Homework for Matrix Manipulations

(MatrixPractice) This practice exercise helps students review the definitions of basic matrix manipulations. It can be used in conjunction with the handout:

Calculate the following quantities for the matrices:

\[ A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix} \quad C = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \]

and the vectors:

\[ |D\rangle = \begin{pmatrix} 1 \\ i \\ -1 \end{pmatrix} \quad |E\rangle = \begin{pmatrix} 1 \\ i \end{pmatrix} \quad |F\rangle = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \]

\[ AB \]
\[ \text{tr}(AB) \]
\[ A^\dagger \]
\[ C^{-1} \]
\[ A|D\rangle \]
\[ |E\rangle^\dagger = \langle E| \]
\[ \langle D|A|D\rangle \]
\[ \det(\lambda I - A) \text{ where } \lambda \text{ is a scalar.} \]
\[ (A|D\rangle)^\dagger \]

Using explicit matrix multiplication (without using a theorem) verify that \( (A|D\rangle)^\dagger = \langle D|A^\dagger \)

(PauliPractice) This practice problem introduces the Pauli Spin matrices. It is a useful early exercise before students begin the quantum content from the Spins paradigm:

The Pauli spin matrices \( \sigma_x, \sigma_y, \) and \( \sigma_z \) are defined by:
These matrices are related to angular momentum in quantum mechanics. Prove, and become familiar with, the identities listed below.

Show that each of the Pauli matrices is hermitian. (A matrix is hermitian if it is equal to its hermitian adjoint.

Show that the determinant of each of the Pauli matrices is $-1$.

Show that $\sigma_i^2 = I$ for each of the Pauli matrices, i.e. for $i \in \{x, y, z\}$.

(Pauli) A follow-up to the previous problem, this exercise introduces the concepts of commutators and anti-commutators

The Pauli spin matrices $\sigma_x, \sigma_y,$ and $\sigma_z$ are defined by:

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
\]

These matrices are related to angular momentum in quantum mechanics. Prove, and become familiar with, the identities listed below.

Show that $\sigma_x \sigma_y = i \sigma_z$ and $\sigma_y \sigma_x = -i \sigma_z$. (Note: These identities also hold under a cyclic permutation of $\{x, y, z\}$, e.g. $x \to y, y \to z, \text{and} z \to x$).

The commutator of two matrices $A$ and $B$ is defined by $[A, B] \overset{\text{def}}{=} AB - BA$. Show that $[\sigma_x, \sigma_y] = 2i \sigma_z$. (Note: This identity also holds under a cyclic permutation of $\{x, y, z\}$, e.g. $x \to y, y \to z, \text{and} z \to x$).

The anti-commutator of two matrices $A$ and $B$ is defined by $\{A, B\} \overset{\text{def}}{=} AB + BA$. Show that $\{\sigma_x, \sigma_y\} = 0$. (Note: This identity also holds under a cyclic permutation of $\{x, y, z\}$, e.g. $x \to y, y \to z, \text{and} z \to x$).

(DiagonalPractice) This easy exercise only requires that students know how to multiply matrices. It can be assigned before the concept of diagonalization is introduced and then used as an early lecture example.

Perform the following matrix multiplications:

\[
\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}, \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}
\]

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