2.22)

We did this problem in class. Evaluate the integral;

\[ V = 2\kappa \int_{0}^{\infty} dx \frac{\lambda}{[x^2 + y^2]^{1/2}} = 2\kappa \lambda \ln(x + \sqrt{x^2 + y^2}) \bigg|_{0}^{\infty} \]

The issue here is understanding how to remove the \( \infty \), which after all is a constant. If integration is taken between \( \pm \infty \) then one needs to consider the limits obtained by expansion of the log terms and cancelling terms to first order.

\[ V = 2\kappa \lambda \ln(y) \]

Figure 1: The coordinate geometry for the problem

2.34)

a) \[ W = (\epsilon/2) \int E^2 d\tau \]

For a spherically symmetric shell, there are no fields on the inside of the charges on each shell. Thus;

\[ E = 0 \quad r < a \text{ and } r > b \]

\[ \vec{E} = \frac{\kappa q}{r^2} \hat{r} \quad a < r < b \]

\[ W = (\epsilon_0/2) \int d\tau \left[ \frac{\kappa q}{r^2} \right]^2 r^2 d\Omega \]

\[ W = (\epsilon_0/2)\kappa^2 q^2 4\pi \int_{a}^{b} dr E^2 / r^2 \]
\[ W = \frac{q^2}{8\pi\epsilon_0} [1/a - 1/b] \]

b) \[ W_1 = \epsilon/2 \int d\tau E_1^2 = \frac{q^2}{8\pi\epsilon_0} (1/a) \]

\[ W_2 = \epsilon/2 \int d\tau E_2^2 = \frac{q^2}{8\pi\epsilon_0} (1/b) \]

\[ A = \epsilon_0 \lim d\tau \vec{E_1} \cdot \vec{E_2} = -\frac{q^2}{4\pi\epsilon_0} (1/b) \]

\[ W_1 + W_2 + A = \frac{q^2}{8\pi\epsilon_0} [1/a - 1/b] \]

Figure 2: The coordinate geometry for the problem

2.35)

a) \[ \sigma = \frac{q}{4\pi R^2} \quad r = R \]

\[ \sigma = -\frac{q}{4\pi a^2} \quad r = a \]

\[ \sigma = \frac{q}{4\pi b^2} \quad r = b \]

b) \[ \vec{E} = 0 \quad 0 < r < R \]

\[ \vec{E} = \kappa \frac{q\hat{r}}{r^2} \quad R < r < a \]

\[ \vec{E} = 0 \quad a < r < b \]

\[ \vec{E} = \kappa \frac{q\hat{r}}{r^2} \quad b < r < \infty \]

\[ V = \kappa q \int \frac{a}{R} dr (1/r^2) = \kappa q [1/R - 1/a] \]
\[ V = \kappa q \int_b^\infty dr (1/r^2) = \kappa q [1/b] \]

\[ V_c = \kappa q [1/R - (1/a - 1/b)] \]

c) Change the zero of the potential. The charge density remains the same. The potential is:

\[ V_c = \kappa q [1/R - 1/a] \]

Figure 3: The coordinate geometry for the problem

2.38)

This is the same as the problem on the 1st exam. The surface charge creates a field only on the outside of the spherical surface. Thus the average field is \((1/2)\epsilon_0 \sigma\). The force/area (from the text) is

\[ \mathcal{F} = (1/2\epsilon_0) \sigma^2 \hat{n} \]

Integrate the z direction of the force.

\[ F = (\sigma^2/2\epsilon) \int d\Omega R^2 \cos(\theta) \]

\[ F = 2\pi \sigma^2 \epsilon_0 R^2 \]