Chapter 3
Vectors in Physics
Units of Chapter 3

• Scalars Versus Vectors
• The Components of a Vector
• Adding and Subtracting Vectors
• Unit Vectors
• Position, Displacement, Velocity, and Acceleration Vectors
• Relative Motion
3-1 Scalars Versus Vectors

Scalar: number with units

Vector: quantity with magnitude and direction

How to get to the library: need to know how far and which way
3-2 The Components of a Vector

Even though you know how far and in which direction the library is, you may not be able to walk there in a straight line:
3-2 The Components of a Vector

Can resolve vector into perpendicular components using a two-dimensional coordinate system:

(a) $r = 1.50 \text{ m}$, $\theta = 25.0^\circ$

(b) $r_y = 0.634 \text{ m}$, $r_x = 1.36 \text{ m}$
3-2 The Components of a Vector

Length, angle, and components can be calculated from each other using trigonometry:
3-2 The Components of a Vector

Signs of vector components:

(a) $A_x > 0$, $A_y < 0$
(b) $A_x < 0$, $A_y < 0$
(c) $A_x < 0$, $A_y > 0$
(d) $A_x > 0$, $A_y > 0$
3-3 Adding and Subtracting Vectors

Adding vectors graphically: Place the tail of the second at the head of the first. The sum points from the tail of the first to the head of the last.
3-3 Adding and Subtracting Vectors

Adding Vectors Using Components:

1. Find the components of each vector to be added.
2. Add the $x$- and $y$-components separately.
3. Find the resultant vector.
3-3 Adding and Subtracting Vectors

(a) 
\[ \vec{A} + \vec{B} = \vec{C} \]

(b) 
\[ \vec{C} = \vec{A} + \vec{B} \]

\[ C_x = A_x + B_x \]

\[ C_y = A_y + B_y \]
3-3 Adding and Subtracting Vectors

Subtracting Vectors: The negative of a vector is a vector of the same magnitude pointing in the opposite direction. Here, \( \vec{D} = \vec{A} - \vec{B} \)
Example: Vector $\mathbf{A}$ has a length of 5.00 meters and points along the $x$-axis. Vector $\mathbf{B}$ has a length of 3.00 meters and points $120^\circ$ from the $+x$-axis. Compute $\mathbf{A}+\mathbf{B} (=\mathbf{C})$. 
Example continued:

\[
\sin \theta = \frac{\text{opp}}{\text{hyp}} \\
\cos \theta = \frac{\text{adj}}{\text{hyp}} \\
\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\text{opp}}{\text{adj}}
\]

\[
\sin 60^{\circ} = \frac{B_y}{B} \Rightarrow B_y = B \sin 60^{\circ} = (3.00 \text{ m}) \sin 60^{\circ} = 2.60 \text{ m}
\]

\[
\cos 60^{\circ} = \frac{-B_x}{B} \Rightarrow B_x = -B \cos 60^{\circ} = -(3.00 \text{ m}) \cos 60^{\circ} = -1.50 \text{ m}
\]

and \(A_x = 5.00 \text{ m}\) and \(A_y = 0.00 \text{ m}\)
Example continued:

The components of \( \mathbf{C} \): 

\[
\begin{align*}
C_x &= A_x + B_x = 5.00 \text{ m} + (-1.50 \text{ m}) = 3.50 \text{ m} \\
C_y &= A_y + B_y = 0.00 \text{ m} + 2.60 \text{ m} = 2.60 \text{ m}
\end{align*}
\]

The length of \( \mathbf{C} \) is:

\[
C = |\mathbf{C}| = \sqrt{C_x^2 + C_y^2} = \sqrt{(3.50 \text{ m})^2 + (2.60 \text{ m})^2} = 4.36 \text{ m}
\]

The direction of \( \mathbf{C} \) is:

\[
\tan \theta = \frac{C_y}{C_x} = \frac{2.60 \text{ m}}{3.50 \text{ m}} = 0.7429
\]

\[
\theta = \tan^{-1}(0.7429) = 36.6^\circ \quad \text{From the } +x\text{-axis}
\]
Example: Margaret walks to the store using the following path: 0.500 miles west, 0.200 miles north, 0.300 miles east. What is her total displacement? Give the magnitude and direction.

Take north to be in the +y direction and east to be along +x.
Example continued:

The displacement is $\Delta \mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$. The initial position is the origin; what is $\mathbf{r}_f$?

The final position will be $\mathbf{r}_f = \mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3$. The components are $r_{fx} = -r_1 + r_3 = -0.2$ miles and $r_{fy} = +r_2 = +0.2$ miles.

Using the figure, the magnitude and direction of the displacement are

$$|\Delta \mathbf{r}| = \sqrt{\Delta r_x^2 + \Delta r_y^2} = 0.283\text{ miles}$$

$$\tan \theta = \frac{|\Delta r_y|}{|\Delta r_x|} = 1 \text{ and } \theta = 45^\circ \text{ N of W.}$$
3-4 Unit Vectors

Unit vectors are dimensionless vectors of unit length.
3-4 Unit Vectors

Multiplying unit vectors by scalars: the multiplier changes the length, and the sign indicates the direction.
3-5 Position, Displacement, Velocity, and Acceleration Vectors

Position vector $\vec{r}_f$ points from the origin to the location in question.

