2.46)

The potential:

\[ V = A e^{-\lambda r}/r \]

then:

\[ \vec{E} = -\vec{\nabla} V = A[\lambda r + 1]e^{-\lambda r}/r^2 \hat{r} \]

The charge density is then:

\[ \vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0 \]

\[ \rho = \epsilon_0 \vec{\nabla} \cdot \vec{E} = \epsilon_0[-\lambda^2 e^{-\lambda r}] \]

Then integrate over a small volume and let \( r \to 0 \).

\[ Q = \int d\tau \rho = \epsilon_0 \int \vec{E} \cdot \vec{\sigma} \]

\[ Q = 4\pi \epsilon_0 A[r \lambda e^{-\lambda r} + e^{-\lambda r}] \to 4\pi \epsilon_0 A \]

This indicated that the charge density contains a delta function \( 4\pi \epsilon_0 \delta(r) \)

2.47)

a) The field and potential are independent of \( z \). The field on the wire comes from Gauss’ law:

\[ E_1 = \frac{\lambda}{2\pi \epsilon_0 \bar{r}_1} \]

\[ \bar{r}_1 = \bar{a} - \bar{R} \]
\[ E_2 = \frac{\lambda}{2\pi\epsilon_0 r_2} \]
\[ \vec{r}_2 = \vec{a} + \vec{R} \]

The potential is obtained by superposition.

\[ V = V_1 + V_2 = \frac{\lambda}{2\pi\epsilon_0} \ln(r_1/r_2) \]

b) This was done in class and shown in Lecture 5

3.1)

Use Gauss’ law to get the average field and then the potential for the charge inside the shell. Add to this the average field due to the charge outside the shell using equations in the text. This gives the expected answer.

3.4)

Use Poisson’s equation. Suppose 2 potentials with the same charge density.

\[ \nabla^2 V_1 = \rho/\epsilon \]
\[ \nabla^2 V_2 = \rho/\epsilon \]

Subtract these equations and integrate over volume.

\[ \int d\tau \ \vec{\nabla} \cdot (\vec{\nabla} V_1 - \vec{\nabla} V_2) = 0 \]

Use Gauss’ theorem changing the volume integral to a surface integral enclosing the volume.

\[ \int d\vec{\sigma} \cdot (\vec{\nabla} V_1 - \vec{\nabla} V_2) = 0 \]

Then the term in brackets is evaluated on the surface and must vanish. The means that \( \vec{\nabla} V_1 = \vec{\nabla} V_2 \), and \( V_1 = V_2 \). The same proof is applied to show that the fields on the surface \( \vec{E} = -\vec{\nabla} V \) must also be equal.