to measure experimentally than τ .

Any circuit containing inductors (and also capacitors) will exhibit time dependant behavior. If such a circuit is driven by a sinusoidal emf, the time dependant elements effect the phase of the current. The current in different elements in the circuit can be out of phase with one another. For a circuit with an applied emf of $\epsilon = \epsilon_m \sin(\omega t)$, we can use a phasor diagram representing the currents to describe the potential difference across each element. Refer to an Introductory Electricity and Magnetism text, for example Halliday, Resnick, and Walker, "Fundamen-

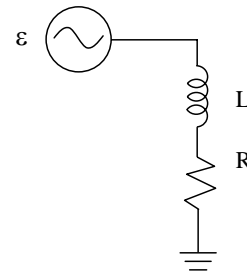


FIG. 3: RL circuit

tals of Physics", Wiley, New York, 2005. For the RL circuit in Fig. 3, we find

$$\epsilon^2 = (IR)^2 + (I\omega L)^2.$$

Either the set of maximum or rms values can be used for the time dependent variables in this equation, ϵ and I . (The value $\sqrt{R^2 + (\omega L)^2}$ is called the impedance of an RL circuit for driving frequency ω .) The above equation can be solved for L . This would be useful if a specialized meter measuring inductance were not available or practical for a particular application.

$$L = \frac{1}{\omega} \sqrt{\left(\frac{\epsilon}{I}\right)^2 - R^2} \quad (4)$$

It is often useful to compare two measured values, or a measured value and an accepted value. In order to compare two experimental values you use the percentage difference, given by $|x_{exp1} - x_{exp2}| / (|x_{exp1} + x_{exp2}|/2) \times 100\%$, where $x_{exp1,2}$ are the two experimentally measured values. In order to compare a measured value with an accepted value you use the percentage error, given by $|x_{exp} - x_{accept}| / x_{accept} \times 100\%$, where x_{accept} is the accepted value of x .

2. MEASUREMENT AND ANALYSIS

Equipment needed:

- Signal generator
- Oscilloscope
- two multimeters
- $\sim 1000 \Omega$ resistor, see below
- ~ 10 mH inductor
- assorted wires and connectors

In all of your measurements, take at least three independent measurements in order to find their average and standard deviation of the mean. The standard deviation of the mean should be used as the uncertainty in a measured value in error propagation, unless it is equal to

zero. In that case, the accuracy of the measurement device should be used as the uncertainty in the measured value.

Use the oscilloscope to display a sinusoidal potential, supplied by the signal generator. Use a frequency of approximately 3000 Hz, and a convenient amplitude. Measure the AC voltage with both multimeters. Compare the AC multimeter readings and $V_m/\sqrt{2}$, where V_m is measured using the oscilloscope. Do the readings agree within their errors? Are both multimeters equally accurate?

Next, view a square wave varying from $-\epsilon_m$ to $+\epsilon_m$. Measure the AC voltage with both multimeters, and compare the measurements with the V_{rms} measured using the oscilloscope. Do the readings agree within their errors? If not, are the multimeters reporting $V_m/\sqrt{2}$ instead?

Measure the inductance, L , and internal resistance, R_L , of your inductor. The actual resistance of the circuit will include both R and R_L . If R_L is not negligible compared to R , then the potential across the inductor will decay to a non-zero constant. Choose an appropriate resistor for your circuit, and measure its resistance. You will be using these components to make an RL circuit. Calculate the expected half decay time for the RL circuit. You will use the signal generator to supply a square wave potential difference. The constant positive amplitude of the square wave is equivalent to the switch in position 1 in Fig. 1, while the zero (or negative) amplitude of the square wave is equivalent to the switch in position 2. Choose a convenient amplitude and period of the square wave. (A period roughly three times $t_{1/2}$ should be adequate.) Use the oscilloscope to view the square wave before connecting the signal generator to the circuit.

Set up the RL circuit shown in Fig. 3. Use the oscilloscope to view the potential across the inductor, and the potential across the resistor. (The circuit element being measured should always be the last element before ground, so you'll need to change the circuit between measurements. The ground of the oscilloscope will effectively short-out any elements following it.) Sketch the observed potentials in your notebook, and sketch or describe them in your report. From the observed V_L , measure the half decay time using the oscilloscope. Compare the experimental value measured using the oscilloscope to the calculated value obtained from Eq. 3. Use error propagation to estimate the error in your calculated value. Do the values agree within their estimated errors? Discuss any sources of error in your lab report.

Now you will use measured values of current and voltage to determine the inductance. Apply a sinusoidal potential across the inductor only (i.e. remove the resistor from the circuit). Use a frequency, f , of approximately 3000 Hz and the maximum potential available from the signal generator. Using a multimeter, measure the potential across and the current in the inductor. After measuring the current, connect the oscilloscope across the inductor and measure the frequency. (The resistance of

the oscilloscope can affect the current.) Using the resistance of the inductor as R in Eq. 4, calculate the inductance of the circuit. Find the percentage error between the calculated value of L and the (accepted) value measured with the multimeter. Use error propagation to estimate the error in your calculated value. Do the values agree within their estimated errors? Discuss any sources of error in your lab report.