The displacement vector $\Delta \vec{r}$ points from the original position to the final position.
3-5 Position, Displacement, Velocity, and Acceleration Vectors

Average velocity vector:

\[ \vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} \]  \hspace{1cm} (3-3)

So \( \vec{v}_{av} \) is in the same direction as \( \Delta \vec{r} \).
3-5 Position, Displacement, Velocity, and Acceleration Vectors

Instantaneous velocity vector is tangent to the path:
3-5 Position, Displacement, Velocity, and Acceleration Vectors

Average acceleration vector is in the direction of the change in velocity:

\[ \mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t} \]
3-5 Position, Displacement, Velocity, and Acceleration Vectors

Velocity vector is always in the direction of motion; acceleration vector can point anywhere:
Example (text problem 3.42): At the beginning of a 3 hour plane trip you are traveling due north at 192 km/hour. At the end, you are traveling 240 km/hour at 45° west of north.

(a) Draw the initial and final velocity vectors.
Example continued:

(b) Find $\Delta \mathbf{v}$.

The components are

$$\Delta v_x = v_{fx} - v_{ix} = -v_f \sin 45^\circ - 0 = -170 \text{ km/hr}$$

$$\Delta v_y = v_{fy} - v_{iy} = +v_f \cos 45^\circ - v_i = -22.3 \text{ km/hr}$$

The magnitude and direction are:

$$|\Delta \mathbf{v}| = \sqrt{\Delta v_x^2 + \Delta v_y^2} = 171 \text{ km/hr}$$

$$\tan \varphi = \frac{|\Delta v_y|}{|\Delta v_x|} = 0.1312 \Rightarrow \varphi = \tan^{-1}(0.1312) = 7.5^\circ$$

South of west
Example continued:

(c) What is $a_{av}$ during the trip?

\[
\begin{align*}
\mathbf{a}_{av} &= \frac{\Delta \mathbf{v}}{\Delta t} \\
\mathbf{a}_{x,av} &= \frac{\Delta v_x}{\Delta t} = \frac{-170 \text{ km/hr}}{3 \text{ hr}} = -56.7 \text{ km/hr}^2 \\
\mathbf{a}_{y,av} &= \frac{\Delta v_y}{\Delta t} = \frac{-22.3 \text{ km/hr}}{3 \text{ hr}} = -7.43 \text{ km/hr}^2
\end{align*}
\]

The magnitude and direction are:

\[
|\mathbf{a}_{av}| = \sqrt{a_{x,av}^2 + a_{y,av}^2} = 57.2 \text{ km/hr}^2
\]

\[
\tan \phi = \frac{|a_{y,av}|}{|a_{x,av}|} = 0.1310 \Rightarrow \phi = \tan^{-1}(0.1310) = 7.5^\circ \quad \text{South of west}
\]
3-6 Relative Motion

The speed of the passenger with respect to the ground depends on the relative directions of the passenger’s and train’s speeds:

(a)

(b)
3-6 Relative Motion

\[ \vec{v}_{pg} = \vec{v}_{pt} + \vec{v}_{tg} \]

This also works in two dimensions:
Example: You are traveling in a car (A) at 60 miles/hour east on a long straight road. The car (B) next to you is traveling at 65 miles/hour east. What is the speed of car B relative to car A?
Example continued:

\[
\begin{align*}
\text{t=0} & \quad \text{t>0} \\
\Delta r_{AG} & \quad \Delta r_{BA} \\
\Delta r_{BG} & \quad \end{align*}
\]

From the picture:

\[
\begin{align*}
\Delta r_{BG} &= \Delta r_{AG} + \Delta r_{BA} \\
\Delta r_{BA} &= \Delta r_{BG} - \Delta r_{AG} \\
\text{Divide by } \Delta t: \quad v_{BA} &= v_{BG} - v_{AG} \\
v_{BA} &= 65 \text{ miles/hr east} - 60 \text{ miles/hr east} \\
&= 5 \text{ miles/hour east}
\end{align*}
\]
Example: You are traveling in a car (A) at 60 miles/hour east on a long straight road. The car (B) next to you is traveling at 65 miles/hour west. What is the speed of car B relative to car A?
Example continued:

\[
\begin{align*}
\Delta r_{BA} &= \Delta r_{BG} - \Delta r_{AG} \\
\text{Divide by } \Delta t: &\quad v_{BA} = v_{BG} - v_{AG} \\
&= 65 \text{ miles/hr west} - 60 \text{ miles/hr east} \\
&= 125 \text{ miles/hr west}
\end{align*}
\]
Summary of Chapter 3

- Scalar: number, with appropriate units
- Vector: quantity with magnitude and direction
- Vector components: $A_x = A \cos \theta$, $B_y = B \sin \theta$
- Magnitude: $A = (A_x^2 + A_y^2)^{1/2}$
- Direction: $\theta = \tan^{-1} (A_y / A_x)$
- Graphical vector addition: Place tail of second at head of first; sum points from tail of first to head of last
Summary of Chapter 3

- Component method: add components of individual vectors, then find magnitude and direction
- Unit vectors are dimensionless and of unit length
- Position vector points from origin to location
- Displacement vector points from original position to final position
- Velocity vector points in direction of motion
- Acceleration vector points in direction of change of motion
- Relative motion: \( \vec{v}_{13} = \vec{v}_{12} + \vec{v}_{23} \)