Figure 1: The coordinate geometry for the problem

From the figure \( \vec{R} + \vec{r} = \vec{z} \) and

\[
 r^2 = |\vec{z} - \vec{R}|^2 = R^2 + z^2 - 2Rz \cos(\theta)
\]

Also;

\[
 \cos(\alpha) = \frac{\hat{z} \cdot \vec{r}}{r} = \frac{z - R \cos(\theta)}{|\vec{z} - \vec{R}|}
\]

We need only the \( z \) component of the electric field. Thus;

\[
 dE_z = \kappa \sigma R^2 \sin(\theta) \, d\theta d\phi \left[ \frac{z - R \cos(\theta)}{|\vec{z} - \vec{R}|} \right] \\
 dE_z = 2\pi \kappa \sigma R^2 z \sin(\theta) \, d\theta \left[ \frac{1 - (R/z) \cos(\theta)}{z^3 \left[ 1 + (R/z)^2 - 2(R/z) \cos(\theta) \right]^{3/2}} \right] d(\cos(\theta))
\]

Then change variables so that \( \eta = 1 + (R/z)^2 - 2(R/z) \cos(\theta) \) We can obtain that

\[
 1 - (R/z) \cos(\theta) = [\eta + 1 - (R/z)^2]/2
\]

\[
 d\eta = -2(R/z) \, d(\cos(\theta))
\]

The integral is then:
\[ E_z = \frac{2\pi \kappa \sigma (R/z)^2}{2(R/z)} \int_{(1-R/z)^2}^{(1+R/z)^2} d\eta \left[ \frac{1}{\eta^{1/2}} - \frac{1-(R/z)^2}{\eta^{3/2}} \right] \]

Now one must be careful to always choose \((R/z - 1) > 0\) for \(z < R\). Integration carefully choosing limits, the result is zero for \(z < R\)

2.10)

\[ \text{Flux out of all sides is equal} \]

![Figure 2: The coordinate geometry for the problem](image)

To get the flux out of one panel of the charge enclosed within a cube of sides \(2a\) we could evaluate the integral, see the figure;

\[ \text{Flux} = \int dx \int dz \, E_y = +\kappa Q \int_0^a dx \int_0^a dz \left[ \frac{a}{x^2 + a^2 + z^2} \right]^{3/2} \]

It is easier to note that the flux through each of the 24 panels is the same. The total flux out of the cube is;

\[ \text{Flux} = Q/\varepsilon_0 \]

The flux out of one panel is 1/24 of this.

2.17)

We find the \(\vec{E}\) field using Coulomb’s law. By symmetry, the only vector component that does not cancel lies along \(\hat{y}\). Thus we evaluate the integral

\[ E_y = 4\kappa \rho \int_{-\frac{d}{2}}^{\frac{d}{2}} dy \int_0^\infty dx \int_0^\infty dz \frac{1}{[x^2 + y^2 + z^2]^{3/2}} \]
The integral is probably easier to evaluate in cylindrical coordinates.

\[
E_y = \kappa \rho \int_{-d/2}^{d/2} dy \int_0^\infty r dr \int_0^{2\pi} d\phi \frac{\cos(\theta)}{r^2 + (y - y_0)^2}
\]

\[
E_y = 2\pi \kappa \rho \int_{-d/2}^{d/2} dy \int_0^\infty r dr \frac{(y - y_0)}{[r^2 + (y - y_0)^2]^{3/2}}
\]

\[
E_y = 2\rho d
\]

Here the thickness is defined as d rather than d/2 as in the problem.

![Figure 3: The coordinate geometry for the problem]

2.20)

The simplest way to show if the vector function can be obtained from a potential function is to see if the curl of the vector function vanishes. Given the statement of the problem here it appears that the expected solution uses a line integral around a closed path. Thus we could chose to integrate around a square loop in the \((x, y)\) plane so take \(z = 0\) and choose the 4 sides as;

1) Along the \(x\) axis, \(dx \hat{x}\) with \(y = 0\) and \(0 \leq x \leq 1\)

2) In the \(\hat{y}\) direction \(dy \hat{y}\) with \(x = 1\) and \(0 \leq y \leq 1\)

1) In the \(-\hat{x}\) direction \(dx - \hat{x}\) with \(y = 1\) and \(0 \leq x \leq 1\)

1) Along the \(y\) axis, \(dy \hat{y}\) with \(x = 0\) and \(0 \leq y \leq 1\)

Apply this for \(\int \vec{E} \cdot d\vec{l} = \int \vec{\nabla} \times \vec{E} \cdot d\vec{\sigma}\) to show the integral on the left hand side does not vanish for the function in a) but does for the function in b). However, as indicated above, it is easier to just look at \(\vec{\nabla} \times \vec{E}\). This does not vanish for the function in a) but does for the function in b). Thus \(\vec{E} = k[y^2 \hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}]\) can be obtained from a
potential. In this case;

\[ V = -(xy^2 + yz^2) \]

Then \( \vec{E} = -\vec{\nabla}V \)

Figure 4: The coordinate geometry for the problem